Dilute BD (homework)

Show that \[ L = \frac{e^{\xi x} - 1}{\xi} \]

\[ \frac{dP_m}{dx} = k_{m-1} P_{m-1} - k_m P_m \]

Let \( k_m = 1 + m \xi \)

\[ \frac{dP_m}{dx} = \left[ 1 + (m-1) \xi \right] P_{m-1} - (1 + m \xi) P_m \]

\[ \sum_{m=1}^{\infty} m \frac{dP_m}{dx} = \sum_{m=1}^{\infty} m P_{m-1} + \xi \sum_{m=1}^{\infty} m^2 P_{m-1} - \xi \sum_{m=1}^{\infty} m P_{m-1} \]

\[ - \sum_{m=1}^{\infty} m P_m - \xi \sum_{m=1}^{\infty} m^2 P_m \]

Let \( L = \sum_{m=1}^{\infty} m P_m \)

\[ \frac{dL}{dt} = \sum_{i=0}^{\infty} (i+1) P_i + \xi \sum_{i=0}^{\infty} (i+1)^2 P_i - \xi \sum_{i=0}^{\infty} (i+1) P_i \]

\[ - L - \xi \sum_{i=0}^{\infty} i^2 P_i \]

\[ \sum_{i=0}^{\infty} i P_i = \sum_{i=0}^{\infty} i P_i = L \quad \text{and} \quad \sum_{i=0}^{\infty} P_i = 1 \]

\[ \frac{dL}{dt} = \xi L + 1 + \xi \left[ 2L + 1 \right] - \xi \left[ L + 1 \right] - L \]

\[ = \xi L + 1 \]

\[ L = \frac{e^{\xi x} - 1}{\xi} \]

Without finite correlation, \( \xi \to 0 \)

\[ L = \frac{(1 + \xi x - 1)}{\xi} = x = \frac{(t-1)}{\tau} \]
Distance between two random points on a line

\[ d = \frac{l}{\Delta}, \quad m = \frac{l}{\Delta} \]

a) \[ P(x_i) = \frac{1}{n \Delta} = \frac{1}{\Delta} \]

\[ P(l) = \sum_{i=1}^{n} P(x_i) \cdot P(x_i + l) + P(x_i) \cdot P(x_i - l) \]

First point at \( x_i \) 2nd point a distance \( l \) apart.

\[ = \left( \frac{1}{n \Delta} \right) \left( \frac{1}{n \Delta} \right)_{i=1} \ldots \ldots \left( \frac{1}{n \Delta} \right) \left( \frac{1}{n \Delta} \right)_{i=\frac{l}{\Delta}} \text{ from left} \]

\[ + \left( \frac{1}{n \Delta} \right) \left( \frac{1}{n \Delta} \right)_{i=\frac{l}{\Delta}} \text{ from left} \ldots \ldots \left( \frac{1}{n \Delta} \right) \left( \frac{1}{n \Delta} \right)_{i=n} \]

\[ = 2 \left( n - \frac{l}{\Delta} \right) \frac{1}{n^2 \Delta^2} = \frac{2 (L - l)}{L^2 \Delta} \]

CDF \[ = \int_{0}^{\frac{L}{\Delta}} \frac{2 (L - l)}{L^2 \Delta} (\Delta \, dl) = \frac{2 L x - x^2}{L^2} \] \( x = L \), \( l = 1 \)

b) If \( l < L_c \) is the correlation length

\[ P(l) = 2 \sum_{i=1}^{n} P(x_i) P(x_i + l) \quad m \Delta = L_c \]

\[ = 2 \left( \frac{1}{n \Delta} \right) \left( \frac{1}{m \Delta} \right)_{i=1} \ldots \ldots \left( \frac{1}{n \Delta} \right) \left( \frac{1}{m \Delta} \right)_{i=m \Delta \text{ from left, end}} \]

\[ = \frac{2}{n m \Delta^2} \left( n - \frac{l}{\Delta} \right) = \frac{2 (L - l)}{L L_c \Delta} \]
C.D.F. = \int_0^{L_c} \frac{2(L-l)}{L_c L} \Delta l = 1

or \quad C = \frac{L_c L}{2LL_c - L_c^2}

C.D.F. = \frac{L_c L}{(2L_c L - L_c^2)} \int_0^x \frac{2(L-x)}{L_c L} dx' = \frac{x(2x-x)}{L_c (2-L_c)}
EE650R: Reliability Physics of Nanoelectronic Devices

Lecture 28: Solution of selected HW Problems on TDDB

Note. In solving these problems, always draw the Weibull plot to capture the information, as we did in the class. Also, let me know if you find any mistakes (alam@purdue.edu)

Part A

a) Using linear fitting of $\log_{10}(T_{BD})$ vs. $V_G$, $\gamma \sim -5.5746$ dec/V

$T_{BD}$ (1.5 V) = $7.237 \times 10^{16}$ sec

$T_{BD}$ (2 V) = $1.181 \times 10^{14}$ sec.

