Unit 3: Equilibrium Carrier Concentrations

Lecture 3.3: Carrier concentration vs. Fermi level

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Carrier concentrations

Electrons

\[ n_0 = N_C \mathcal{F}_{1/2} \left[ \left( E_F - E_C \right)/k_B T \right] \text{ m}^{-3} \]

\[ N_C = \frac{1}{4} \left( \frac{2m_n^* k_B T}{\pi \hbar^2} \right)^{3/2} \]

nondegenerate:

\[ n_0 = N_C e^{(E_F - E_C)/k_B T} \]

Holes

\[ p_0 = N_V \mathcal{F}_{1/2} \left[ \left( E_V - E_F \right)/k_B T \right] \text{ m}^{-3} \]

\[ N_V = \frac{1}{4} \left( \frac{2m_p^* k_B T}{\pi \hbar^2} \right)^{3/2} \]

nondegenerate:

\[ p_0 = N_V e^{(E_V - E_F)/k_B T} \]
Electron concentration

Electrons

\[ n_0 = N_C \mathcal{F}_{1/2} \left[ \left( E_F - E_C \right) / k_B T \right] \text{ m}^{-3} \]

\[ N_C = \frac{1}{4} \left( \frac{2m_D^* k_B T}{\pi \hbar^2} \right)^{3/2} \]

nondegenerate:

\[ n_0 = N_C e^{(E_F - E_C) / k_B T} \]

For Si at \( T = 300 \text{K} \):

\[ m_D^* = 1.182 m_0 \text{ (DOS effective mass)} \]

\[ N_C = 3.23 \times 10^{19} \text{ cm}^{-3} \]
Hole concentration

For Si at $T = 300$K:

$m^*_p = 0.81m_0$ (DOS effective mass)

$N_V = 1.83 \times 10^{19}$ cm$^{-3}$

\[ p_0 = N_V \mathcal{F}_{1/2} \left[ \left( \frac{E_V - E_F}{k_BT} \right) \right] \text{ m}^{-3} \]

\[ N_V = \frac{1}{4} \left( \frac{2m^*_p k_B T}{\pi \hbar^2} \right)^{3/2} \]

nondegenerate:

\[ p_0 = N_V e^{(E_V - E_F)/k_BT} \]
Given the Fermi level, we can deduce the electron and hole concentrations.

\[ n_0 = N_C \exp\left(\frac{E_F - E_C}{k_B T}\right) \]

\[ N_C = 3.23 \times 10^{19} \text{ cm}^{-3} \]

(silicon at 300 K)
Given the Fermi level, we can deduce the electron and hole concentrations.

\[ p_0 = N_V \exp\left(\frac{E_V - E_F}{k_B T}\right) \]

\[ N_V = 1.83 \times 10^{19} \text{ cm}^{-3} \]

(silicon at 300 K)
From carrier concentration to Fermi level

If we are given \( n \):

\[
E_F = E_C + k_B T \ln \left( \frac{n_0}{N_C} \right)
\]

If we are given \( p \):

\[
E_F = E_V - k_B T \ln \left( \frac{p_0}{N_V} \right)
\]
The equilibrium product of the electron and hole concentrations is a very important quantity for a semiconductor.
\[ n_0 p_0 = N_C e^{(E_F - E_C)/k_B T} N_V e^{(E_V - E_F)/k_B T} \]

\[ n_0 p_0 = N_C N_V e^{(E_F - E_C)/k_B T} \]

\[ n_0 p_0 = N_C N_V e^{-E_G/k_B T} = n_i^2 \]

\[ n_i = \sqrt{N_C N_V e^{-E_G/2k_B T}} \]

\[ n_0 = N_C e^{(E_F - E_C)/k_B T} \]

\[ p_0 = N_V e^{(E_V - E_F)/k_B T} \]

\[ N_C = 2 \left[ \frac{(m_n^* k_B T)}{2\pi \hbar^2} \right]^{3/2} \]

\[ N_V = 2 \left[ \frac{(m_p^* k_B T)}{2\pi \hbar^2} \right]^{3/2} \]
np product

- Independent of Fermi level (for nondegenerate semiconductor)
- Depends exponentially on band gap
- Depends exponentially on temperature
- For Si at 300 K

\[ n_i = 1.0 \times 10^{10} \text{ cm}^{-3} \]
Recall: Fermi level and hole concentration

