NAME:______________________________ **PUID: :**______________________________

SOLUTIONS: ECE 656 Exam 6: Fall 2013

December 9, 2013

Mark Lundstrom
Purdue University

This is a closed book exam. You may use a calculator and the formula sheet at the end of this exam.

There are three equally weighted questions. To receive full credit, you must **show your work** (scratch paper is attached).

The exam is designed to be taken in 50 minutes.

Be sure to fill in your name and Purdue student ID at the top of the page.

DO NOT open the exam until told to do so, and stop working immediately when time is called.

The last page is an equation sheet, which you may remove, if you want.

30 points possible, 10 per question

1) 2 points per part – 10 points total

2a) 3 points
2b) 4 points
2c) 3 points

3a) 3 points
3b) 2 points
3c) 3 points
3d) 2 points

Answer the **five multiple choice questions** below by choosing the **one, best answer**.

1.1) In practice, determine the high field diffusion coefficient from $D_n(\mathcal{E}) = (k_B T_L / q)\mu_n(\mathcal{E})$. What happens as a result of this approach?

a) The resulting $D_n(\mathcal{E})$ is too small.

b) The resulting $D_n(\mathcal{E})$ is about right.

c) The resulting $D_n(\mathcal{E})$ is too large.

d) The resulting $D_n(\mathcal{E})$ is too small for Si and too large for GaAs.

e) The resulting $D_n(\mathcal{E})$ is too large for Si and too small for GaAs.

1.2) How does the drift energy in a 3D, bulk semiconductor under high electric field compare to the thermal energy?

a) The drift energy is much smaller than the thermal energy.
b) The drift energy is about one-third of the thermal energy..
c) The drift energy is about two-thirds of the thermal energy...
d) The drift energy is about equal to the thermal energy...
e) The drift energy is much larger than the thermal energy...

1.3) How does the peak velocity in a temporal velocity overshoot compare to the peak velocity in a steady-state, spatial velocity overshoot?

a) Temporal velocity overshoot is typically smaller that steady-state velocity overshoot.
b) Temporal velocity overshoot is typically larger than steady-state velocity overshoot.
c) Temporal velocity overshoot is typically about the same as steady-state velocity overshoot.
d) Temporal velocity overshoot is typically larger than s.s. velocity overshoot in Si and smaller than s.s. velocity overshoot in GaAs.
e) Temporal velocity overshoot is typically smaller than s.s. velocity overshoot in Si and larger than s.s. velocity overshoot in GaAs.

1.4) Which of the following is true about a ballistic device with two, ideal Landauer contacts at different voltages?

a) The distribution function in the device is a Fermi-Dirac distribution with the average Fermi level of the two contacts .
b) The distribution function in the device is a Fermi-Dirac distribution with the Fermi level of the contact with the more positive potential.
c) The distribution function in the device is a Fermi-Dirac distribution with the Fermi level of the contact with the more negative potential.
d) Each state in the device is in equilibrium with one of the two contacts.
e) Each state in the device is in equilibrium with both the two contacts

1.5) If we write the on-current of a MOSFET as $I_{ON} = WQ_n \langle \upsilon \rangle$. What is the average velocity, $\langle \upsilon \rangle$?

a) The velocity at the source end of the channel.
b) The velocity at the drain end of the channel.
c) The average velocity in the channel.
d) The peak velocity in the channel.
e) The thermal velocity in the source.

2) In this problem, we will try to estimate the average kinetic energy and the energy relaxation time of electrons in bulk, <111> oriented Si under an electric field of 20 kV/cm. Assume that the velocity is saturated at 10^7 cm/s and that the conductivity effective mass is $m_C^* = 0.26 m_0$. You should also assume that the dominant scattering mechanism is optical (or intervalley) phonon scattering with $\hbar\omega_0 = 0.063$ eV and that momentum relaxation is dominated by ADP scattering with $\mu_n = \mu_{n0}\sqrt{T_L/T_e}$, where $\mu_{n0} = 1400$ cm²/V-s, $T_L = 300$ K is the lattice temperature and T_e is the electron temperature.

HINT: Some of this information may be useful in solving this problem.

2a) Determine the electron temperature.

Solution:

$$\mu_n = \frac{\langle \upsilon \rangle}{\mathcal{E}} = \frac{10^7}{2\times 10^5} = 500 \text{ cm}^2/\text{V-s}$$

$$\mu_n = \mu_{n0}\sqrt{T_L/T_e}$$

$$500 = 1400\sqrt{300/T_e}$$

$$T_e = 300\left(\frac{1400}{500}\right)^2 = 2352 \text{ K}$$

$$\boxed{T_e = 2352 \text{ K}}$$

2b) Develop an expression for the energy relaxation time of an average electron in terms of the momentum relaxation time (you will not be able to get a numerical answer at this point).

