Unit 1: Transistor Fundamentals

Quick Review of Semiconductor Fundamentals

Mark Lundstrom

Electrical and Computer Engineering
Birck Nanotechnology Center
Purdue University, West Lafayette, Indiana USA

lundstro@purdue.edu
Essentials of semiconductor physics

1) Energy bands
2) Doping
3) Fermi function and Fermi level
4) Carrier densities
5) Drift-diffusion equation
6) Energy band diagrams
7) Quasi-Fermi levels
Silicon energy levels

Si atom (At. no. 14)

4S^0
3P^2
3S^2
2P^6
2S^2
1S^2

4 valence electrons
8 valence states

“core levels”

Lundstrom: Nanotransistors 2015
Silicon energy levels / energy bands

- Only the valence states are of interest to us.
- The 8 valence states give rise to $8N_{\text{atoms}}$ states per cm$^3$ in the solid.
- But the interaction of the electron wavefunctions alters the discrete energy levels of the isolated Si atoms.

$N_{\text{atoms}} \approx 5 \times 10^{22}$ cm$^{-3}$

4 nearest neighbors
Silicon energy levels → energy bands

Si atom (At. no. 14)  Si crystal

3P^2

3S^2

energy

$T = 0 \text{ K}$

“forbidden gap”

conduction “band”

valence “band”

$4N_{\text{atoms}}$ states
Silicon energy levels $\rightarrow$ energy bands

Si atom (At. no. 14) $\rightarrow$ Si crystal

- $3P^2$ levels
- $3S^2$ levels

Energy levels at $T = 300$ K:

$$\langle E \rangle = \frac{3}{2} k_B T = 0.026 \text{ eV}$$

Forbidden gap:

$4N_{\text{atoms}}$ states

Conduction "band":

$4N_{\text{atoms}}$ states

Valence "band":

$4N_{\text{atoms}}$ states

Lundström: Nanotransistors 2015
Intrinsic Si

\[ E_C \quad n = n_i \approx 10^{10} \text{ cm}^{-3} \]

\[ E_G = 1.1 \text{ eV} \]

\[ E_V \quad p = n_i \approx 10^{10} \text{ cm}^{-3} \]

\[ k_B T = 0.026 \text{ eV} \quad (T = 300 \text{ K}) \]

\[ P \sim e^{-E_G/k_B T} \]

\[ n = p = n_i \]
Intrinsic silicon

Si crystal structure

4 nearest neighbors

\[ n = p = n_i \propto e^{-E_G/k_BT} \]
Metals, insulators and semiconductors

**Insulators:** don’t conduct electricity well
usually don’t conduct heat well

**Metals:** conduct electricity (and heat) well.

**Semiconductors:** in-between, **but**
their properties can be controlled
insulators              metals       semiconductors

empty states

$E_C$

$E_G \approx 9 \text{ eV (SiO}_2\text{)}$

$k_B T \approx 0.026 \text{ eV (300K)}$

$E_V$

filled states

$E_{TOP}$

empty states

filled states

$Lundstrom: \text{Nanotransistors 2015}$

$E_C$

$E_G \approx 1.1 \text{ eV (Si)}$

$E_V$

filled states

$Lundstrom: \text{Nanotransistors 2015}$
Covalent (column IV) semiconductors

[Image of the periodic table with annotations]

- **Col. III dopant**: Elements in column III
- **Col. V dopant**: Elements in column V

Col. IV:
- B (11th element)
- C (6th element)
- N (7th element)
- O (8th element)
- F (9th element)
- Ne (10th element)
- Al (13th element)
- Si (14th element)
- P (15th element)
- S (16th element)
- Cl (17th element)
- Ar (18th element)

Col. III dopants: Li, Be, Na, K, Rb, Cs

http://en.wikipedia.org/wiki/Periodic_table
Doping

Lundstrom: Nanotransistors 2015
N-type doping (donors)

\[ E_H = -\frac{13.6}{n^2} \text{eV} \quad n = 1, 2, 3, \ldots \]

\[ E_D = -\frac{m_0 q^4}{2\left(4\pi \varepsilon_0 \hbar n\right)^2} \text{eV} \]

\[ E_D < 0.05 \text{eV} \]

Weakly bound

Easily broken at room temperature
Ionized donors

\[ n = N_D^+ \approx N_D \]
P-type doping (acceptors)
Ionized acceptors

\[ p = N_A^- \approx N_A \]
III-V semiconductors

http://en.wikipedia.org/wiki/Periodic_table
Filling states

What is the probability that states in the conduction band are filled?

\[ E_C \quad n \quad \text{cm}^{-3} \]

\[ E_G = 1.1 \text{eV} \]

What is the probability that states in the valence band are empty?

\[ E_V \quad p \quad \text{cm}^{-3} \]
Occupation of states

Si atom (At. no. 14)

States way above have very little probability of being occupied.

