Modern Physics

Unit 1: Classical Models and the Birth of Modern Physics

Lecture 1.5: Discrete Line Spectra and Bohr's Model

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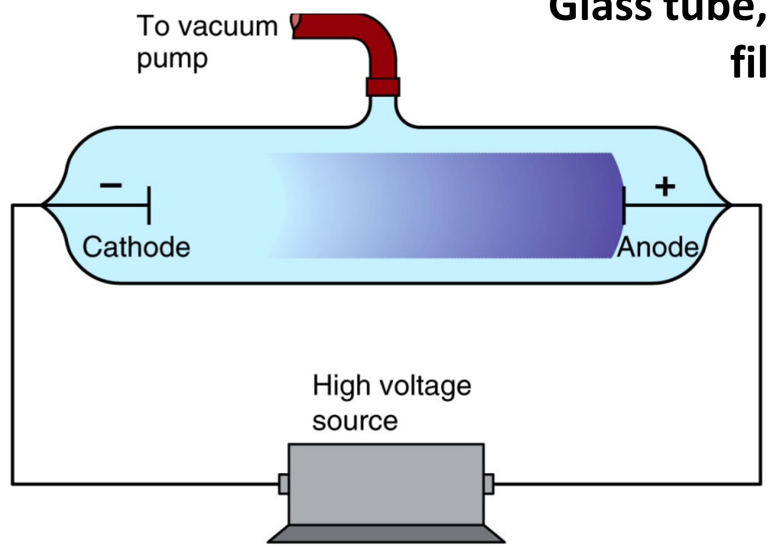
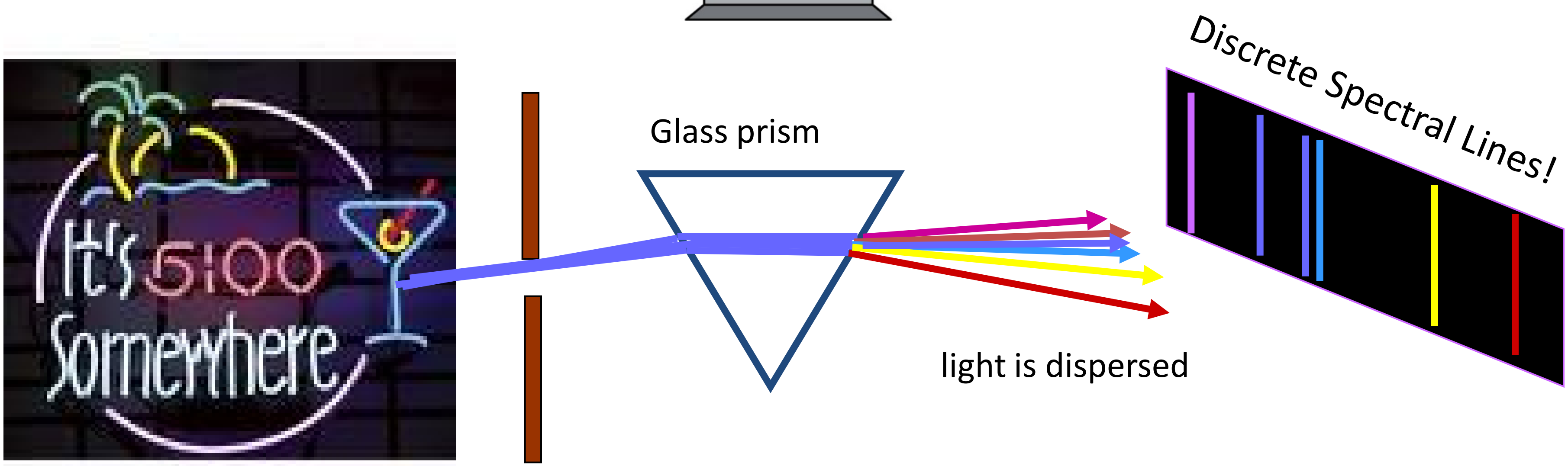
III. Emission of Light by Atoms in Gas Discharge Tubes