Solution:

Input energy from the field = power dissipated by scattering

$$J_{nx}\mathcal{E}_x = nq\langle \upsilon \rangle \mathcal{E}_x = \frac{n(u-u_0)}{\langle \tau_E \rangle} = \frac{n\frac{3}{2}k_B(T_e - T_L)}{\langle \tau_E \rangle}$$

$$\langle \tau_E \rangle = \frac{3}{2}\frac{k_B(T_e - T_L)}{q\langle \upsilon \rangle \mathcal{E}_x} = 1.5\times\frac{1.38\times 10^{-23}}{1.6\times 10^{-19}}\times\frac{2352-300}{10^7\times 2\times 10^4} = 1.33\times 10^{-12}$$

$$\boxed{\langle \tau_E \rangle = 1.33 \text{ ps}}$$

2c) Compare (quantitatively) the energy relaxation time to the momentum relaxation time for this example.

Solution:

$$\mu_n = \frac{\langle \upsilon \rangle}{\mathcal{E}} = \frac{10^7}{2\times 10^5} = 500 \text{ cm}^2/\text{V-s}$$

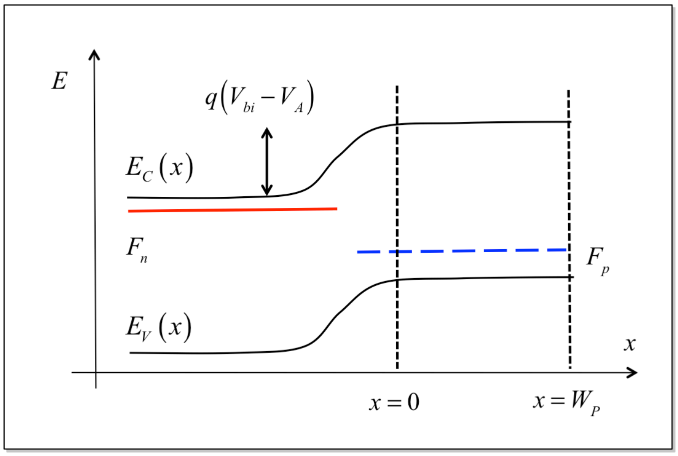
$$\mu_n = \frac{q\langle \tau_m \rangle}{m^*}$$ (use MKS units for the calculation below)

$$\langle \tau_m \rangle = \frac{m^* \mu_n}{q} = \frac{0.26\times 9.11\times 10^{-31}}{1.6\times 10^{-19}} 500\times 10^{-4} = 7.4\times 10^{-14}$$

$$\frac{\langle \tau_E \rangle}{\langle \tau_m \rangle} = \frac{133\times 10^{-14}}{7.4\times 10^{-14}} = 18$$

$$\boxed{\frac{\langle \tau_E \rangle}{\langle \tau_m \rangle} = 18}$$

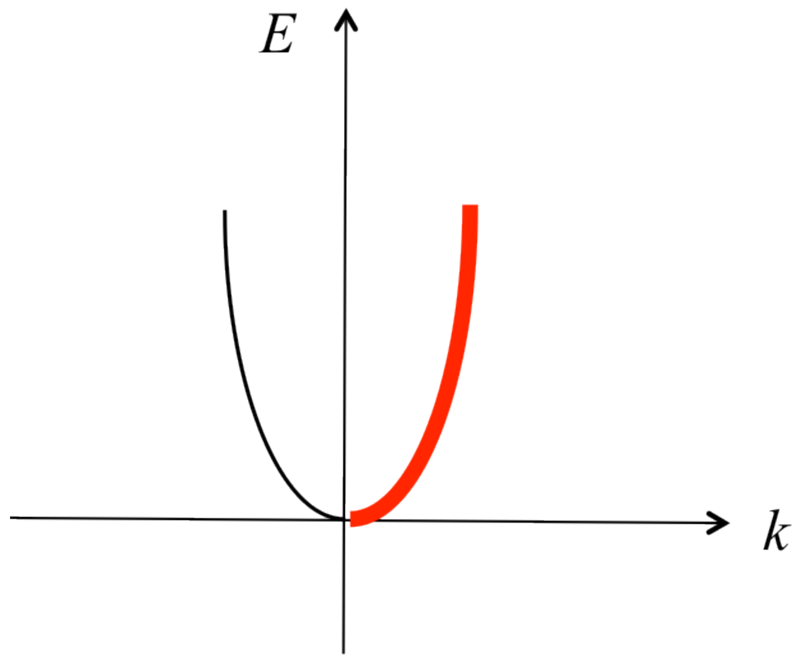
3) Consider the forward biased n⁺p junction sketched below. The classic, short-base theory of the diode gives the current as $I_D = qA\frac{n_i^2}{N_A}\frac{D_n}{W_P}\left(e^{qV_A/k_BT}-1\right)$. (Non-degenerate carrier statistics have been assumed).



