Principles of Electronic Nanobiosensors

Unit 2: Settling Time
Lecture 2.1: Shape of a Surface

By Muhammad A. Alam
Professor of Electrical and Computer Engineering
Purdue University
alam@purdue.edu
Outline

• Recap of lecture 1.3
• Shape of a object described by fractals
  – Regular fractals
  – Irregular fractals
• Outline of the course
• Summary and conclusion
Nanobiosensors have high sensitivity

Is there something fundamental about the geometry?
Geometry is Important, but exactly how?

Geometry of electrostatics

Geometry of diffusion

Berg, 1963

Alam, Principles of Nanobiosensors, 2013
Basic concepts: dimension of a surface

D=2

D=1

D=0

D=?
Classification of surfaces

Fractal Dimension ($D_F$)- Box counting technique

- Plane $D_F=2$
- Line $D_F=1$
- Dot $D_F=0$
- Random NW $1 < D_F < 2$

$N(h) \sim h^2$
$N(h) \sim h^1$
$N(h) \sim h^0$

$log(N(h))$ vs $log(1/h)$ graph:
- Plane $D_F=2$
- Random NW $1 < D_F < 2$
- Line $D_F=1$
- Dot $D_F=0$
Example: Regular 1D fractals

\[
D_{F,1} = \frac{\log(N)}{\log(1/h)} = \frac{\log(2^n)}{\log(3^n)} = 0.63
\]

Bigger than a point, but smaller than a line

In general, \( D_{F,1} = \frac{\log(m)}{\log(n)} \) .... keep \( m \) piece

of \( n \) pieces

Alam, Principles of Nanobiosensors, 2013
Regular and irregular 1D fractals

\[ D_{F,1} = \frac{\log(m)}{\log(n)} \quad \text{…. keep m piece} \]

of n pieces

regular

irregular

Alam, Principles of Nanobiosensors, 2013
Dimension of quasi-2D Fractal

\[ D_{F,2} = \frac{\log(N)}{\log(1/h)} = \frac{\log(3^n) + \log(2^n)}{\log(3^n)} = 1 + \frac{\log(2^n)}{\log(3^n)} = 1 + DF_x \]

In general, \( D_{F,2} = DF_x + DF_y \)

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Same DF, but different geometry

same dimension, because $DF = 1 + \frac{\log(m)}{\log(n)}$

What about this irregular fractal?
Dimension of a irregular fractal surface

\[ \text{Slope}=D_F=1.54. \]
Irregular to regular surfaces

\[ D_{F,CT} = 1 + \frac{\log(m)}{\log(n)} \]

For \( D_{F,stick} = 1.5 \)

Let \( m = 2 \), solve for \( n \):

\[ \log(n) = \frac{\log(2)}{D_{F,stick} - 1} \]

Result: \( n = 4 \)

Generation algorithm:

Take a line segment

Remove the fraction \( \frac{n-2}{n} \) from its centre (result: \( \frac{1}{2} \))

repeat …
Outline

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Settling time defines the fundamental limits of detection (Lectures 5-10)

Heisenberg principle

\[ \Delta E \times \Delta t \sim \hbar \]

Settling time principle

\[ \rho_0 \times t_s^{(3-D_F)/2} \sim c \]

The result is technology agnostic
Defined by the geometry of diffusion
Sensitivity defines transducer-specific limits of detection (Lectures 11-22)

Mass, charge, and electron affinity – based biosensors

Noise limits of surrounding media

Geometry of the sensor

Defined by the geometry of screening
Selectivity defines the practical limits of detection (Lectures 22-30)

Selectivity defines the ability to differentiate between noise and signal.

Related to fundamental issues of random sequential absorption.

Technology-agnostic sensor metric.

Alam, Principles of Nanobiosensors, 2013
Geometry is the key for nanobiosensing

Geometry of diffusion for theory of selectivity

Geometry of screening for sensitivity

Geometry of random sequential adsorption for selectivity

Alam, Principles of Nanobiosensors, 2013
Conclusions

• Fractal dimension characterizes the shape of a sensor surface quantitatively.
• Both regular or random surfaces may be characterized by fractals.
• A random surface may be converted into a regular surface for ease of analysis.
• The geometry of a sensor surface dictates a number of its properties, including settling time, sensitivity, and selectivity.