1) Consider a semiconductor with a slowly varying effective mass, \( m^*(x) \). Following the procedure in Lecture 23 (Lecture 14, Fall 2011) derive the equation of motion for an electron in \( k \)-space analogous to the result for a constant effective mass.

\[
\frac{d(hk_x)}{dt} = F_e = -\frac{dE_C(x)}{dx}
\]

2) Consider a semiconductor with a position dependent effective mass and electron affinity, \( \chi(x) \), so that

\[
E_C(x) = E_{\text{vac}} - \chi(x) - qV(x),
\]

where \( E_{\text{vac}} \) is a constant, reference energy (the vacuum level) and \( V(x) \) is the electrostatic potential.

Solve the steady-state BTE in the relaxation time approximation and compare your result to the result obtained in Lecture 23 (Lecture 14, Fall 2011).

3) In Lecture 24 (Lecture 15, Fall 2011) we stated that for power law scattering in 2D

\[
\langle \langle \tau_m \rangle \rangle = \tau_0 \frac{\Gamma(s + 2)}{\Gamma(2)},
\]

where \( s \) is the characteristic exponent in the expression

\[
\tau(E - E_C) = \tau_0 \left[ \left( \frac{E - E_C}{k_B T} \right)^s \right]^
u,
\]

and nondegenerate conditions are assumed. Prove this result.

3a) Work out the corresponding result in 1D.
3b) Work out the corresponding result in 3D.
4) On slide 32 of Lecture 24 (Lecture 15, Fall 2011) expressions for the tensors, $\sigma_{ij}$ and $[s_T]_{ij}$, were given. Determine the corresponding expression for $[\kappa_0]_{ij}$.

5) Solve the two equations below for the electric fields in the $x$- and $y$-directions and show that the results on slide 23 of Lecture 25 (Lecture 16, Fall 2011) are correct.

$$J_x = \sigma_n E_x - (\sigma_n \mu_n r_n^e) E_y B_z$$
$$J_y = \sigma_n E_y + (\sigma_n \mu_n r_n^e) E_x B_z$$

6) This homework exercise will help you become familiar with how $B$–fields affect transport.

Consider the equation of motion for an average electron,

$$\dot{\mathbf{F}}_e = -q\mathbf{E} - q\mathbf{v} \times \mathbf{B} = \frac{d\mathbf{p}}{dt}. \quad \text{(i)}$$

Assume that the electron moves for a time, $\tau_m$, then scatters, returning the average momentum to zero, so

$$\frac{d\mathbf{p}}{dt} = -\frac{\mathbf{p}}{\tau_m}. \quad \text{(ii)}$$

Assuming that $\bar{p} = m\bar{\mathbf{v}}$, we find an equation for the average velocity as

$$\bar{\mathbf{v}} = -\frac{q\tau_m}{m} \mathbf{E} - \frac{q\tau_m}{m} \bar{\mathbf{v}} \times \mathbf{B}. \quad \text{(iii)}$$

This equation can be solved exactly for the velocity (see prob. 4.18, Lundstrom, *Fundamentals of Carrier Transport*, 2000), but let’s take a different approach.

6a) Assume carriers move in 2D and that only a z-directed $B$-field is present.

Evaluate eqn. (iii) and find two coupled equations for $v_x$ and $v_y$.

6b) Solve the two equations for $v_x$ and $v_y$ in terms of the electric field and the $B$-field.
6c) Write the current densities as

\[ J_x = -n_s q v_x \]  
\[ J_y = -n_s q v_y \]

and use the results of problem 6b) to find the current densities as

\[ J_x = \frac{\sigma_n}{1 + (\mu_n B_z)^2} \left( \mathcal{E}_x - \mu_n B \mathcal{E}_y \right) \]  
\[ J_y = \frac{\sigma_n}{1 + (\mu_n B_z)^2} \left( \mathcal{E}_x + \mu_n B \mathcal{E}_y \right), \]

which can also be written as

\[ \begin{pmatrix} J_x \\ J_y \end{pmatrix} = \frac{\sigma_n}{1 + (\mu_n B_z)^2} \begin{pmatrix} 1 & -\mu_n B_z \\ \mu_n B_z & 1 \end{pmatrix} \begin{pmatrix} \mathcal{E}_x \\ \mathcal{E}_y \end{pmatrix} \]  

or as

\[ J_i = \sigma_{ij} (B_z) \mathcal{E}_j. \]

Note that the magnetic field affects both the diagonal and off-diagonal components of the magnetoconductivity tensor. **Explain** why there is no Hall factor, \( r_H \), in the result.

6d) Solve eqn. (via) for the electric field and show that

\[ \begin{pmatrix} \mathcal{E}_x \\ \mathcal{E}_y \end{pmatrix} = \frac{1}{\sigma_n} \begin{pmatrix} 1 & \mu_n B_z \\ -\mu_n B_z & 1 \end{pmatrix} \begin{pmatrix} J_x \\ J_y \end{pmatrix} \]

According to eqn. (vii), the longitudinal magnetoconductivity is independent of the \( B \)-field (while the longitudinal magnetoconductivity depends on \( B \) as shown in eqn. (via)). Equation (vii) shows that the Hall voltage is proportional to \( B \).

6e) Show that for small \( B \)-fields, eqn. (via) can be written as

\[ \mathcal{J}_n = \sigma_n \mathbf{E} - (\sigma_n \mu_n) \mathbf{E} \times \mathbf{B} \]

Note that while this analysis is simpler than solving the BTE, by beginning with an average electron with an average momentum, \( \bar{p} \), we have missed the averaging of the distribution of momenta which leads to a non-unity Hall factor, \( r_H \).
7) Hall factors are important to consider when analyzing experiments. Answer the following questions.

7a) Derive an expression for the Hall factor in 3D and show that for ionized impurity scattering, it gives $r_H = 1.93$.

7b) Develop an expression for the Hall factor in 2D. Is it the same for parabolic energy bands and for graphene?

7c) Ionized impurity scattering in graphene is said to vary as $E$. What is the Hall factor for graphene?

8) We saw from the solution to the Boltzmann equation (Lecture 24 (Lecture 15, Fall 2011)), that the diffusion coefficient is given by

$$D_n = \left\langle v_i^2 \tau_m \right\rangle$$

Use this result from the BTE to find the definition of the mean-free-path for backscattering in the Landauer picture, i.e. what is the definition of $\lambda(E)$?