Homework 1: Variability, Reliability, and Accelerated Testing

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In Lectures 1-3, we have discussed (i) the distinction between variability and reliability, (ii) three classes of reliability analysis – empirical, statistical, and physical, and (iii) the issues of projection, accelerated testing, and solution of a general class of problems called “stochastic process with a threshold.

Part I: Review Questions for Self-Test

1. I suggested in Lecture 1 that ‘Industrial Revolution’ depended on standardization, in particular of screw-threads, and the same is true for integrated circuits. The difference in standards of screw threads between America and UK nearly caused the Allied Army to lose the African front. Find the story to convince yourselves regarding the importance of standardization.

2. If transistors based on poly-silicon has such a large variability issue, then why do people use such transistors for displays and other technologies? Why can they not use crystalline silicon to solve the variability issue?

3. The analysis of variability and reliability often use the same mathematical approaches and theoretical tools. What is the fundamental distinction between variability and reliability that one has to be careful about?

4. What is the difference between front-end and back-end reliability? Can you give some examples? Why do the call some reliability front-end, while other back-end?

5. Why was the first IC from Intel made entirely of PMOS transistors?

6. Why was the HCI problem so dominant in early 1980s and what technique solved the HCI problem?

7. Define three approaches to Reliability Physics? Can you give one example from personal experience that will fall under the respective groups.

8. Reliability became a big issue after WWII. Give three examples that motivated the change.

9. Explain how political/social considerations might help define the threshold or acceptable risk levels for reliability.
10. What does it mean that Reliability is an extreme non-equilibrium problem? Why is device physics that you learn in EE606 and EE612 is not so ‘extreme’ non-equilibrium problem?

11. For the problem of BFRW, why is physical model absolutely essential? Why can I not use statistical or empirical modeling?

12. What do we mean by accelerated testing? What is the danger of accelerated testing?

13. What is difference between Reliability and Risk?

14. In which example in the class did we discuss the statistical reliability model based on ‘fault-tree’ analysis? What type of logic synthesis algorithm can help the analysis of fault trees?

15. What mistaken assumption led the AT&T Bell Labs forgo the research of integrated circuits in 1960s?

Part 2: Descriptive and Statistical Reliability

A system consists of three components A, B, and C. And it has been designed for a 10-day critical mission. You need to decide if you will buy single copies of more expensive components ($15k a piece, 5000 days MTTF, failure probability 0.002), or pair of copies of each ($2k a piece, 1000 days MTTF, failure probability 0.01) and put the pair in parallel, and connect the three pairs in series.

(a) Show that n components connected in series has a probability of not failing is $P_s = p^n$, where $p$ is the probability of not failing for individual components.

(b) Show that the corresponding result for n components connected in parallel is given by $P_s = 1 - (1 - p)^n$.

(c) Use the results in parts (a) and (b) to find the reliability is individual system. Which system is more reliable? Ans. $P_s = 0.994$ and $0.9997$.

(d) Higher reliability is cheaper, but it has an important hidden cost. Can you explain what it could be (Hint: Does replacement only involve product cost?).

(e) If the failure probability of each component is Poisson distributed, $f(t) = \lambda e^{(-\lambda t)}$, so that $MTTF = 1/\lambda$. The survival probability is $R(t) = 1 - \int_0^t f(z)dz = e^{-\lambda t}$. Find the failure probability for more and less expensive components. How these numbers compare with part (c).

(f) How would the result change if we need reliability for 1 year operation.

Part 3: Simulating BFRW Problem

(1) Analytical part:

(a) While the solution in the presence of flow
appears correct and satisfies the boundary condition at \( c(0,t) = 0 \). The image contribution \( (2^{nd} \) term on the right) however moves with the opposite velocity necessary to satisfy the solution for \( x > 0 \). Instead, the following solution satisfies the right boundary condition and velocity for \( x > 0 \), i.e.,

\[
C(x,t) = (1/\sqrt{4\pi D \ t}) \left[ e^{-\frac{(x-x_0-vt)^2}{4Dt}} - e^{-\frac{(x+x_0-vt)^2}{4Dt}} \right]
\]

Prove analytically and plot \( C(x,t) \) for \( x_0 = 2, D = 1, \) and \( v = -0.1 \) to establish that the boundary condition is satisfied at all times.

(b) Use the condition that \( f(t) = (\nu c - D \frac{dc}{dx}) \) at \( x=0 \) to find the arrival time distribution

\[
f(t) = \frac{x_0}{\sqrt{(4 \pi D \ t)^3}} e^{-\frac{(x_0-v \times t)^2}{4Dt}}
\]

Use the previous MATLAB code to confirm particle conservation.

(c) Calculate the moment of the distribution to show that

\[
< t^k > = \int_0^\infty t^k f(t) \ dt
\]

\[
< t > = x_0/\nu,
\]

The average is the same as the one derived by the equation for mean time, while

\[
\sigma^2 = \langle t^2 \rangle - \langle t \rangle^2 = \frac{Dx_0}{\nu^2} = \langle t \rangle^2 / \nu^2
\]

Argue that, as a result, a decoupled analysis of the statistics and mean arrival time is not appropriate.

(d) Confirm numerically that the maximum arrival time is not velocity sensitive and is given by

\[
T_{\text{max}} = \left( \frac{3D}{\nu^2} \right) \left( \sqrt{\left( 1 + \frac{Pe^2}{9} \right)} - 1 \right)
\]
Where Pecht number \( \text{Pe} = \frac{x_0 v}{2D} \equiv \frac{v}{2v_{diff}} \).

(e) What is the purpose of Shockley-Haynes experiment? Convince yourself that the problem above solves the Shockley-Haynes experiment analytically.

**Numerical part:**

Download the MATLAB code called ‘Fish in a River with a waterfall.txt’ and make necessary modifications to answer the following questions.

a) Plot the spatial probability distribution of fishes going over the waterfall as a function of \( p \) to see how the distribution changes with velocity of the flow, i.e., \( p = 0.5 + a \times v \) where \( a \) is some constant and \( v \) is the velocity. How does the numerical solution compare with the analytical solution of the same problem. (Hint. Start with \( p \sim 1 \)).

b) Do you see that as \( p \) approaches 0.5, that the distribution diverges as \( 1/t^n \) for large \( t \)? What is the value of \( n \) do you find from the numerical simulation. Does it compare with the analytical solution of the problem. (Hint. You may need a large number of particles to see this distribution.)

c) Why does the run-time increases so much as we approach \( p=0.5 \)? What does the rise in run time tell you in terms of number of samples needed for statistically relevant accelerated testing close to operating conditions for real ICs?

d) As \( p \) approaches 1, the distribution approaches a delta function. Explain. What is the ballistic transport time for such a system?

c) Determine how the average and standard deviation of the distribution changes with time and sample size.

**References:**