SOLUTIONS: ECE 656 Homework (Weeks 15-16)

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1) Monte Carlo simulations of high-field transport in bulk silicon with an electric field of 100,000 V/cm show the following results for the average velocity and kinetic energy:

$$\upsilon_d = \langle \upsilon \rangle = 1.04 \times 10^7 \text{ cm/s}$$
$$u = \langle KE \rangle = 0.364 \text{ eV}$$

Estimate the average momentum relaxation time, $\langle \tau_m \rangle$ and energy relaxation time, $\langle \tau_E \rangle$.

Solution:

$$\mu_n = \langle \upsilon \rangle / \mathcal{E} = 104 \text{ cm}^2/\text{V-s}$$
$$\mu_n = \frac{q\langle \tau_m \rangle}{m_c^*},$$

where m_c^* is the conductivity effective mass. Use MKS units for the calculation:

$$\langle \tau_m \rangle = \frac{q}{\mu_n m_c^*} = \frac{\mu_n m_c^*}{q} = \frac{104 \times 10^{-4} \times 0.23 \times 9.11 \times 10^{-31}}{1.6 \times 10^{-19}} 1.36 \times 10^{-14}$$

$$\boxed{\langle \tau_m \rangle = 1.36 \times 10^{-14} \text{ s}}$$

From the energy balance equation:

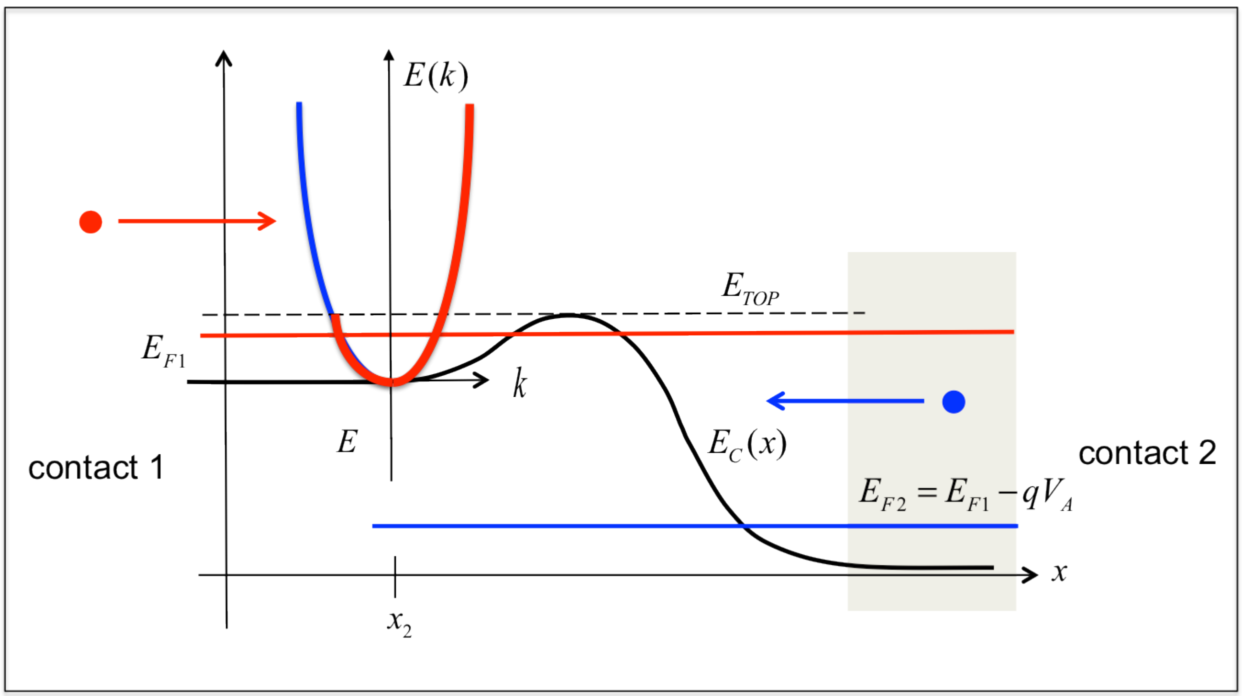
$$J_x \mathcal{E}_x = nq\langle \upsilon \rangle \mathcal{E}_x = \frac{n(u-u_0)}{\langle \tau_E \rangle}$$
$$\langle \tau_E \rangle = \frac{(u-u_0)}{q\langle \upsilon \rangle \mathcal{E}_x}$$
$$\frac{u_0}{q} = 1.5\frac{k_B T_L}{q} = 0.039$$
$$\langle \tau_E \rangle = \frac{(u-u_0)}{q\langle \upsilon \rangle \mathcal{E}_x} = \frac{(0.364-0.039)}{1.04 \times 10^7 \times 10^5} = 0.313 \times 10^{-12}$$

$$\boxed{\langle \tau_E \rangle = 0.3 \text{ ps}}$$

2) The suggested exercise on slide 32 of Lecture 39a (Lecture 29 Fall 2011) asks how the states are occupied in the source of a ballistic nanowire (1D) MOSFET. Answer the following questions.

