Unit 3: Equilibrium Carrier Concentrations

Lecture 3.4: Carrier concentration vs. doping density

Mark Lundstrom
lundstro@purdue.edu
Electrical and Computer Engineering
Purdue University
West Lafayette, Indiana USA
Carrier concentrations vs. Fermi level

Electrons

\[ n_0 = N_C \mathcal{F}_{1/2} \left[ \frac{(E_F - E_C)}{k_B T} \right] \text{ m}^{-3} \]

\[ N_C = \frac{1}{4} \left( \frac{2m^*_n k_B T}{\pi \hbar^2} \right)^{3/2} \]

nondegenerate:

\[ n_0 = N_C e^{(E_F - E_C)/k_B T} \]

\[ n_0 = n_i e^{(E_F - E_i)/k_B T} \]

Holes

\[ p_0 = N_V \mathcal{F}_{1/2} \left[ \frac{(E_V - E_F)}{k_B T} \right] \text{ m}^{-3} \]

\[ N_V = \frac{1}{4} \left( \frac{2m^*_p k_B T}{\pi \hbar^2} \right)^{3/2} \]

nondegenerate:

\[ p_0 = N_V e^{(E_V - E_F)/k_B T} \]

\[ p_0 = n_i e^{(E_i - E_F)/k_B T} \]
What is the net charge in this region?

\[ \rho = q \left[ p - n + N_D^+ - N_A^- \right] \quad \text{C/m}^3 \]

bulk, uniform semiconductor
Space charge neutrality

Nature abhors a vacuum. Nature also abhors a charge.

Mobile charges (electrons and holes) will be attracted to the immobile ionized dopants), so that the net charge is zero.

$$\rho = q \left[ p - n + N_D^+ - N_A^- \right] = 0$$

Almost uniform semiconductors will be nearly neutral, but with strong non-uniformities (e.g. PN junctions), there will be a space charge.
Fully ionized dopants

All donors have donated their electrons to the conduction band and are now positively charged.

\[ N^+_D = N_D \quad N^-_A = N_A \]

All acceptors have accepted an electron from the valence band and are now negatively charged.

Lundstrom: 2018
Assume fully ionized dopants (this will typically be the case for good dopants near and above room temperature).

\[
\rho = q \left[ p - n + N_D^+ - N_A^- \right] = 0
\]

\[
\rho = q \left[ p_0 - n_0 + N_D - N_A \right] = 0
\]

\[n_0 p_0 = n_i^2\]

Assuming that we know how many dopants we introduced into the semiconductor, these are two equations in two unknowns – \( p \) and \( n \).
Solving for the carrier density

1) charge neutrality: \[ p_0 - n_0 + N_D^+ - N_A^- = 0 \]

2) Fully ionized dopants: \[ p_0 - n_0 + N_D - N_A = 0 \]

3) np product: \[ n_0 p_0 = n_i^2 \]

4) result: \[ \frac{n_i^2}{n_0} - n_0 + N_D - N_A = 0 \]
\[ p_0 - \frac{n_i^2}{p_0} + N_D - N_A = 0 \]
Result: N-type

\[ \frac{n_i^2}{n_0} - n_0 + N_D - N_A = 0 \]

\[ n_0 = \frac{N_D - N_A}{2} + \left[ \left( \frac{N_D - N_A}{2} \right)^2 + n_i^2 \right]^{1/2} \]

\[ p_0 = \frac{n_i^2}{n_0} \]
Result: P-type

\[ p_0 - \frac{n_i^2}{p_0} + N_D - N_A = 0 \]

\[ p_0 = \frac{N_A - N_D}{2} + \left[ \left( \frac{N_A - N_D}{2} \right)^2 + n_i^2 \right]^{1/2} \]

\[ n_0 = \frac{n_i^2}{p_0} \]
Example 1

Consider Si doped with phosphorus at $N_D = 2.00 \times 10^{15}$ cm$^{-3}$

The temperature is 300 K. What are $n$ and $p$?

Recall that at 300 K in Si, $n_i = 1.00 \times 10^{10}$ cm$^{-3}$

Assume that the donors are fully ionized.

$$n_0 = \frac{N_D - N_A}{2} + \left[ \left( \frac{N_D - N_A}{2} \right)^2 + n_i^2 \right]^{1/2} = \frac{N_D}{2} + \left[ \left( \frac{N_D}{2} \right)^2 + n_i^2 \right]^{1/2}$$

$N_D >> n_i$

$n_0 = N_D = 2.00 \times 10^{15}$ cm$^{-3}$

$p_0 = n_i^2 / n_0$

$p_0 = \left( 10^{10} \right)^2 / 2 \times 10^{15} = 5 \times 10^4$ cm$^{-3}$
Example 2

Consider Si doped with phosphorus at $N_D = 2.00 \times 10^{15} \text{ cm}^{-3}$ and Boron at $N_A = 1.00 \times 10^{15} \text{ cm}^{-15}$. The temperature is 300 K. What are $n$ and $p$?

\[
    n_0 = \frac{N_D - N_A}{2} + \left[ \left( \frac{N_D - N_A}{2} \right)^2 + n_i^2 \right]^{1/2}
\]

\[
    N_D - N_A \gg n_i \quad n_0 = N_D - N_A
\]

\[
    n_0 = 1.00 \times 10^{15} \text{ cm}^{-3}
\]

\[
    p_0 = \frac{n_i^2}{n_0} \quad p_0 = \left(10^{10}\right)^2 / 1 \times 10^{15} = 1 \times 10^5 \text{ cm}^{-3}
\]
Conclusion

When the net doping density is much greater than the intrinsic carrier concentration and the dopants are fully ionized, then

\[
\begin{align*}
n_0 &= N_D - N_A & p_0 &= n_i^2 / \left( N_D - N_A \right) & \text{N-type} & N_D > N_A \\
p_0 &= N_A - N_D & n_0 &= n_i^2 / \left( N_A - N_D \right) & \text{P-type} & N_A > N_D
\end{align*}
\]
Summary

\[ n_0 = N_D - N_A \]
\[ p_0 = n_i^2 / (N_D - N_A) \]
\[ p_0 = N_A - N_D \]
\[ n_0 = n_i^2 / (N_A - N_D) \]

N-type

P-type

freeze out

extrinsic

intrinsic

\[ n \text{ or } p \]