Essentials of MOSFETs

Unit 3: MOS Electrostatics

Lecture 3.1: Energy Band Diagram Approach

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Before developing analytical expressions, we should understand MOS electrostatics from an energy band perspective.

Unit 3: electrostatics

\[ I_D/W = |Q_n(V_{GS}, V_{DS})| \langle v_x(V_{GS}, V_{DS}) \rangle \]

Unit 4: transport
The potential, $\Psi(x,y)$, in the semiconductor is controlled by the voltages applied to the terminals.

\[
E_C(x,y) = E_{C0} - q\Psi(x,y)
\]

\[
E_V(x,y) = E_{V0} - q\Psi(x,y)
\]
Goal: Find: \( \psi(x, y) \)

Solve the Poisson equation:

\[
\nabla \cdot \vec{D}(x, y) = \rho(x, y)
\]

\[
\vec{D} = \varepsilon_s \vec{E} = \kappa_s \varepsilon_0 \vec{E}
\]

\[
\vec{E} = -\nabla \psi
\]

\[
\rho(x, y) = q \left[ p(x, y) - n(x, y) + N_D(x, y) - N_A(x, y) \right]
\]

Drawing an energy band diagram provides us with a qualitative solution to the Poisson equation.
The 1D MOS Capacitor

\[ V_D = 0 \]

\[ V_G \]

\[ V_S = 0 \]

“MOS capacitor”

n-Si

p-Si

Lundstrom: 2018
Electrostatic potential vs. position

$V_G > 0$

$p_0(x) < N_A^-$

$p_0(x) \approx N_A^-$

$E_C(y) = E_{C0} - q\psi(y)$

$\psi(x \to \infty) = 0$

Lundstrom: 2018
Band bending

\[ n_0(y) = N_C e^{(E_F - E_C(y)) / k_B T} \]

\[ E_C(y) = E_{C0} - q\psi(y) \]

\[ E_F \]

\[ E_V(y) = E_C(y) - E_G \]

\[ p_0(y) = N_V e^{(E_V(y) - E_F) / k_B T} \]

slope = \( q \times \) electric field

increasing electrostatic potential

Lundstrom: 2018
“Flat-band” conditions

Fermi levels line up at the “flat-band voltage”,

\[ V'_G = 0 \]

(Ignore metal-semi workfunction differences for now.)
Applied gate voltage

\[
E_C(x \to \infty) = E_{C0}
\]

\[\psi(y) = 0 \quad \text{(arbitrary reference for zero potential)}\]

\[
E_C(y) = E_{C0} - q\psi(y)
\]

\[
\psi(y) = \frac{E_{C0} - E_C(y)}{q}
\]

Lundstrom: 2018
\( V'_G < 0: \) “accumulation”

- surface potential < 0
- bands bend up
- hole density increases exponentially near the surface.

\[
p_0(y) = N_V e^{(E_V(y) - E_F)/k_B T}
\]

\[
Q_S = +q \int_0^\infty \left( p_0(y) - N_A^- \right) dy \quad \text{C/cm}^2
\]

(accumulation charge piles up very near the interface)
$V'_G > 0$: “depletion”

- surface potential $> 0$
- bands bend down
- space charge density $y < W_D$:

$$p_0(y) = N_V e^{(E_V(y) - E_F)/k_B T} \approx 0$$

$$n_0(y) = N_C e^{(E_F - E_C(y))/k_B T} \approx 0$$

$$\rho(y) \approx -q N_A^- \left( y < W_D \right) \text{ C/cm}^3$$

“depletion charge”

$$\rho(y) \approx 0 \left( y \geq W_D \right) \text{ C/cm}^3$$
\[ V'_G = V'_T: \text{ onset of “inversion”} \]

Electron concentration in the bulk:

\[ n_B = n_i^2 / N_A \ll p_B \]

Electron concentration at the surface:

\[ n_0(y = 0) = N_C e^{(E_F - E_C(0))/k_B T} = n_B e^{q \psi_s / k_B T} \]

Band bending to make electron concentration at the surface = hole concentration in the bulk:

\[ n_B e^{q \psi_s / k_B T} = N_A \text{ surface is “inverted”} \]
Onset of “inversion”

\[ n(0) = p_B \]
\[ V'_G = V'_T \]
\[ q\psi_S = 2 q\psi_B \]

\[ p_B = n_i e^{(E_i - E_F)/k_B T} = N_A \]
\[ p_B = n_i e^{q\psi_B/k_B T} = N_A \]

\[ \psi_S = 2\psi_B \]
\[ \psi_B = \frac{k_B T}{q} \ln \left( \frac{N_A}{n_i} \right) \]

Lundström: 2018
\[ V'_G > V_T: \text{“inversion”} \]

\[ Q_S = Q_D + Q_n \]

\[ \psi_S \approx 2\psi_B \quad \psi_B = \frac{k_B T}{q} \ln \left( \frac{N_A}{n_i} \right) \]

Hard to bend the bands further.

\[ W_T = \sqrt{2 \varepsilon_S (2\psi_B) / qN_A} \]

Electron charge piles up very near to the surface.

Inversion (mobile) charge

-qn(y)

depletion charge

-\(qN_A^-\)
Total charge in semiconductor vs. surface potential

\[
\begin{align*}
\log_{10} |Q_S(\psi_S)| &\sim e^{-q\psi_S/2k_BT} \\
C/cm^2 &\sim \sqrt{\psi_S} \\
\end{align*}
\]

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Exercise

Re-do the previous two slides for an n-type semiconductor.
Summary

accumulation

$\psi_S < 0$

$V'_G < 0$

$E_C$

$E_F$

$E_V$

flatband

$\psi_S = 0$

$V'_G$

$E_C$

$E_F$

$E_V$

depletion/inversion

$\psi_S > 0$

$V'_G > 0$

$E_C$

$E_F$

$E_V$

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Our goal is to solve the Poisson equation for $\psi(x, y)$.

In general, a numerical solution is required, but

In depletion, we can solve the problem analytically using the **depletion approximation**.