Glass tube, evacuated and back-filled with gas

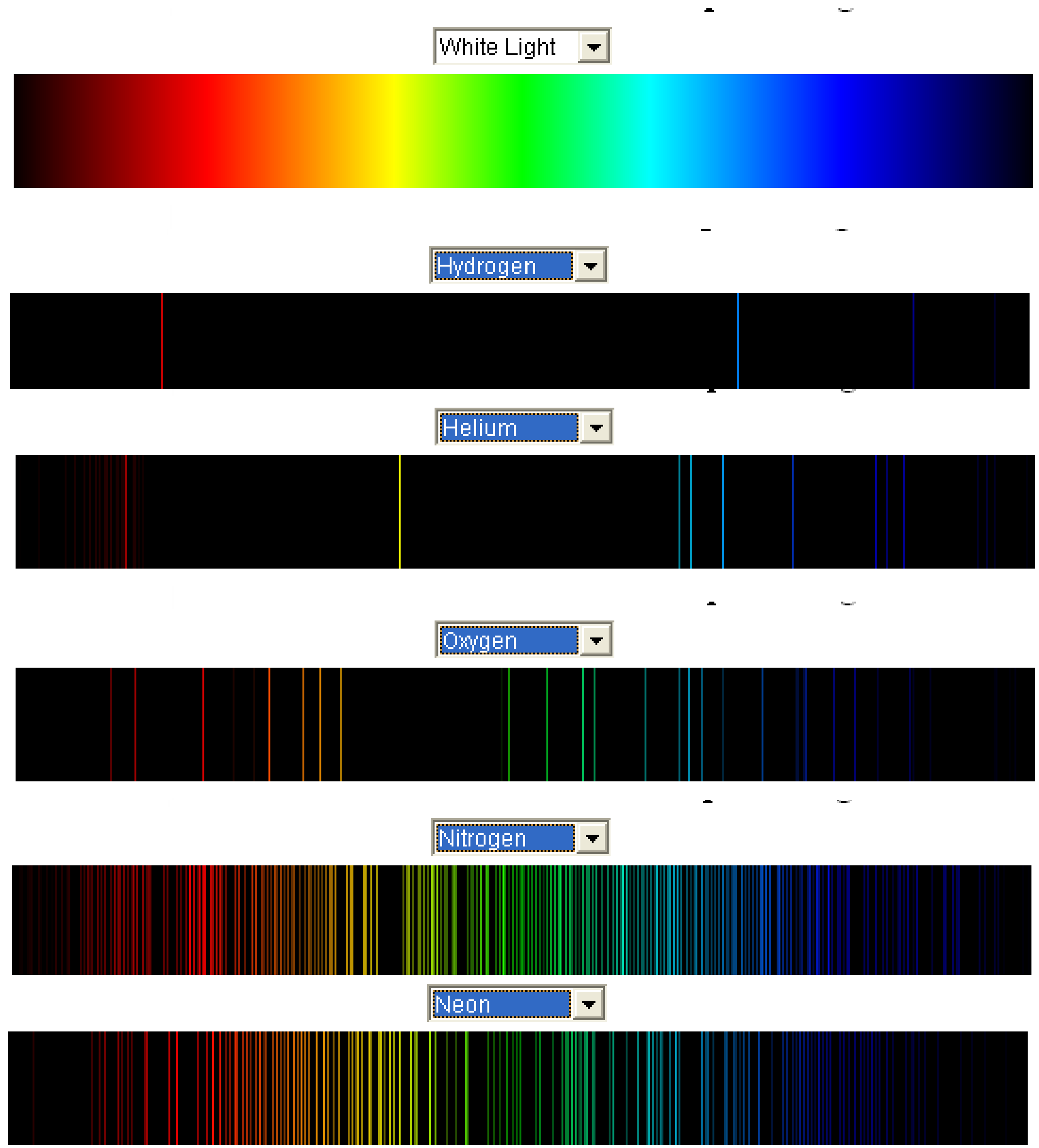




Visible Spectral Lines for Common Gases

Check out http://www.colorado.edu/physics/2000/applets/a2.html

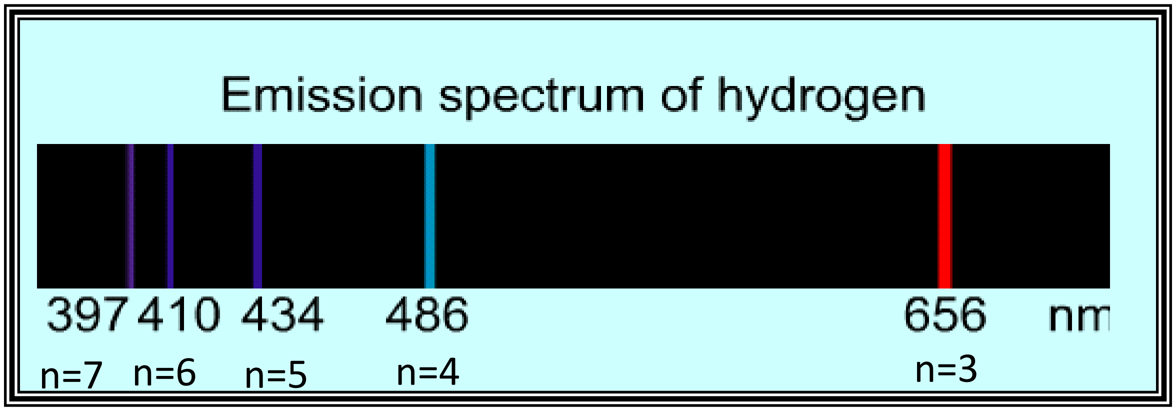
Wavelengths accurately measured



Why are there so many lines?

Focus on the Simplest Gas - Hydrogen

Balmer's empirical formula (1885) explains the observed wavelengths from <u>hydrogen</u> gas with <u>*high accuracy*</u>



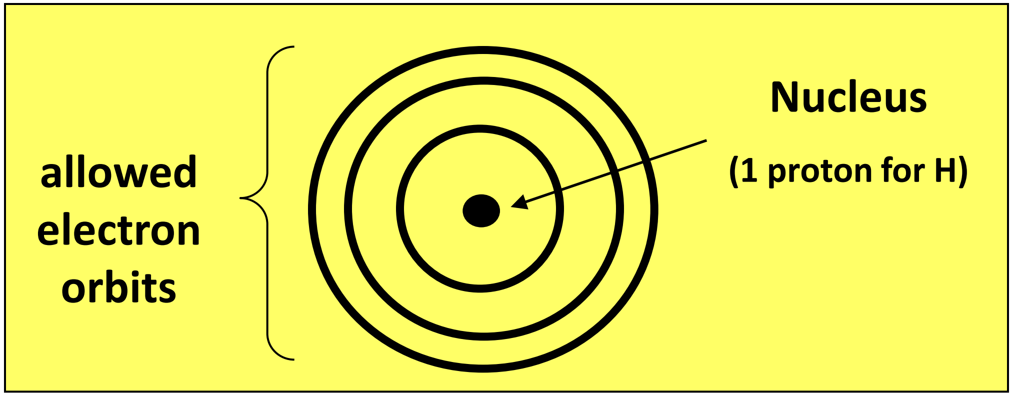
$$\frac{1}{\lambda} = R_H \left[\frac{1}{2^2} - \frac{1}{n^2} \right]; \;\; n = 3, 4, 5\ldots$$

$$R_H = 1.097 \times 10^7 \text{ m}^{-1}$$

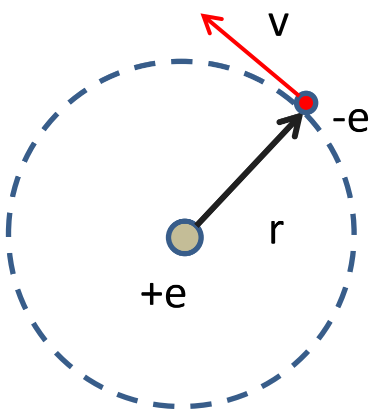
IV. Bohr Model (1911): Assumptions

(hybrid model combining classical and quantum physics)

- Electron moves in circular orbits
- Only certain orbits are allowed; quantization of angular momentum determines radius of orbit
- Electron gives off no radiation when in allowed orbit
- Radiation only emitted when electron makes <u>transition</u> from one allowed orbit to another



Kinematical Considerations



$$\vec{F} = \frac{1}{4\pi\varepsilon_o}\frac{e^2}{r^2}(-\hat{r})$$

$$|F| = ma = m\frac{v^2}{r}$$

$$K = \frac{1}{2}mv^2 = \frac{1}{2}r|F| = \frac{1}{2}\frac{1}{4\pi\varepsilon_o}\frac{e^2}{r}$$

remember virial theorem;
L1.02: (<K> = -½ <V>)

$$V = -\frac{1}{4\pi\varepsilon_o}\frac{e^2}{r} \quad \Rightarrow E = K + V = -\frac{1}{8\pi\varepsilon_o}\frac{e^2}{r}$$

total energy

Quantize Angular Momentum

$$mvr = n\hbar; \quad n = \text{positive integer}; \quad \hbar \equiv h/2\pi$$

$$\boxed{r = n^2\frac{4\pi\varepsilon_o\hbar^2}{me^2}}$$

$$(mv)^2 r^2 = n^2\hbar^2$$ (convenient assumption!)