The numbers are very large – in other words, the mean-time-to-failure (MTTF) for a small capacitor at operating voltages is negligibly small. Sometimes people developing new technologies would stress a few small transistors close to operating voltages and find that nothing is happening to the oxides. They may therefore conclude that the technology is reliable – often, the conclusion is wrong because they neglected the effects of statistics in determining technology lifetime (see below).

b) $A_0 = 2.5 \ \mu m^2$ \quad $A_1 = 0.25 \ \mu m^2 \times 100 \times 10^6 = 25 \times 10^6 \ \mu m^2$

$$T_{BD,IC}^{BD,IC} = \left( \frac{A_{IC}}{A_i} \right)^{1/\beta}; \text{ where } \beta = 1.8$$

$T_{BD,IC}$ (1.5 V) = $9.3469 \times 10^{12}$ sec

$T_{BD,IC}$ (2 V) = $1.5253 \times 10^{10}$ sec

You can see that the IC lifetime is dramatically different from individual small-area transistor lifetime. This is the consequence of low $b$ or reduced oxide thickness. If oxide were really thick, $b=8$ for example, the $T$

c) $T_{BD,IC}$ (2 V) = $1.5253 \times 10^{10}$ sec

$T_{life} = 10 \text{ years } = 3.1 \times 10^8 \text{ sec}$

Therefore, $\Delta V = \log(T_{BD,IC}/T_{life}/g = \log(1.52 \times 10^{10}/3.1 \times 10^8))/5.57 = 0.3 V$

So, $V_{safe} = 2V + 0.3V = 2.3 \text{ volts}$

d) One can not accept more than 1 in 10000 ICs returned from field (also called 100 ppm – parts per million), therefore $F_2 = 10^{-4}$.

At mean lifetime, $F_1 = 1 - \exp\left[-\Gamma(1+1/\beta)\right] = 0.555$ (mean of Weibull, Lec. 5)

$$\frac{T_{BD,IC}^{F_2}}{T_{BD,IC}^{F_1}} = \left( \frac{\ln(1-F_2)}{\ln(1-F_1)} \right)^{1/\beta} = 136.07$$

$T_{BD,IC}$ (1.5 V) @ 100 ppm = $6.86 \times 10^{10}$ sec.

$T_{BD,IC}$ (2.0 V) @ 100 ppm = $1.117 \times 10^8$ sec.

Use the same procedure to calculate $\Delta V$ as in c) which gives $V_{safe} = 1.93 \text{ V}$.
Note that compared to part (a), statistics has reduced the overall lifetime by six orders of magnitude. This effect is even more pronounced with lower $\beta$ (see below).

e) As $T_{BD}$ will increase from case b) because $A_{IC}$ is small, therefore $V_{Safe}$ will increase.

Part C

a) $\beta = 1.2$ from the $\beta$ vs. Tox figure. Then you read off the lifetime of a 1.4 nm oxide at a given voltage from appropriate $T_{BD}$ vs. $V_G$ plot (e.g. $T_{BD1}(4V) \sim 0.1$ sec) The same plot gives you $\gamma = 6.8$ dec/volts.

b) $A_0 = 2.5 \mu m^2$ 
   $A_{IC} = 0.13 \mu m^2 \times 100 \times 10^6 = 13 \times 10^6 \mu m^2$

   $\frac{T_{BD1}}{T_{BD}} = \left( \frac{A_0}{A_1} \right)^{\frac{1}{\beta}} = 3.95 \times 10^5$ where $\beta = 1.2$. This scaling is sometimes called ‘Area Scaling’.

   and $\frac{T_{BD,IC}^{F_1}}{T_{BD,IC}^{F_2}} = \left( \frac{\ln(1 - F_1)}{\ln(1 - F_2)} \right)^{\frac{1}{\beta}} = 40643.0$. This is also called ‘Percentile Scaling’.

Integrated scaling = $3.95 \times 10^5 \times 40643.0 = 1.6 \times 10^{10}$

Since $\gamma = 6.8$ dec/volts. $T_{BD1}(4V) = 0.1$ sec for single $2.5 \mu m^2$ transistor.
$T_{BD1}(1.5V) = 0.1 \times 10^{6.89(4-1.5)} / 1.6 \times 10^{10} = 6.25 \times 10^5$ sec
$\Delta V = 0.39$ volts (see part A), therefore $V_{Safe} = 1.11$ V

You have to be careful in using the technology. If you account for the uncertainty in $\beta$, the safe operating voltage will go down further. All this before consideration of Plasma damage and ESD!

c) $\beta$ will be (almost) same.
   but voltage acceleration factor will be lower as observed from Fig. 1.
   So for PMOS, $V_{Safe}$ will be lower compared to NMOS.
Part D

a) From Part C-b,

For three SDB in an IC ...

\[
\left( \frac{T_{BD,IC}^{n=3}}{T_{BD,IC}^{n=1}} \right)^{\beta} = \frac{3}{e} \left( 6\pi \right)^{1/6} \frac{1}{F^{1/3}} = 830.0 \quad \text{with } n=3 \text{ and } F_n=10^{-4}
\]

For \( \beta=1.2 \), the lifetime improves by a factor \( \sim 270.0 \). Therefore, with \( \gamma=6.8 \) dec/volt, the safe operating voltage goes up by 0.35 V, i.e. \( V_{safe}=1.46 \) volts.

b) \( F = 10^{-12} \) for single transistor with two breakdowns

\[
\left( \frac{T_{BD,IC}^{n=2}}{T_{BD,IC}^{n=1}} \right)^{\beta} = \frac{2}{e} \left( 4\pi \right)^{1/4} \frac{1}{F^{1/2}} = 1.38 \times 10^6
\]

For \( \beta=1.2 \), lifetime improves by a factor of \( 1.3 \times 10^5 \). Does it mean that the safe operating voltage will now go up by 0.75 volts to 1.86 volts? Not really, because above \( \sim 1.5 \) volts, the theory of multiple soft-breakdown does not apply – for many transistors, the first breakdown will be hard breakdown.