3a) Treat the n⁺ region as a Landauer contact and assume that the depletion region is ballistic. Also assume an absorbing contact at $x=W_P$. On the sketch below, which shows the k-states at the beginning of the p-type quasi-neutral region, indicate which k-states are populated from the n⁺ region.

Solution:

All of the +k-states are occupied by electrons coming from the n⁺ region.



3b) Using the results of problem, 3a), present an expression for the magnitude of the electron current injected into the p-type quasi-neutral region, $I_n^+(0)$. (You do not need to solve this expression).

Solution:

$$\boxed{I_n^+(0) = qA\frac{1}{\Omega}\sum_{\vec{k},k_x>0} \upsilon_x f_0(F_n)}$$

where F_n is the electron quasi-Fermi level in the neutral n^+ region.

3c) The diode current is all carried by electrons in an n^+p diode, so $I_D = \mathcal{T} I_n^+(0)$. Evaluate this expression in the classic, diffusive limit and use it to solve for $I_n^+(0)$. HINT: You could get the same answer by evaluating the expressionin 3b), but this approach is easier.

Solution:

$$I_D = \mathcal{T} I_n^+(0) = qA\frac{n_i^2}{N_A}\frac{D_n}{W_P}\left(e^{qV_A/k_BT} - 1\right)$$

$$I_n^+(0) = \frac{1}{\mathcal{T}} qA \frac{n_i^2}{N_A} \frac{D_n}{W_P} \left(e^{qV_A/k_BT} - 1 \right)$$

In the diffusive limit:

$$\mathcal{T} = \frac{\lambda}{\lambda + W_P} \rightarrow \frac{\lambda}{W_P}$$

so

$$I_n^+(0) = \frac{W_P}{\lambda} qA \frac{n_i^2}{N_A} \frac{D_n}{W_P} \left(e^{qV_A/k_BT} - 1 \right) = \frac{D_n}{\lambda} qA \frac{n_i^2}{N_A} \left(e^{qV_A/k_BT} - 1 \right)$$

$$I_n^+(0) = \frac{\upsilon_T \lambda}{2\lambda} qA \frac{n_i^2}{N_A} \left(e^{qV_A/k_BT} - 1 \right) = qA \frac{n_i^2}{2N_A} \upsilon_T \left(e^{qV_A/k_BT} - 1 \right)$$

$$\boxed{I_n^+(0) = qA \frac{n_i^2}{2N_A} \upsilon_T \left(e^{qV_A/k_BT} - 1 \right)}$$

Exam 3: ECE 656 Fall 2013

3d) Use the result of 3c) to develop an expression for the diode current, I_D, that is valid from the ballistic to diffusive limit. Compare your answer to the classic, short base result and discuss the differences.

Solution:

$$I_D = \mathcal{T} I_n^+(0)$$

$$I_D = \frac{\lambda}{\lambda + W_P} I_n^+(0)$$

$$I_D = \frac{\lambda}{\lambda + W_P} qA \frac{n_i^2}{2N_A} \upsilon_T \left(e^{qV_A/k_BT} - 1 \right)$$

$$I_D = \frac{\lambda \upsilon_T}{2(\lambda + W_P)} qA \frac{n_i^2}{N_A} \left(e^{qV_A/k_BT} - 1 \right) = \frac{D_n}{(\lambda + W_P)} qA \frac{n_i^2}{N_A} \left(e^{qV_A/k_BT} - 1 \right)$$

$$I_D == \frac{W_P}{(\lambda + W_P)} qA \frac{n_i^2}{N_A} \frac{D_n}{W_P} \left(e^{qV_A/k_BT} - 1 \right)$$

$$\boxed{I_D = \frac{1}{(1 + \lambda/W_P)} \times \left\{ qA \frac{n_i^2}{N_A} \frac{D_n}{W_P} \left(e^{qV_A/k_BT} - 1 \right) \right\}}$$

i) diffusive limit: $W_P >> \lambda$: $I_D = qA \frac{n_i^2}{N_A} \frac{D_n}{W_P} \left(e^{qV_A/k_BT} - 1 \right)$ (classic result)

ii) ballistic limit: $W_P << \lambda$: $I_D = \frac{W_P}{\lambda} qA \frac{n_i^2}{N_A} \frac{D_n}{W_P} \left(e^{qV_A/k_BT} - 1 \right)$

$$I_D = qA \frac{n_i^2}{2N_A} \upsilon_T \left(e^{qV_A/k_BT} - 1 \right) = I_n^+(0)$$