Principles of Electronic Nanobiosensors

Unit 2: Settling Time
Lecture 2.3: Classical Sensors II

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Geometry of diffusion/sensor response

Planar:

\[ t_s \sim \frac{2N_S^2}{D} \frac{1}{\rho_0^2} \]

SiNW:

\[ t_s \sim \frac{N_S a_0}{D} \frac{1}{\rho_0} \]
A ‘Mendeleev table’ for biosensors

For other geometries, we need a slightly better technique based on ‘diffusion-equivalent capacitance …’
Outline

• Introduction
• So many sensors ... How to classify them
• Geometry of diffusion defines response time
  – Approach based on ‘diffusion triangle’
  – Approach based on ‘diffusion capacitance’
• Conclusion
Steady state: analogy to electrostatics

\[ D \nabla^2 \rho = 0 \]

\[ \varepsilon \nabla^2 \phi = 0 \]

\[ I = C_0 (\rho_0 - \rho_s) \]

\[ C_0 = \frac{D}{W} \quad \text{(Planar)} \]
\[ = \frac{2\pi D}{\log\left(\left(W + a_0\right)/a_0\right)} \quad \text{(NW)} \]
\[ = \frac{4\pi D}{a_0^{-1} - \left(W + a_0\right)^{-1}} \quad \text{(ND)} \]

\[ Q = C_0 (\phi - \phi_s) \]

\[ C_0 = \frac{\varepsilon}{W} \quad \text{(Planar)} \]
\[ = \frac{2\pi \varepsilon}{\log\left(\left(W + a_0\right)/a_0\right)} \quad \text{(Cylinder)} \]
\[ = \frac{4\pi \varepsilon}{a_0^{-1} - \left(W + a_0\right)^{-1}} \quad \text{(Sphere)} \]
Diffusion and capture in steady-state

\[ I = C_0 \left( \rho_o - \rho_s \right) \sim C_0 \rho_o \]

\[ I = A \frac{dN}{dt} = C_0 \rho_0 \]

\[ N(t) = C_0 \rho_0 t / A \]

\[ C_{0,1} = \frac{D}{W} \]

\[ C_{0,2} = \frac{2\pi D}{\log \left( \frac{W + a_0}{a_0} \right)} \]

\[ C_{0,3} = \frac{4\pi D}{a_0^{-1} - \left( W + a_0 \right)^{-1}} \]

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Time-dependent capture

\[ N(t) = C_0 \rho_0 t / A \]

\[ C_{0,1} = \frac{D}{W} \]
\[ C_{0,2} = \frac{2\pi D}{\log{(W + a_0)/a_0)} \]
\[ C_{0,3} = \frac{4\pi D}{a_0^{-1} - (W + a_0)^{-1}} \]

\[ W(t) = \sqrt{2nDT} \]

\[ C_{D(t)} = \frac{D}{\sqrt{2Dt}} \]
\[ C_{D(t)} = \frac{2\pi D}{\log{((\sqrt{4Dt} + a_0)/a_0)} \]
\[ C_{D(t)} = \frac{4\pi D}{a_0^{-1} - (\sqrt{6Dt} + a_0)^{-1}} \]

Alam, Principles of Nanobiosensors, 2013
Integer dimensional sensors

\[ N(t) = C_{D(t)} \rho_0 t \]

\[ C_{D(t)} = \frac{D}{\sqrt{2Dt}} \quad C_{D(t)} \sim \frac{2 \pi D}{\ln(\sqrt{4Dt}/a_0)} \quad C_{D(t)} \sim \frac{4 \pi D}{a_0^{-1} - \left(1/\sqrt{6Dt}\right)} \]

\[ N(t) = k \rho_0 t_s^{1/2} \quad N(t) = k \rho_0 t_s^{1} \quad N(t) = k \rho_0 t_s^{1} \]

Alam, Principles of Nanobiosensors, 2013
Exact solutions in 3D

\[ \frac{d\rho}{dt} = D \nabla^2 \rho \]

\[ \frac{dN}{dt} = k_F (N_0 - N) \rho_s \]

\[ k_F \to \infty, \rho_s = 0 \]

\[ J(t) = \frac{D \rho_0}{a_0} \left( 1 + \frac{a_0}{\sqrt{6Dt}} \right) \]

\[ N(t) = 4\pi a_0^2 \int_0^t J(t')dt' \]

\[ N(t) = 4\pi \rho_0 Da_0 \left[ t + \left( \frac{a_0}{\sqrt{6D}} \right) \sqrt{t} \right] \]

\[ N(t) \sim 4\pi \rho_0 Da_0 t^1 \]

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Summary: For integer sensors

\[ N(t) \propto \rho_0 t_s^{\left(\frac{3-D_F}{2}\right)} \quad (1 \leq D_F \leq 2) \]

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<tr>
<th>( D_F = 2 )</th>
<th>( D_F = 1 )</th>
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\[ N(t) \propto \rho_0 t_s^{1} \quad (0 \leq D_F \leq 1) \]

| \( D_F = 0 \) |

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Nanodots sensor offer no significant additional advantage!
Conclusions

- Settling time defines the average time needed for a sensor to capture the minimum number of molecules required for definitive response.

- This time defines a fundamental limit to detection limit – almost analogous to the Heisenberg uncertainly principle.

- Size of the biomolecule is hidden in diffusion coefficient.

- Geometry of diffusion determines the response time.

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