Linear and saturation currents

Can we derive an expression that goes smoothly from the linear to saturation region?

\[ I_{DSAT} = \left( \frac{T_0}{2 - T_0} \right) W |Q_n(V_{GS}, V_{DS})| v_T \]

\[ I_{DLIN} = T_0 W |Q_n(V_{GS}, V_{DS})| \frac{v_T}{2k_B T / q} V_{DS} \]
Transmission theory of the MOSFET

\[ I_{DS} = \frac{2q}{h} \int \mathcal{T}(E) M(E) (f_S - f_D) \, dE \]

\[ \mathcal{T}(E) = \mathcal{T}_0 \]

\[ I_{DS} = \mathcal{T}_0 \frac{2q}{h} \int M(E) (f_S - f_D) \, dE \]
\[ I_{DS} = \mathcal{T}_0 \frac{2q}{h} \int M(E)(f_S - f_D) dE \]

\[ I_{DS} = \mathcal{T}_0 \frac{2q}{h} \int WM_{2D}(E)(f_S - f_D) dE \quad M_{2D}(E) = g_v \frac{\sqrt{2m^*(E - E_C)}}{\pi \hbar} \]

\[ f_S(E) = \frac{1}{1 + e^{(E-E_{FS})/k_B T}} \quad f_D(E) = \frac{1}{1 + e^{(E-E_{FD})/k_B T}} \quad E_{FD} = E_{FS} - qV \]

**Result for MB statistics:**

\[ I_{DS} = \mathcal{T}_0 W \frac{q}{h} \left( g_v \frac{\sqrt{2\pi m^* k_B T}}{\pi \hbar} \right) k_B T \ e^{n_{FS}} \left( 1 - e^{-qV_{DS}/k_B T} \right) \]
Relating current to charge

\[ I_{DS} = T_0 W \frac{q}{\hbar} \left( \frac{g_V \sqrt{2 \pi m^* k_B T}}{\pi \hbar} \right) k_B T e^{n_{FS}} \left( 1 - e^{-qV_{DS}/k_B T} \right) \]

The result is just transmission times the ballistic result, but we want to relate the current to the charge at the VS.

In the ballistic case for small \( V_{DS} \):

\[ Q_n(x = 0) = q n_S(x = 0) = qN_{2D} e^{\eta_{FS}} = qN_{2D} e^{(E_{FS} - E_C(0))/k_B T} \quad (E_{FD} \approx E_{FS}) \]

In the ballistic case for large \( V_{DS} \):

\[ Q_n(x = 0) = q n_S(x = 0) = \frac{qN_{2D}}{2} e^{\eta_{FS}} = \frac{qN_{2D}}{2} e^{(E_{FS} - E_C(0))/k_B T} \quad (E_{FD} \ll E_{FS}) \]
Charge and transmission (i)

\[
\begin{align*}
E & \quad I^+(0) \quad \mathcal{T}I^-(L) \quad I^-(L) \\
(1-\mathcal{T})I^+(0) & \quad Q_n(0) \quad E_{FD}
\end{align*}
\]
(1) Charge injected from the source

\[ Q_{n}^{(1)}(x = 0) = q \frac{N_{2D}}{2} e^{(E_{FS} - E_{C}(0))/k_BT} \]

(2) Charge injected from the source that backscatters

\[ Q_{n}^{(2)}(x = 0) = (1 - \mathcal{R}_0) q \frac{N_{2D}}{2} e^{(E_{FS} - E_{C}(0))/k_BT} \]

(3) Charge injected from the drain that transmits to the VS

\[ Q_{n}^{(3)}(x = 0) = \mathcal{R}_0 q \frac{N_{2D}}{2} e^{(E_{FD} - E_{C}(0))/k_BT} \]
Charge-based current expression

\[ I_{DS} = T_0 W \frac{q}{h} \left( \frac{g_v \sqrt{2 \pi m^* k_B T}}{\pi \hbar} \right) k_B T \ e^{\eta_{FS}} \left( 1 - e^{-q V_{DS}/k_B T} \right) \]

\[ Q_n(x = 0) = -q \frac{N_{2D}}{2} \left[ (2 - T_0) e^{(E_{FS} - E_C(0))/k_B T} + T_0 e^{(E_{FD} - E_C(0))/k_B T} \right] \quad N_{2D} = g_v \frac{m_n^*}{\pi \hbar^2} k_B T \]

\[ I_{DS} = T_0 W \left| \frac{Q_n}{Q_n} \right| \frac{q}{h} \left( \frac{g_v \sqrt{2 \pi m^* k_B T}}{\pi \hbar} \right) k_B T \ e^{\eta_{FS}} \left( 1 - e^{-q V_{DS}/k_B T} \right) \]

Refer to Lecture Notes, Lecture 17.6 for more discussion.
Current for arbitrary drain voltage

\[ I_{DS} = W \left| Q_n(V_{GS}, V_{DS}) \right| \left[ \frac{1 - e^{-qV_{DS}/k_BT}}{1 + \left( \frac{\tau_0}{2 - \tau_0} \right)e^{-qV_{DS}/k_BT}} \right] \nu_{inj} \]

\[ \nu_{inj} = \left( \frac{\tau_0}{2 - \tau_0} \right) \nu_{inj}^{ball} \]

\[ \nu_{inj}^{ball} = v_T = \sqrt{\frac{2k_BT}{\pi m_n^*}} \]

Missing one thing:

