Unit 5: The Semiconductor Equations

Lecture 5.5:
Unit 5 Recap

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We need to formulate 3 equations in 3 unknowns.

\[ p(\vec{r}), n(\vec{r}), V(\vec{r}) \]

or

\[ F_p(\vec{r}), F_n(\vec{r}), V(\vec{r}) \]
Continuity equation

\[ \frac{\partial p}{\partial t} = -\nabla \cdot \frac{\vec{J}_p}{q} + G_p - R_p \]

\[ \vec{J}_p = pq \mu_p \vec{E} - qD_p \nabla p \]

Need an equation for the electric field

optical generation or impact ionization

Radiative, Auger, or defect-assisted
Electrostatics

\[ \int \vec{D} \cdot d\vec{S} = Q \quad \leftrightarrow \quad \nabla \cdot \vec{D} = \rho(x) \]

Gauss’s Law \quad Poisson equation

\[ \vec{D} = K_s \varepsilon_0 \vec{E} \]

\[ \nabla \cdot (K_s \varepsilon_0 \vec{E}) = \rho(\vec{r}) \]

\[ \rho(\vec{r}) = q \left[ p(\vec{r}) - n(\vec{r}) + N_D^+(\vec{r}) - N_A^-(\vec{r}) \right] \]
The “semiconductor equations”

Three equations in three unknowns:

\[
\begin{align*}
\frac{\partial p}{\partial t} &= -\nabla \cdot \left( \frac{\mathbf{J}_p}{q} \right) + G_p - R_p \\
\frac{\partial n}{\partial t} &= -\nabla \cdot \left( \frac{\mathbf{J}_n}{-q} \right) + G_n - R_n \\
\nabla \cdot (K_s \varepsilon_0 \mathbf{E}) &= \rho
\end{align*}
\]

\[
\begin{align*}
\mathbf{J}_p &= pq \mu_p \mathbf{E} - qD_p \nabla p = p \mu_p \nabla F_p \\
\mathbf{J}_n &= nq \mu_n \mathbf{E} + qD_n \nabla n = n \mu_n \nabla F_n \\
\rho &= q \left( p - n + N_D^+ - N_A^- \right) \\
\mathbf{E} (\mathbf{r}) &= \nabla V (\mathbf{r})
\end{align*}
\]
An energy band diagram is a plot of the bottom of the conduction band and the top of the valence band vs. position.

An energy band diagram is a powerful tool for understanding semiconductor devices because they provide **qualitative solutions to the semiconductor equations**.
An important principle

The Fermi level is constant in equilibrium.

The starting point for drawing energy band diagrams.
Band diagrams

Drawing the band diagram

Reading the band diagram

\[ V(x) \propto -E_C(x) \]
\[ \mathcal{E} \propto \frac{dE_C(x)}{dx} \]
\[ \log n(x) \propto E_F - E_i(x) \]
\[ \log \rho(x) \propto E_i(x) - E_F \]
\[ \rho(x) \propto \frac{d^2E_C}{dx^2} \]
Drawing equilibrium band diagrams

1) Begin with $E_F$

2) Draw the E-bands where you know the carrier density

3) Then add the rest
Equilibrium vs. non-equilibrium

**Equilibrium**

\[ n_0 = n_i e^{(E_F - E_i) / k_B T} \]
\[ p_0 = n_i e^{(E_i - E_p) / k_B T} \]
\[ n_0 p_0 = n_i^2 \]
\[ f_0 = \frac{1}{1 + e^{(E - E_F) / k_B T}} \]

**Non-equilibrium**

\[ n = n_i e^{(F_n - E_i) / k_B T} \]
\[ p = n_i e^{(E_i - F_p) / k_B T} \]
\[ n p = n_i^2 e^{(F_n - F_p) / k_B T} \]
\[ f_c = \frac{1}{1 + e^{(E - F_n) / k_B T}} \]
\[ 1 - f_v = 1 - \frac{1}{1 + e^{(E - F_p) / k_B T}} \]

Lundstrom: 2018
Solving the semiconductor equations

\[ \frac{\partial p}{\partial t} = -\nabla \cdot \left( \frac{\vec{J}_p}{q} \right) + G_p - R_p \]

\[ \frac{\partial n}{\partial t} = -\nabla \cdot \left( \frac{\vec{J}_n}{-q} \right) + G_n - R_n \]

\[ \nabla \cdot \left( K_s \varepsilon_0 \vec{E} \right) = \rho \]

For problems that focus on minority carriers in low-level injections, these equations can be simplified, so that we only need to solve the minority carrier diffusion equation (MCDE).
Example: Minority hole diffusion equation

\[
n = n_0 = N_D
\]

\[
\frac{\partial p}{\partial t} = -\nabla \cdot \left( \frac{J_p}{q} \right) + G_p - R_p
\]

\[
\frac{\partial p}{\partial t} = -\frac{d}{dx} \left( \frac{J_{px}}{q} \right) + G_L - R_p
\]

\[
\frac{\partial \Delta p}{\partial t} = -\frac{d}{dx} \left( -qD_p \frac{d\Delta p}{dx} / q \right) + G_L - \frac{\Delta p}{\tau_p}
\]

\[
\frac{\partial \Delta p}{\partial t} = D_p \frac{d^2 \Delta p}{dx^2} - \frac{\Delta p}{\tau_p} + G_L
\]

(N-type semiconductor in low level injection)

(hole continuity equation)

(1D, generation by light)

(low-level injection, no electric field)

\((D_p \text{ spatially uniform})\)

Lundstrom: 2018
General procedure

1) Write down the MCDE

\[ \frac{\partial \Delta p(x,t)}{\partial t} = D_p \frac{d^2 \Delta p(x,t)}{dx^2} - \frac{\Delta p(x,t)}{\tau_p} + G_L \]

N-type

\[ \frac{\partial \Delta n(x,t)}{\partial t} = D_n \frac{d^2 \Delta n(x,t)}{dx^2} - \frac{\Delta n(x,t)}{\tau_n} + G_L \]

P-type

2) Simplify the MCDE for the specific problem

3) Solve the MCDE for the excess minority carrier density

4) Deduce QFL from the excess minority carrier density
General features of MCDE solutions

Transient solutions goes as $\exp[-t/T_n]$

For long regions, steady-state spatial solutions go as $\exp[-x/L_n]$ in a long region

For short regions, steady-state solutions are linear.
Summary

1) Direct, numerical solutions
2) Qualitative solutions with energy band diagrams
3) Simplified, analytical solutions.

\[
\frac{\partial p}{\partial t} = -\nabla \cdot \left( \frac{\vec{J}_p}{q} \right) + G_p - R_p
\]

\[
\frac{\partial n}{\partial t} = -\nabla \cdot \left( \frac{\vec{J}_n}{-q} \right) + G_n - R_n
\]

\[
\nabla \cdot (K_S \varepsilon_0 \vec{E}) = \rho
\]

\[
\vec{J}_p = p\mu_p \nabla F_p = pq\mu_p \vec{E} - qD_p \nabla p
\]

\[
\vec{J}_n = n\mu_n \nabla F_n = nq\mu_n \vec{E} + qD_n \nabla n
\]

\[
\rho = q \left( p - n + N^+_D - N^-_A \right)
\]

\[
\vec{E} (\vec{r}) = \nabla V (\vec{r})
\]
Vocabulary

Built-in potential
Continuity equation
Diffusion length
Einstein relation
Energy band diagram
Gauss’s Law
Low level injection
Minority carrier diffusion equation
Poisson equation
Relative dielectric constant
Quasi-Fermi levels
Quasi-neutrality
Semiconductor equations