Equilibrium and non-equilibrium

In equilibrium:

\[ n_0 = N_C e^{(E_F - E_C)/k_B T} \quad p_0 = N_V e^{(E_V - E_F)/k_B T} \]

Out of equilibrium, there can be additional “excess carriers”:

\[ n = n_0 + \Delta n \quad p = p_0 + \Delta p \]

(The excess carrier concentrations can be positive or negative.)

Question: How do the excess carrier concentrations vary with time?
Carrier recombination

P-type, equilibrium

\[ \Delta n(t = 0) \]

\[ p_0 = 10^{17} \text{ cm}^{-3} \quad n_0 = n_i^2 / p_0 = 10^3 \text{ cm}^{-3} \]

Expect:

\[ \Delta n(t) = \Delta n(t = 0) e^{-t/\tau} \]  \( \Delta n \) may be either positive or negative.

Goal: Understand the recombination lifetime, \( T_n \).
How can excess carriers recombine?

We will discuss three different ways:

1) Band-to-band (radiative) recombination

2) Auger recombination

3) SRH (defect-assisted) recombination
1) Band-to-band (radiative) recombination

\[ \frac{\partial n}{\partial t} \bigg|_{b-b} = \frac{\partial p}{\partial t} \bigg|_{b-b} = -B(np - n_i^2) \]

\[ n_0 p_0 = n_i^2 \]

(Note that this is zero in equilibrium – as it should be.)
The term “low level injection” means that the excess carrier concentration is orders of magnitude smaller than the equilibrium majority carrier concentration but orders of magnitude larger than the equilibrium minority carrier concentration.
Example: Low level injection in a p-type semiconductor

\[ n = n_0 + \Delta n \]
\[ p = p_0 + \Delta p \]
\[ \Delta n \approx \Delta p \]
\[ \Delta n = 10^8 \text{ cm}^{-3} \gg n_0 \]
\[ \Delta p \approx \Delta n = 10^8 \text{ cm}^{-3} \ll p_0 \]

filled states

\[ n_0 = 10^3 \text{ cm}^{-3} \]

\[ p_0 = 10^{17} \text{ cm}^{-3} \]

Lundström: 2018
Low level injection in a p-type semi

\[ n = n_0 + \Delta n \approx \Delta n \]

\[ p = p_0 + \Delta p \approx N_A \]

\[ \frac{\partial \Delta n}{\partial t}_{b-b} \approx -BN_A \Delta n \approx -\frac{\Delta n}{\tau_{bb}} \]

\[ \tau_{bb} = \frac{1}{BN_A} \]

Lundstrom: 2018
Excess carrier concentration vs. time

\[ \frac{\partial n}{\partial t} \bigg|_{b-b} = -B(np - n_i^2) \]

\[ \frac{\partial \Delta n}{\partial t} \bigg|_{b-b} \approx \frac{-\Delta n}{\tau_{bb}} \]

\[ \tau_{bb} = \frac{1}{BN_A} \]

\[ \Delta n(t) = Ce^{-t/\tau_{bb}} \]

\[ \Delta n(t) = \Delta n(t = 0)e^{-t/\tau_{bb}} \]
2) Auger recombination

Energy is given to a second electron

\[
\frac{\partial p}{\partial t}_{\text{Auger}} = C_n n (np - n_i^2) + C_p p (np - n_i^2)
\]

(Note that this is zero in equilibrium – as it should be.)
Low level injection in an n-type semiconductor

\[ \frac{\partial p}{\partial t}_{\text{Auger}} = -C_n n (np - n_i^2) \]

\[ E_G \]

\[ E_C \]

\[ E_V \]

\[ n = n_0 + \Delta n \approx N_D \]

\[ p = p_0 + \Delta p \approx \Delta p \]

\[ \frac{\partial \Delta p}{\partial t}_{\text{Auger}} \approx -C_n N_D^2 \Delta p = -\frac{\Delta p}{\tau_{\text{Auger}}} \]

\[ \tau_{\text{Auger}} = \frac{1}{C_n N_D^2} \]