\( \tau(V_{DS}) \)
Check the ballistic limit

\[ I_{DS} = W Q_n(V_{GS}, V_{DS}) \left[ \frac{1 - e^{-qV_{DS}/k_BT}}{1 + \left( \frac{T_0}{2 - T_0} \right) e^{-qV_{DS}/k_BT}} \right] v_{inj} \]

\[ v_{inj} = \left( \frac{T_0}{2 - T_0} \right) v_{inj}^{ball} \]

\[ v_{inj}^{ball} = v_T = \sqrt{\frac{2k_BT}{\pi m_n}} \]

\[ T_0 = 1 \]

\[ v_{inj} = v_{inj}^{ball} = v_T \] ✓

\[ I_{DS} = W Q_n(V_{GS}, V_{DS}) \left[ \frac{1 - e^{-qV_{DS}/k_BT}}{1 + e^{-qV_{DS}/k_BT}} \right] v_T \] ✓
Check the small $V_{DS}$ limit

$$I_{DS} = W \left| Q_n(V_{GS}, V_{DS}) \right| \left[ \frac{1 - e^{-qV_{DS}/k_BT}}{1 + \left( \frac{T_0}{2 - T_0} \right) e^{-qV_{DS}/k_BT}} \right] \nu_{inj}$$

$$\nu_{inj} = \left( \frac{T_0}{2 - T_0} \right) \nu_{inj}^{ball}$$

$$\nu_{inj}^{ball} = \nu_T = \sqrt{\frac{2k_BT}{\pi m_n^*}}$$

In this case, $T_0 = T_{LIN}$.

$$e^{-qV_{DS}/k_BT} \approx 1 - qV_{DS}/k_BT$$

$$I_{DS} = T_{LIN} W \left| Q_n(V_{GS}, V_{DS}) \right| \left( \frac{\nu_T}{2(k_BT/q)} \right) V_{DS}$$
Check the large $V_{DS}$ limit

$$I_{DS} = W \left| Q_n (V_{GS}, V_{DS}) \right| \left[ \frac{1 - e^{-qV_{DS}/k_B T}}{1 + \left( \frac{T_0}{2 - T_0} \right) e^{-qV_{DS}/k_B T}} \right] v_{inj}$$

$$v_{inj} = \left( \frac{T}{2 - T_0} \right) v_{inj}^{ball}$$

$$v_{inj}^{ball} = v_T = \sqrt{\frac{2k_B T}{\pi m_n^*}}$$

In this case, $T_0 = T_{SAT}$.

$$e^{-qV_{DS}/k_B T} \rightarrow 0 \quad I_{DS} = W \left| Q_n (V_{GS}, V_{DS}) \right| \left( \frac{T_{SAT}}{2 - T_{SAT}} \right) v_T$$
Drain bias dependent transmission

\[ \mathcal{T}_0(V_{GS}, V_{DS}) \]

\[ \mathcal{T}_0(V_{GS}, V_{DS} \rightarrow 0) = \mathcal{T}_{LIN} \quad \mathcal{T}_0(V_{GS}, V_{DS} \gg V_{DSAT}) = \mathcal{T}_{SAT} \]

The calculation of the transmission for arbitrary drain voltage is still a research problem. It has been argued that it can be determined from the channel potential, \( V(x) \) according to:

\[ \mathcal{T}_0(V_{DS}) = \frac{\lambda_0}{\lambda_0 + L_C(V_{DS})} \quad L_C = \int_0^L e^{q[V(x)-V(g)]/k_BT} \, dx \]
The model

\[ I_{DS} = W \left| Q_n(V_{GS}, V_{DS}) \right| \langle \nu(V_{DS}) \rangle \]

\[ F_{SAT}(V_{DS}) = \begin{bmatrix}
1 - e^{-qV_{DS}/k_BT} \\
1 + \left( \frac{T_0}{2 - T_0} \right) e^{-qV_{DS}/k_BT}
\end{bmatrix} \]

\[ \langle \nu(V_{DS}) \rangle = F_{SAT}(V_{DS}) \nu_{inj} \]

\[ \nu_{inj} = \left( \frac{T_0}{2 - T_0} \right) \nu_T \]

The major challenge in using this model is the need to know \( T_0(V_{DS}) \)
A model that uses approximate calculations of the channel potential to compute the bias-dependent transmission has been reported. It is less empirical than the MVS model.

Finally: Fermi-Dirac statistics

\[ I_{DS} = W \left| Q_n(V_{GS}, V_{DS}) \right| \langle \nu(V_{GS}, V_{DS}) \rangle \quad \langle \nu(V_{GS}, V_{DS}) \rangle = F_{SAT}(V_{GS}, V_{DS}) \nu_{inj} \]

\[ F_{SAT}(V_{GS}, V_{DS}) = \left[ \frac{1 - F_{1/2}(\eta_{FD})/F_{1/2}(\eta_{FS})}{1 + \left( \frac{\tau_0}{2 - \tau_0} \right) F_0(\eta_{FD})/F_0(\eta_{FS})} \right] \nu_{inj}(V_{GS}) \]

\[ \nu_{inj}^{ball}(V_{GS}) = \nu_T \frac{F_{1/2}(\eta_{FS})}{F_0(\eta_{FS})} \]

\[ \nu_{inj} = \left( \frac{\tau_0(V_{GS}, V_{DS})}{2 - \tau_0(V_{GS}, V_{DS})} \right) \nu_{inj}^{ball} \]

\[ \eta_{FD} = \eta_{FS} - \frac{V_{DS}}{(k_B T / q)} \]