Given the Fermi level, we can deduce the electron and hole concentrations.

\[ p_0 = N_V \exp\left(\frac{(E_V - E_F)}{k_B T}\right) \]

\[ N_V = 1.83 \times 10^{19} \text{ cm}^{-3} \]

(silicon)

Lundstrom: 2018
Another way

Find the electron concentration first.

\[ n_0 = 1.47 \times 10^{16} \text{ cm}^{-3} \]

Then use

\[ n_0 p_0 = n_i^2 \]

\[ n_i = \sqrt{N_C N_V} e^{-E_G/2k_B T} \neq 1.0 \times 10^{10} \text{ cm}^{-3} \]

\[ p_0 = \frac{n_i^2}{n_0} \]

\[ = \left(10^{10}\right)^2 / 1.47 \times 10^{16} \]

\[ = 0.68 \times 10^3 \text{ cm}^{-3} \]

\[ \text{(vs. } 1.14 \times 10^4 \text{ cm}^{-1}) \]
Fermi level is closer to the conduction band than to the valence band.

Electron concentration is greater than the hole concentration. $n_0 > p_0$

But the $np$ product does not change. $n_0 p_0 = n_i^2$
E-band diagram for P-type semiconductor

Fermi level is closer to the valence band than to the conduction band.

Hole concentration is greater than the electron concentration. \( p_0 > n_0 \)

But the \( np \) product does not change. \( n_0 p_0 = n_i^2 \)
Intrinsic semiconductor

Fermi level is near the middle of the gap.

Hole concentration is equal to the electron concentration. $p_0 = n_0$

The $np$ product is still the same. $n_0 p_0 = n_i^2$

Exactly where is the intrinsic Fermi level?
The intrinsic Fermi level

\[ n_0 = N_C e^{(E_F - E_C)/k_B T} \]

\[ p_0 = N_V e^{(E_V - E_F)/k_B T} \]

\[ n_0 = p_0 = n_i \quad E_F = E_i \]

\[ N_C e^{(E_i - E_C)/k_B T} = N_V e^{(E_V - E_i)/k_B T} \]

\[ E_i = \frac{E_C + E_V}{2} + \frac{k_B T}{2} \ln \left( \frac{N_V}{N_C} \right) \]
The intrinsic level is very near the middle of the band gap.
Alternative expression for carrier densities

\[ n_0 = N_C e^{(E_F - E_C)/k_B T} \]
\[ p_0 = N_V e^{(E_V - E_F)/k_B T} \]

\[ n_i = N_C e^{(E_i - E_C)/k_B T} \quad \rightarrow \quad N_C = n_i e^{-(E_i - E_C)/k_B T} \]

\[ p_i = N_V e^{(E_V - E_i)/k_B T} \quad \rightarrow \quad N_V = n_i e^{-(E_V - E_i)/k_B T} \]
“Reading” an E-band diagram

$n_0 = n_i e^{(E_F - E_i)/k_BT}$

$p_0 = n_i e^{(E_i - E_F)/k_BT}$

- Fermi level above $E_i$, n-type
- Fermi level below $E_i$, p-type
Summary

\[ n_0 = N_C e^{(E_F - E_C)/k_B T} \quad \quad n_0 = n_i e^{(E_F - E_i)/k_B T} \]

\[ p_0 = N_V e^{(E_V - E_F)/k_B T} \quad \quad p_0 = n_i e^{(E_i - E_F)/k_B T} \]

\[ N_C = \frac{1}{4} \left( \frac{2 m_n^* k_B T}{\pi \hbar^2} \right)^{3/2} \]

\[ N_V = \frac{1}{4} \left( \frac{2 m_p^* k_B T}{\pi \hbar^2} \right)^{3/2} \]

\[ n_i = \sqrt{N_C N_V} e^{-E_G/2k_B T} \]

\[ n_0 p_0 = n_i^2 \]