States below this energy have a high probability of being occupied.

Lundstrom: Nanotransistors 2015
Fermi level / Fermi function

\[ f_0(E) = \frac{1}{1 + e^{(E - E_F)/k_BT}} \]

*(Fermi function)*

- Small probability of being filled
- Even smaller probability of being empty

\(E_C\), \(E_V\), \(E_F\), Fermi level or electrochemical potential
Carrier densities and density-of-states

\[ n_0(E) dE = D_C(E) dE \, f_0(E) \]

\[ n_0 = \int_{E_C}^{E_{C\text{top}}} n_0(E) dE = \int_{E_C}^{E_{C\text{top}}} D_C(E) f_0(E, E_F) dE \]

\[ p_0 = \int_{E_{V\text{bot}}}^{E_V} p_0(E) dE = \int_{E_{V\text{bot}}}^{E_V} D_V(E) \left[ 1 - f_0(E, E_F) \right] dE \]

\[ p_0(E) dE = D_V(E) dE \left[ 1 - f_0(E) \right] \]
Carrier densities and Fermi level

\begin{align*}
n_0 &= N_C \mathcal{F}_{1/2} \left\{ \left( E_F - E_C \right) / k_B T \right\} \text{cm}^{-3} \\
p_0 &= N_V \mathcal{F}_{1/2} \left\{ \left( E_V - E_F \right) / k_B T \right\} \text{cm}^{-3}
\end{align*}

\( N_C, N_V: \) “effective DOS”

Non-degenerate semiconductors

\[ n_0 = N_C \mathcal{F}_{1/2} \left\{ \left( E_F - E_C \right) / k_B T \right\} \text{cm}^{-3} \]

\[ E_C \]

\[ E_F < E_C, \quad E_F > E_V \]

\[ E_V \]

\[ p_0 = N_V \mathcal{F}_{1/2} \left\{ \left( E_V - E_F \right) / k_B T \right\} \text{cm}^{-3} \]

\[ n_0 = N_C \exp \left\{ \left( E_F - E_C \right) / k_B T \right\} \text{cm}^{-3} \]

\[ p_0 = N_V \exp \left\{ \left( E_V - E_F \right) / k_B T \right\} \text{cm}^{-3} \]

\[ n_0 p_0 = n_t^2 = N_C N_V e^{-E_G / k_B T} \]
N-type semiconductor

\[ E_C \]

\[ n_0 \gg n_i, p_0 \text{ cm}^{-3} \]

\[ E = E_F \]

\[ E_V \]

\[ p_0 \ll n_i \text{ cm}^{-3} \]

\[ n_0 = N_D \text{ cm}^{-3} \]

\[ n_0 = N_C \exp\left\{ \frac{(E_F - E_C)}{k_B T} \right\} \text{ cm}^{-3} \]

\[ p_0 = N_V \exp\left\{ \frac{(E_V - E_F)}{k_B T} \right\} \text{ cm}^{-3} \]

\[ n_0 p_0 = n_i^2 \]

\[ p_0 = \frac{n_i^2}{n_0} \]
P-type semiconductor

\[ n_0 \ll n_i \text{ cm}^{-3} \]

\[ E = E_F \]

\[ p_0 > n_i, n_0 \text{ cm}^{-3} \]

\[ p_0 = N_A \text{ cm}^{-3} \]

\[ p_0 = N_V \exp \left\{ \left( \frac{E_V - E_F}{k_B T} \right) \right\} \text{ cm}^{-3} \]

\[ n_0 = N_C \exp \left\{ \left( \frac{E_F - E_C}{k_B T} \right) \right\} \text{ cm}^{-3} \]

\[ n_0 p_0 = n_i^2 \]

\[ n_0 = \frac{n_i^2}{p_0} \]
Intrinsic semiconductor

\[ n_0 = n_i \text{ cm}^{-3} \]

\[ E_C \]

\[ E_F = E_i \]

\[ p_0 = n_0 = n_i \text{ cm}^{-3} \]

\[ E_V \]

\[ p_0 = n_i \text{ cm}^{-3} \]
Essentials of semiconductor physics

1) Energy bands
2) Doping
3) Fermi function and Fermi level
4) Carrier densities

5) Drift-diffusion equation
6) Energy band diagrams
7) Quasi-Fermi levels
Classical particle: electron in free space

\[ F(t) \quad m_0 \quad \nu(t) \quad x(t) \quad F = m_0 a \]

\[ E = \frac{1}{2} m_0 \nu^2 = \frac{p^2}{2m_0} \]

\[ p = m_0 \nu \]
Energy and “crystal momentum”

\[ E = E_C + \frac{p^2}{2m_n^*} \]

