Unit 5: The Semiconductor Equations

Lecture 5.3: Quasi-Fermi levels

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### Equilibrium vs. non-equilibrium

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<th>Equation</th>
<th>Equilibrium</th>
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<td>( n_0 )</td>
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<td>( n_0 p_0 = n_i^2 )</td>
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<td>( np \neq n_i^2 )</td>
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<td>( 1 - f_v = 1 - \frac{1}{1 + e^{(E - F_p)/k_BT}} )</td>
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Lundstrom: 2018
The semiconductor equations

\[
\frac{\partial p}{\partial t} = -\nabla \cdot \left( \frac{\vec{J}_p}{q} \right) + G_p - R_p
\]

\[
\frac{\partial n}{\partial t} = -\nabla \cdot \left( \frac{\vec{J}_n}{-q} \right) + G_n - R_n
\]

\[
\nabla^2 V = -\frac{\rho}{K_s \varepsilon_0}
\]

Three equations in three unknowns:

\[
p(\vec{r}), \ n(\vec{r}), \ V(\vec{r})
\]

or:

\[
F_p(\vec{r}), \ F_n(\vec{r}), \ V(\vec{r})
\]

\[
\vec{J}_p = p\mu_p \vec{V} F_p
\]

\[
\vec{J}_n = n\mu_n \vec{V} F_n
\]

\[
\rho = q \left( p - n + N_D^+ - N_A^- \right)
\]

\[
\vec{E}(\vec{r}) = \nabla V(\vec{r})
\]
Where is the Fermi level?

N-type silicon in equilibrium

\[ n_0 = N_D = 10^{17} \text{ cm}^{-3} \]

\[ n_0 = n_i e^{(E_F - E_i)/k_B T} \]

\[ p_0 = n_i e^{(E_i - E_F)/k_B T} \]

\[ p_0 = \frac{n_i^2}{n_0} = 10^3 \text{ cm}^{-3} \]

Given either carrier density, we can determine the Fermi level.

Lundstrom: 2018
Now where is the Fermi level?

Now assume that there are $10^{14}$ cm$^{-3}$ excess carriers.

$$n \approx n_0 = 10^{17} \text{ cm}^{-3}$$

$E_C$

$E_G$

$E_V$

$$p = \Delta p = 10^{14} \text{ cm}^{-3} \gg p_0$$

a) Where it was in equilibrium
b) Closer to the conduction band
c) Closer to the valence band
d) Near the middle of the band
e) None of the above

Same # of electrons, more holes -> need 2 Fermi levels!
Quasi-Fermi levels

\[ n = 10^{17} \text{ cm}^{-3} \]
\[ E_C \quad F_n \approx E_F \]
\[ E_i \quad F_p \]
\[ E_V \quad p = 10^{14} \text{ cm}^{-3} \]

Non-equilibrium (low level injection)

\[ n = n_i e^{(F_n - E_i)/k_BT} \approx n_0 \]
\[ F_n = E_F \]
\[ p = n_i e^{(E_V - F_p)/k_BT} \gg p_0 \]
\[ F_p < E_F \]

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Non-equilibrium

When the QFL’s are split, the semiconductor is out of equilibrium.

In equilibrium: \( F_n = F_p = E_F \)

\[ n = 10^{17} \text{ cm}^{-3} \]
\[ p = 10^{14} \text{ cm}^{-3} \]
Equilibrium

\[ n_0 = n_i e^{(E_F - E_i)/k_B T} \]

\[ p_0 = n_i e^{(E_i - E_p)/k_B T} \]

\[ n_0 p_0 = n_i^2 \]

Out of equilibrium

\[ n = n_i e^{(E_n - E_i)/k_B T} \]

\[ p = n_i e^{(E_i - E_p)/k_B T} \]

\[ n p = n_i^2 e^{(E_n - E_p)/k_B T} \]

\[ F_n > F_p \rightarrow np > n_i^2 \]

\[ F_n < F_p \rightarrow np < n_i^2 \]
Some numbers (equilibrium)

N-type silicon in equilibrium

\[ n_0 = N_D = 10^{17} \text{ cm}^{-3} \]

\[ E_C \quad E_F \quad E_i \quad E_G \]

\[ p_0 = \frac{n_i^2}{n_0} = 10^3 \text{ cm}^{-3} \]

\[ n_0 = n_i e^{(E_F - E_i)/k_B T} \]

\[ p_0 = n_i e^{(E_i - E_F)/k_B T} \]

\[ (E_F - E_i) = k_B T \ln\left(\frac{n}{n_i}\right) \]

\[ (E_F - E_i) = 0.026 \ln\left(\frac{10^{17}}{10^{10}}\right) \text{ eV} \]

\[ (E_F - E_i) = 0.42 \text{ eV} \]

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Some numbers (LL injection)

\[ n = n_i e^{(F_n - E_i)/k_B T} \]

\[ p = n_i e^{(E_i - F_p)/k_B T} \]

\[ (F_n - E_i) = k_B T \ln \left( \frac{n}{n_i} \right) = 0.026 \ln \left( 10^7 \right) \]

\[ (E_i - F_p) = 0.42 \text{ eV} \]

\[ (E_i - F_p) = k_B T \ln \left( \frac{p}{n_i} \right) = 0.026 \ln \left( 10^4 \right) \]

\[ (E_i - F_p) = 0.24 \text{ eV} \]

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Note that it is possible for a semiconductor to be out of equilibrium (e.g. with current flowing), but with $F_n = F_p$.

$$n_0 = N_D = 10^{17} \text{ cm}^{-3}$$

$$p_0 = \frac{n_i^2}{n_0} = 10^3 \text{ cm}^{-3}$$

N-type semiconductor in equilibrium

N-type semiconductor resistor under bias
E-band diagram of an N-type resistor

Note that it is possible for a semiconductor to be out of equilibrium (e.g. with current flowing), but with $F_n = F_p$.

$$F_n = F_p$$

$E_C$  

$E_i$  

$E_V$  

$p = p_0$  

$E_C - F_n = E_C - E_F$  

$n = n_0 = N_D$  

$np = n_i^2$  

No excess carriers
Assumptions

What assumption is involved in replacing the equilibrium Fermi level with two quasi-Fermi levels out of equilibrium?

In equilibrium, the probability that a state is occupied is given by:

\[ f_0(E) = \frac{1}{1 + e^{(E-E_F)/k_BT}} \]

We assume that the same function describes the occupation of states in the conduction and valence bands if we simply replace the Fermi level by the appropriate QFL:

\[ f_C(E) = \frac{1}{1 + e^{(E-E_n)/k_BT}} \]

\[ 1 - f_V(E) = 1 - \frac{1}{1 + e^{(E-E_p)/k_BT}} \]

Lundstrom: 2018
Summary

Equilibrium:

\[ E_F \]

\[ f_0(E) = \frac{1}{1 + e^{(E-E_F)/k_BT}} \]

Out of Equilibrium:

\[ F_n \]

\[ f_C(E) = \frac{1}{1 + e^{(E-F_n)/k_BT}} \]

\[ F_p \]

\[ 1 - f_V(E) = 1 - \frac{1}{1 + e^{(E-F_p)/k_BT}} \]

This assumption can be expected to work if we are close enough to equilibrium.