2a) Draw a sketch like that in slide 29 of the lecture, but illustrate how the states in the $E(k)$ are occupied from contact 1 (left) or contact 2 (right).

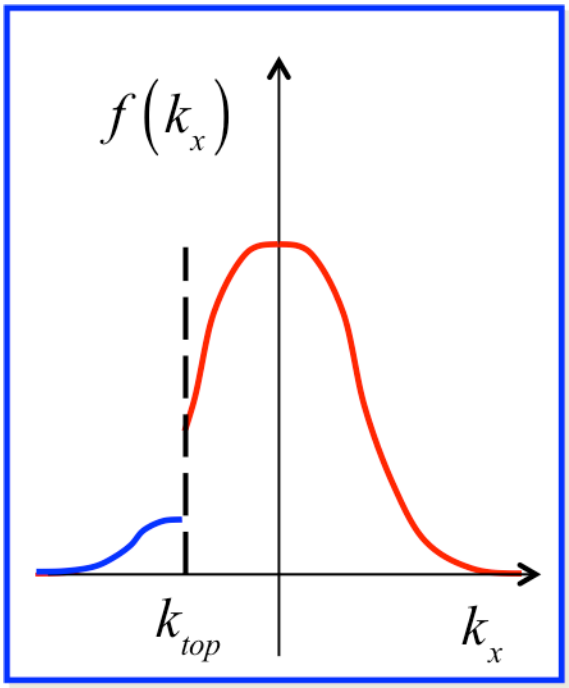
Solution:



red states: field by contact 1
blue states: filled by contact 2

2b) Sketch the corresponding distribution function, $f(k_x)$, assuming Boltzmann statistics.

Solution:



2c) Give analytical expressions for the local density of states in the source. Assume a 2D density of states and express your answer in terms of E_{TOP}, the energy at the top of the barrier.

Solution:
The 1D density-of-states when both +k and –k states are filled is:

$$D_{1D}(E) = \frac{1}{\pi\hbar}\sqrt{\frac{2m^*}{\left[E - E_C(x)\right]}}$$

in this case, for contact 1:

$$E_C < E < E_{TOP}: \quad D_{1D}^1(E) = \frac{1}{\pi\hbar}\sqrt{\frac{2m^*}{\left[E - E_C(x)\right]}} \qquad D_{1D}^2(E) = 0$$

$$E > E_{TOP}: \quad D_{1D}^1(E) = \frac{1}{2\pi\hbar}\sqrt{\frac{2m^*}{\left[E - E_C(x)\right]}} \qquad D_{1D}^2(E) = \frac{1}{2\pi\hbar}\sqrt{\frac{2m^*}{\left[E - E_C(x)\right]}}$$

3) The classic expression for the base transit time of a bipolar transistor is $W_B^2/2D_n$. This expression does not comprehend ballistic or quasi-ballistic transport. High frequency transistors have thin base widths for which these effects could become important. Answer the following questions.

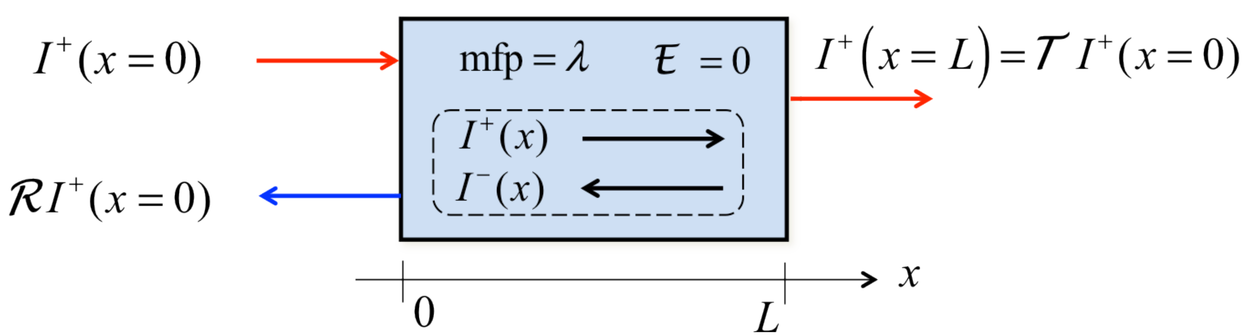
3a) Derive an expression for the base transit time when the base is one mean-free-path thick and compare it to the classic expression, $W_B^2/2D_n$.

3b) Estimate the thickness of a Si base doped at $N_A = 10^{18}\ \text{cm}^{-3}$ if it is one mean-free-path thick.

HINT: review the discussion in Sec. III and IV of Lecture 12 Fall 2011.

Solution:

Consider a slab of length, *L*, with an absorbing contact at *x* = *L* and a flux, $I^+(0)$, injected from the left.