“band structure”

\[ \psi (\vec{r}) = u_k (\vec{r}) e^{i\vec{k} \cdot \vec{r}} \]

Electron energy

\[ E = E_C \]

Hole energy

\[ E = E_V - \frac{p^2}{2m_p^*} \]

\[ p = \hbar k \]

\[ k = \frac{2\pi}{\lambda} \]

Lundstrom: Nanotransistors 2015
Drift current

\[ F(t) \rightarrow m_n^* \rightarrow F = m_0 a \]

\[ \nu(t) \times (t) \]

\[ E \rightarrow F_e = -qE \]

\[ \nu_{dn} = -\mu_n E \quad \mu_n = \left( \frac{q \tau}{m_n^*} \right) \text{cm}^2/\text{V-s} \]

\[ J_n = -nq\nu_{dn} = +nq\mu_n E \text{A/cm}^2 \]

Drift current
Thermal velocity

\[ \langle KE \rangle = \frac{3}{2} k_B T \]

\[ \langle KE \rangle = \frac{1}{2} m_n^* \langle v^2 \rangle \]

\[ \sqrt{\langle v^2 \rangle} = v_{rms} = \sqrt{\frac{3k_B T}{m_n^*}} \]

\[ v_{rms} \approx 10^7 \text{ cm/s} \]
Diffusion current

\[ F_e = -D \frac{dp}{dx} \text{ #/cm}^2\text{-s} \]

\( D \text{ cm}^2/\text{s} \)

(diffusion coefficient)

(Adolph Fick, 1855)

\[ J_n = -q F_e = +qD \frac{dn}{dx} \text{ A/cm}^2 \]

diffusion current
Drift-diffusion equation

\[ J_{n,\text{drift}} = -n q v_{dn} = +n q \mu_n \mathcal{E} \text{ A/cm}^2 \]

drift current

\[ J_{n,\text{diff}} = -q \mathcal{F}_e = +q D_n \frac{dn}{dx} \text{ A/cm}^2 \]

diffusion current

\[ J_n = J_{n,\text{drift}} + J_{n,\text{diff}} = n q \mu_n \mathcal{E} + q D_n \frac{dn}{dx} \]

\[ \frac{D_n}{\mu_n} = \frac{k_B T}{q} \]

(Einstein relation)
Energy bands can bend

But the Fermi level is constant (in equilibrium)!

$Lundstrom: Nanotransistors 2015$
Voltage and electron potential energy

\[ E = -qV \]

\[ +V \]
Example: electrostatic potential vs. position

\[ V(x) \]

+\( V_G \)

\[ V = 0 \]

Lundstrom: Nanotransistors 2015
Bandbending in semiconductors

\[ E_C(0) = E_C(+\infty) - qV(0) \]

\[ E_C(x) = E_C(+\infty) - qV(x) \]

\[ \frac{dE_C(x)}{dx} = -q \frac{dV(x)}{dx} = qE \]

\[ V = 0 \]
Quasi-Fermi levels

\[ E_C \quad n_0 = N_D \quad \text{cm}^{-3} \]

\[ V = 0 \quad E = E_F \quad V > 0 \]

\[ E_V \quad p_0 \ll n_i \quad \text{cm}^{-3} \]

\[ \rightarrow x \]
Quasi-Fermi levels

\[ V = 0 \]

\[ E_C \]

\[ F_n(x) \]

\[ n = N_D \text{ cm}^{-3} \]

\[ p \ll n_i \text{ cm}^{-3} \]

\[ E_V \]

\[ V > 0 \]

\[ x \]
Out of equilibrium, the Fermi level must be replaced by two QFL’s – one for electrons and one for holes.

\[ n_0 = N_C \exp \left\{ \frac{(E_F - E_C)}{k_B T} \right\} \rightarrow n = N_C \exp \left\{ \frac{(F_n - E_C)}{k_B T} \right\} \text{ cm}^{-3} \]

\[ p_0 = N_V \exp \left\{ \frac{(E_V - E_F)}{k_B T} \right\} \rightarrow p = N_V \exp \left\{ \frac{(E_V - F_p)}{k_B T} \right\} \text{ cm}^{-3} \]
Current and QFL’s

The Fermi level is constant in equilibrium.

The quasi-Fermi levels can be position dependent.

\[ J_n = n q \mu_n E + q D_n \frac{dn}{dx} \quad J_n = n \mu_n \frac{dF_n}{dx} \]
Essentials of semiconductor physics

1) Energy bands
2) Doping
3) Fermi function and Fermi level
4) Carrier densities
5) Drift-diffusion equation
6) Energy band diagrams
7) Quasi-Fermi levels
References: Basic semiconductor physics


ECE 305: Semiconductor Devices: https://nanohub.org/groups/ece305lundstrom

ECE 606: Solid State Devices https://nanohub.org/groups/ece606lundstrom