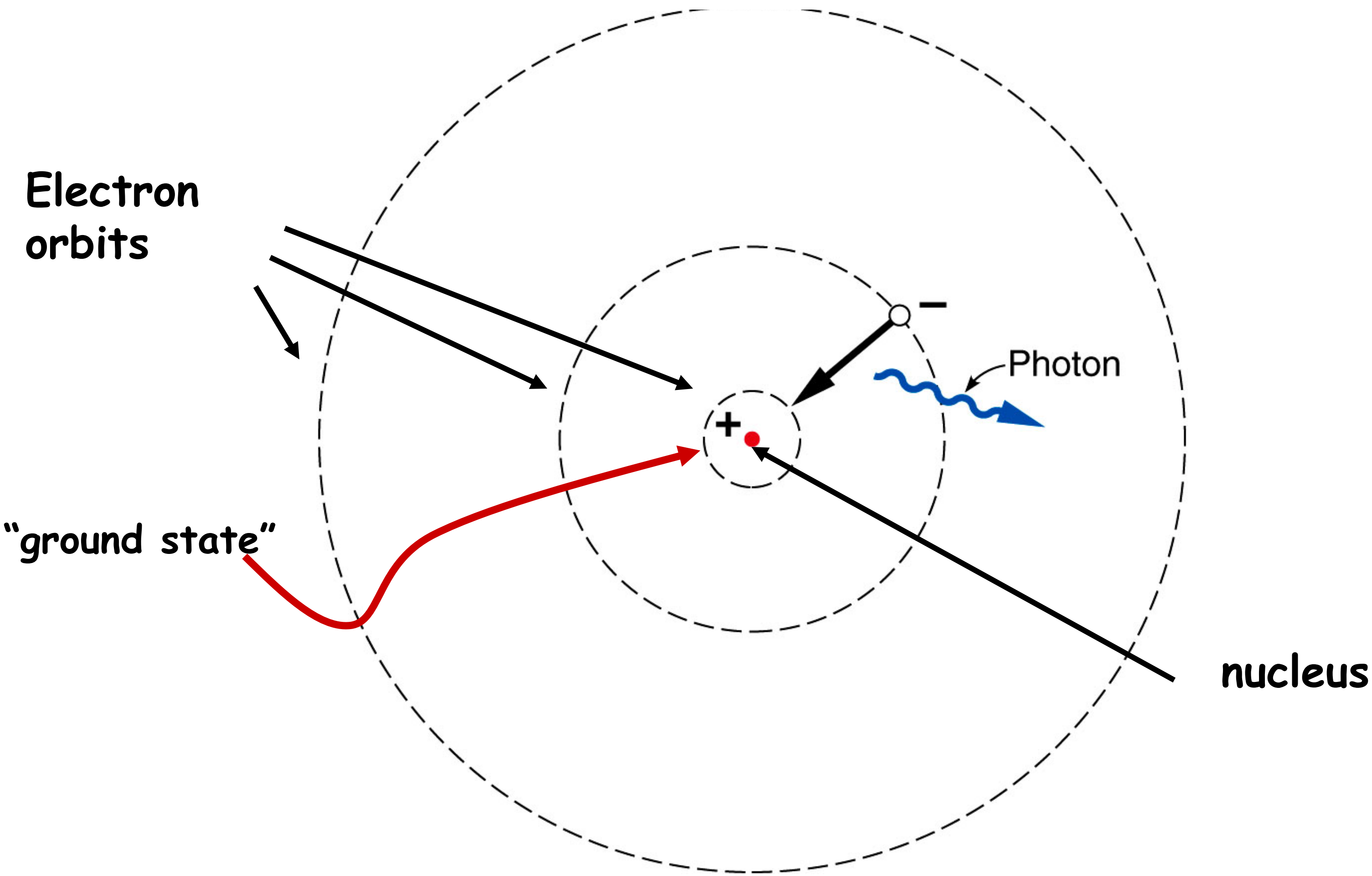
$$(mv)^2 = \frac{1}{4\pi\varepsilon_o}\frac{me^2}{r}$$

$$\boxed{E_n = -\frac{1}{2}\left[\frac{1}{4\pi\varepsilon_o}\frac{e^2}{a_o}\right]\frac{1}{n^2} = -\frac{13.6\,eV}{n^2}}$$ where $a_o \equiv r|_{n=1}$

What is an eV ? See Appendix at end of lecture.