The flux that emerges at the right is

$$I^+(x=L)=\mathcal{T}\,I^+(0) \tag{i}$$

where

$$\mathcal{T}(E)=\frac{\lambda(E)}{\lambda(E)+L} \tag{ii}$$

At the right:

$$n^+\left(0\right) = I^+(0)\Big/\left\langle \upsilon_x^+ \right\rangle \qquad \text{(iii)}$$

$$n^-\left(0\right) = I^-(0)\Big/\left\langle \upsilon_x^+ \right\rangle \qquad \text{(iv)}$$

$$n\left(0\right) = \left(1+\mathcal{R}\right)I^+(0)\Big/\left\langle \upsilon_x^+ \right\rangle \qquad \text{(v)}$$

$$n\left(0\right) = \left(2-\mathcal{T}\right)I^+(0)\Big/\left\langle \upsilon_x^+ \right\rangle \qquad \text{(vi)}$$

At the left:

$$n^+\left(L\right) = I^+(L)\Big/\left\langle \upsilon_x^+ \right\rangle \qquad \text{(vii)}$$

$$n^-\left(0\right) = 0 \qquad \text{(viii)}$$

$$n\left(L\right) = I^+(L)\Big/\left\langle \upsilon_x^+ \right\rangle \qquad \text{(ix)}$$

$$n\left(L\right) = \mathcal{T}\, I^+(0)\Big/\left\langle \upsilon_x^+ \right\rangle \qquad \text{(x)}$$

$$n\left(0\right) - n\left(L\right) = \frac{\left(2-\mathcal{T}\right)I^+(0)}{\left\langle \upsilon_x^+ \right\rangle} - \frac{\mathcal{T}\, I^+(0)}{\left\langle \upsilon_x^+ \right\rangle}$$

$$n\left(0\right) - n\left(L\right) = \frac{I^+(0)}{\left\langle \upsilon_x^+ \right\rangle} 2\left(1-\mathcal{T}\right) \qquad \text{(xi)}$$

According to (i) $I = \mathcal{T}\, I^+(0)$

In a charge control model

$$I = \frac{N}{t_t} \qquad \text{(xii)}$$

where N is the total number of electrons in the slab (per unit cross-sectional area) and t_t is the transit time.

$$N = \frac{1}{2}\left[n\left(0\right) + n\left(L\right)\right]L = \frac{1}{2}\left[\left(2-\mathcal{T}\right)I^+(0)\Big/\left\langle \upsilon_x^+ \right\rangle + \mathcal{T}\, I^+(0)\Big/\left\langle \upsilon_x^+ \right\rangle\right]L = \frac{I^+(0)L}{\left\langle \upsilon_x^+ \right\rangle}$$

(xiii)

Now from the charge control model

$$t_t = \frac{N}{I} = \frac{\left(\dfrac{I^+(0)L}{\left\langle \upsilon_x^+ \right\rangle}\right)}{\mathcal{T}\, I^+(0)} = \frac{L}{\left\langle \upsilon_x^+ \right\rangle}\frac{1}{\mathcal{T}} = \frac{L}{\left\langle \upsilon_x^+ \right\rangle}\frac{\lambda + L}{\lambda} = \frac{L}{2D_n}\left(\lambda + L\right) \qquad \text{(xiv)}$$

where

$$D_n = \frac{\langle \upsilon_x^+ \rangle \lambda}{2} \qquad \text{(xv)}$$

From (xiv), we write:

$$t_t = \frac{L^2}{2D_n}\left(1 + \lambda/L\right). \qquad \text{(xvi)}$$

In the diffusive limit: $L >> \lambda$, we get the diffusive transit time:

$$t_t = t_{diff} = \frac{L^2}{2D_n}. \qquad \text{(xvii)}$$

In the ballistic limit: $L << \lambda$, and we get the ballistic transit time:

$$t_t = t_{ball} = \frac{L\lambda}{2D_n} = \frac{L}{\langle \upsilon_x^+ \rangle}. \qquad \text{(xviii)}$$

In general,

$$\boxed{t_t = t_{diff} + t_{ball}}\ . \qquad \text{(xix)}$$

The longer of the ballistic or diffusive transit times is the one that matters.

3b) Estimate the thickness of a Si base doped at $N_A = 10^{18}\ \text{cm}^{-3}$ if it is one mean-free-path thick.

Solution:

From Pierret, Advanced Semiconducor Fundamentals, 2nd Ed., p. 184, we find

$$\mu_n \approx 250\ \text{cm}^2/\text{V-s}$$

$$D_n = \frac{k_B T}{q}\mu_n \approx 6.5\ \text{cm}^2/\text{s}$$

$$D_n = \frac{\upsilon_T \lambda}{2}$$

$$\lambda = \frac{2D_n}{\upsilon_T} = \frac{2 \times 6.5}{1.04 \times 10^7} = 12.5\ \text{nm}$$

$$\boxed{W_B = \lambda = 12.5\ \text{nm}}$$