Week 4 Summary

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1) About phonons
2) Electron-phonon coupling
3) Other scattering mechanisms
harmonic oscillator: mass and spring

\[ U = \frac{1}{2} k (x - x_0)^2 \]

\[ F = -\frac{dU}{dx} = -k(x - x_0) \]

\[ \frac{d^2x}{dt^2} = -k(x - x_0) \]

\[ x(t) - x_0 = Ae^{i\omega t} \]

\[ \omega = \sqrt{k/M} \]

\[ E \sim A^2 \]
LA phonons

![Graph showing phonon dispersion](image)

\[ \tilde{u}(x) = A \hat{e}_v e^{i(\beta x - \omega t)} \]

- Dispersion relation:
  \[ \omega(\beta) = 2 \sqrt{\frac{k}{M}} \frac{|\sin \frac{\beta a}{2}|}{a} \]

- Velocity:
  \[ v_s = a \sqrt{\frac{k}{M}} \]

- Approximate velocity:
  \[ \frac{\omega}{\beta} = v_s = \sqrt{\frac{c_s}{\rho}} \approx 8.4 \times 10^5 \text{ cm/s} \]

(Si)
LO phonons

\[ \ddot{u}(x) = A \dot{e}_\nu e^{i(\beta x - \omega t)} \]

the two atoms in a unit cell oscillate out of phase
phonon dispersion

\[ \sqrt{2k/m} \]
\[ \sqrt{2k/M} \]

\( \omega \)

\( \sqrt{2k \left( \frac{1}{m} + \frac{1}{M} \right)} \)

(LO)

(LA)

\( \pi/a \)

\( \pi/a \)

Lundstrom ECE-656 F13
simplified phonon dispersion

1) Longitudinal modes couple most strongly with electrons.

2) **Intra**valley scattering requires small β.

3) **Inter**valley requires β near the Brillouin zone boundary.

\[ \omega = v_s \beta \]

“intervalley phonons”
amplitude of a lattice (phonon) vibration

\[ \langle E_{\text{tor}} \rangle = \Omega \rho 2|A|^2 \omega^2 \]  \hspace{1cm} \text{(classically)}

quantum mechanics says:  \( \langle E_{\text{tor}} \rangle = \hbar \omega \)

\[ |A|^2 = \frac{\hbar}{2 \Omega \rho \omega} \]

\[ u_j = (A e^{i(\beta_j - \omega t)} + c.c.) = 2 |A| \cos(\beta_j \omega - \omega_j t) \]

\[ u_j = \sqrt{\frac{\hbar}{2 \Omega \rho \omega}} \cos(\beta_j \omega - \omega_j t) \quad x = aj \quad j = 1, 2, 3, \ldots \]
1) About phonons

2) Electron-phonon coupling

3) Other scattering mechanisms
electron-phonon coupling (LA)

the bandgap depends on lattice constant: \( \delta E_{\text{a}} = D \frac{\delta a}{a} \)

"deformation potential"

\( \delta E_c = D_c \frac{\delta a}{a} \)

LA phonons:

near \( \beta = 0 \):

\[ \delta a = u(x) - u(x-a) = u(x) - \left( u(x) - \frac{\partial u}{\partial x} a \right) = \frac{\partial u}{\partial x} a \]

\[ \frac{\delta a}{a} = \frac{\partial u}{\partial x} \quad \text{"strain"} \]

\[ U_s = \delta E_c = D_c \frac{\delta u}{a} \]
deformation potential scattering

\[ \frac{\delta a}{a} = \frac{\partial u}{\partial x} \quad \text{“strain”} \quad U_s = \delta E_c = D_c \frac{\delta a}{a} = D_c \frac{\partial u_\beta}{\partial x} \]

\[ u_\beta(x,t) = A_\beta e^{i(\beta t - \omega t)} \]

\[ U_s = D_A \frac{\partial u_\beta}{\partial x} = \pm i \beta D_A u_\beta = K_\beta u_\beta \]

\[ |K_\beta|^2 = \beta^2 D_A^2 \]

“acoustic deformation potential scattering (ADP)”
electron-phonon coupling (LO)

\[ \delta E_c = D_c \frac{\delta a}{a} \]

\[ \delta a(x) = u_\beta(x) \]

\[ u_\beta(x,t) = A_\beta e^{-i(\beta z - \omega t)} \]

\[ U_s \approx D_\beta u_\beta = K_\beta u_\beta \]

\[ |K_\beta|^2 = D_\beta^2 \]

“optical deformation potential scattering (ODP)”

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optical phonons in polar semiconductors

\[ U_s = K_\beta u_\beta \]

\[ u_\beta(x,t) = A_\beta e^{\pm i(b_2 - c_2)} \]

small angle scattering dominates

“polar optical phonon scattering”

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acoustic phonons in polar semiconductors

\[ U_{\delta} = K_{\delta} \mu_{\delta} \]

\[ u_{\beta}(x,t) = A_{\beta} e^{\pm i(\beta z - \omega t)} \]

\[ |K_{\beta}|^2 = \frac{q^2 \varepsilon_{p\beta}^2}{\kappa_0^2 \varepsilon_0^2} \]

“piezoelectric scattering”
electron-phonon scattering potentials

\[ u_\beta(\vec{r},t) = A_\beta e^{\pm i(\vec{\beta}\cdot\vec{r} - \omega_\beta t)} \]

\[ U_s = K_\beta u_\beta \]

- ADP \[ |K_\beta|^2 = \beta^2 D_A^2 \]
- ODP \[ |K_\beta|^2 = D_0^2 \]
- PZ \[ |K_\beta|^2 = \left(\frac{q e_{PZ}}{\kappa_0 \varepsilon_0}\right)^2 \]
- POP \[ |K_\beta|^2 = \frac{\rho q^2 \omega_0^3}{\beta^2 \kappa_0 \varepsilon_0} \left(\frac{\kappa_0}{\kappa_{\infty}} - 1\right) \]
topics

1) About phonons

2) Electron-phonon coupling

3) Other scattering mechanisms
other scattering mechanisms

1) Neutral impurity
2) Alloy scattering
3) Surface / edge roughness scattering
4) Plasmon scattering
5) Electron-electron scattering
6) Electron-hole