Lecture 3.1: The Fermi function
Occupation of states

Si atom (At. no. 14)

States above this energy have a low probability of being occupied.

States below this energy have a high probability of being occupied.

Lundstrom: 2018
Fermi level

\[ D_c(E) = g_v \frac{m_n^* \sqrt{2m_n^*(E - E_c)}}{\pi^2 \hbar^3} \]

\[ D_v(E) = g_v \frac{m_p^* \sqrt{2m_p^*(E_v - E)}}{\pi^2 \hbar^3} \]

\[ f_0(E) = \frac{1}{1 + e^{(E - E_F)/k_B T}} \]

Electrochemical potential

Fermi function

Lundstrom: 2018
The Fermi function

\[ f_0(E) = \frac{1}{1 + e^{(E - E_F)/k_B T}} \]

**Fermi function**

Probability that a state at energy, \( E \), is occupied in equilibrium.

**Graph:**
- Small probability of being filled.
- Small probability of being empty.

\[ f_0(E_F) = \frac{1}{2} \]

Lundstrom: 2018
More about the Fermi function

\[ f_0(E) = \frac{1}{1 + e^{(E-E_F)/k_BT}} \]

Fermi level

\[ k_BT = 0.026 \text{ eV} \]
Effect of temperature

\[ f_0(E) = \frac{1}{1 + e^{(E-E_F)/k_BT}} \]

\[ k_BT \]

\[ T_1 > T_0 \]

\[ E_F \]

\[ T_2 < T_0 \]

\[ E \]
Electrons and holes

These states are way above the Fermi level.

Typically, we will find the Fermi level somewhere inside the bandgap.

These states are way below the Fermi level.

← A few states near $E_C$ may be occupied.

← A few states near $E_V$ may be empty.
Conduction band

Nondegenerate N-type semiconductor

\[ f_0(E) = \frac{1}{1 + e^{(E - E_F)/k_B T}} \]

\[ E >> E_F \]

\[ f_0(E) \approx e^{(E_F - E)/k_B T} \]

small probability of being full

Lundstrom: 2018
Valence band

small probability of being empty

\[ f_0(E) = \frac{1}{1 + e^{(E - E_F)/k_BT}} \]

\[ 1 - f_0(E) \approx e^{(E - E_F)/k_BT} \]

\[ E << E_F \]

nondegenerate P-type semiconductor

Lundstrom: 2018
Nondegenerate semiconductors

In a nondegenerate semiconductor, the Fermi level does not get too close to the band edges.

\[
f_0(E_C) = \frac{1}{1 + e^{(E_C - E_F)/k_B T}}
\]

\[
f_0(E_C) \approx e^{(E_F - E_C)/k_B T}
\]

\[
f_0(E_V) = \frac{1}{1 + e^{(E_V - E_F)/k_B T}}
\]

\[
1 - f_0(E_V) \approx e^{(E_V - E_F)/k_B T}
\]

Lundstrom: 2018
Energy band diagram of an intrinsic semiconductor

Energy gap: $E_G = 1.1 \text{ eV}$

Conduction band: $E_C$, $n = n_i$

Valence band: $E_V$, $p = n_i$

Fermi level: $E_F = E_i$

Fermi-Dirac distribution function:

$$f_0(E) = \frac{1}{1 + e^{(E-E_F)/k_BT}}$$
Energy band diagram and carrier densities

\[ f_0(E) \approx e^{(E_F - E)/k_B T} \]

\[ n \propto e^{(E_F - E_C)/k_B T} \]

\[ 1 - f_0(E) \approx e^{(E_V - E_F)/k_B T} \]

\[ p \propto e^{(E_V - E_F)/k_B T} \]

Lundstrom: 2018
The Fermi function gives the probability that a state (if it exists) is occupied in equilibrium.

\[ f_0(E) = \frac{1}{1 + e^{(E-E_F)/k_B T}} \]

(Fermi function)

The two key parameters in the Fermi function are the Fermi level and the temperature.