Lundstrom: 2018
3) SRH (defect-assisted) recombination

energy released as thermal energy

\[
\frac{\partial n}{\partial t}_{SRH} = \frac{\partial p}{\partial t}_{SRH} = \frac{-(np - n_i^2)}{\tau_p (n + n_1) + \tau_n (p + p_1)}
\]

\( \tau_n = 1/c_n N_T \)

\( \tau_p = 1/c_p N_T \)

\( n_1, p_1 \approx n_i \)

(Shockley Read Hall recombination)
Low level injection in a p-type semiconductor

\[ \frac{\partial n}{\partial t}_{SRH} = \frac{\partial p}{\partial t}_{SRH} = \frac{-(np - n_i^2)}{\tau_p (n + n_1) + \tau_n (p + p_1)} \]

\[
\begin{align*}
n &= n_0 + \Delta n \approx \Delta n \\
p &= p_0 + \Delta p \approx N_A \\
np &\approx \Delta n N_A >> n_i^2 \\
p + p_1 &>> n + n_1 \\
p &>> p_1
\end{align*}
\]

\[
\frac{\partial \Delta n}{\partial t}_{SRH} = -\frac{\Delta n}{\tau_n}
\]

\[
\tau_{SRH} = \frac{1}{c_n N_T}
\]
The minority carrier lifetime is a key parameter for solar cells, bipolar transistors, etc.
Multiple recombination processes

\[ \frac{\partial \Delta n}{\partial t} \bigg|_{tot} = - \frac{\Delta n}{\tau_{eff}} \]

\[ \frac{1}{\tau_{eff}} = \frac{1}{\tau_{b-b}} + \frac{1}{\tau_{Auger}} + \frac{1}{\tau_{SRH}} \]

\[ \Delta n(t) = \Delta n(0) e^{-t/\tau_{eff}} \]
Discussion

When are the various recombination processes dominant?

1) Auger: For heavily doped semiconductors

\[ \tau_{\text{Auger}} = \frac{1}{C_p N_A^2} \]

2) SRH: When defects are present and other effects don’t dominate.

3) Radiative: Only for direct gap semiconductors.
BB recombination in **direct gap** semiconductors

Conservation of energy:

\[ E_{ph} = hf \approx E_G \]

Conservation of momentum:

\[ \hbar k_1 - \hbar k_2 = \hbar k_{ph} \approx 0 \]

\[ k_1 \approx k_2 \]

(“vertical transitions” photons have very little momentum)
BB recombination in **indirect gap** semiconductors

Conservation of energy:

\[ E_{ph} = hf = E_G \pm \hbar \omega_{lv} \]

Conservation of momentum:

\[ \hbar k_1 - \hbar k_2 = \hbar k_{ph} + \hbar k_{lv} \]

(must involve a lattice vibration with the right momentum)

**BB recombination in indirect semiconductors is very weak!**
Three type of recombination

1) Band-to-band radiative recombination
   dominates in direct gap semiconductors
   makes lasers and LEDs possible

2) Auger recombination
   dominates when the carrier densities are very high
   (heavily doped semiconductors or lasers)

3) SRH recombination
   dominates in indirect gap semiconductors
   and in low quality direct gap semiconductors
Recombination-generation

\[ R = X(np - n_i^2) \]

\[ X = B \quad \text{band-to-band radiative} \]

\[ X = C_n n + C_p p \quad \text{Auger} \]

\[ X = \frac{1}{\tau_p (n + n_1) + \tau_n (p + p_1)} \quad \text{SRH} \]

\[ np > n_i^2 \quad \text{net recombination} \]

\[ np < n_i^2 \quad \text{net generation} \]
3) SRH (defect-assisted) generation

$R > 0$: recombination

$R < 0$: generation

$np > n_i^2$ releases thermal energy

$np < n_i^2$ requires thermal energy
Summary

When excess carriers are introduced, a semiconductor responds by trying to restore equilibrium.

In the simplest, and quite common case, the perturbation decays exponentially with time.

\[ \Delta n(t) = \Delta n(t = 0) e^{-t/\tau_n} \]

(low level injection)

The minority carrier lifetime is controlled by radiative, Auger, or defect-assisted process – or by some combination of these.