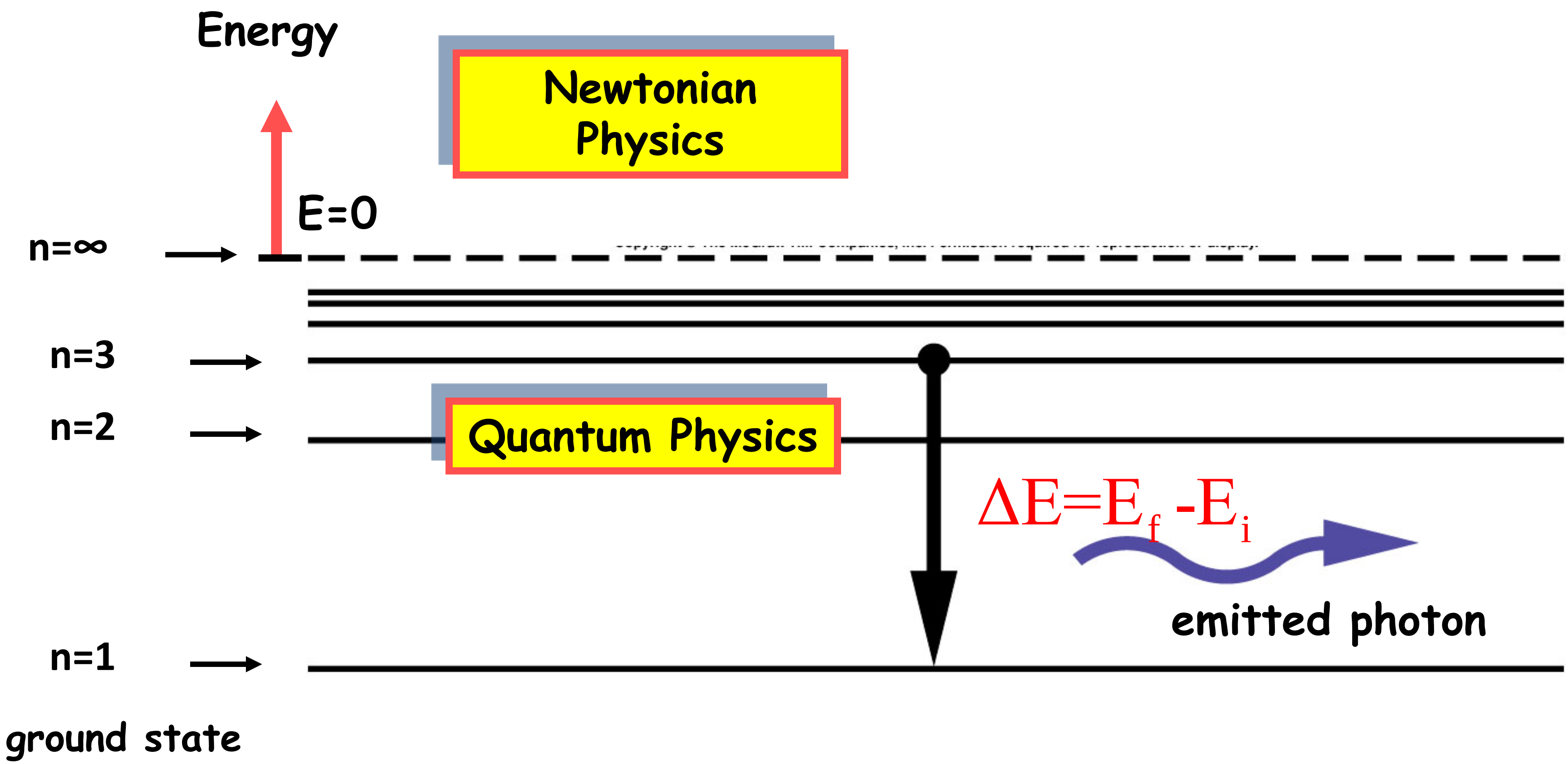
Bohr's model for light emission from hydrogen

(Bohr 1911)



Using this model, you can derive Balmer's empirical formula

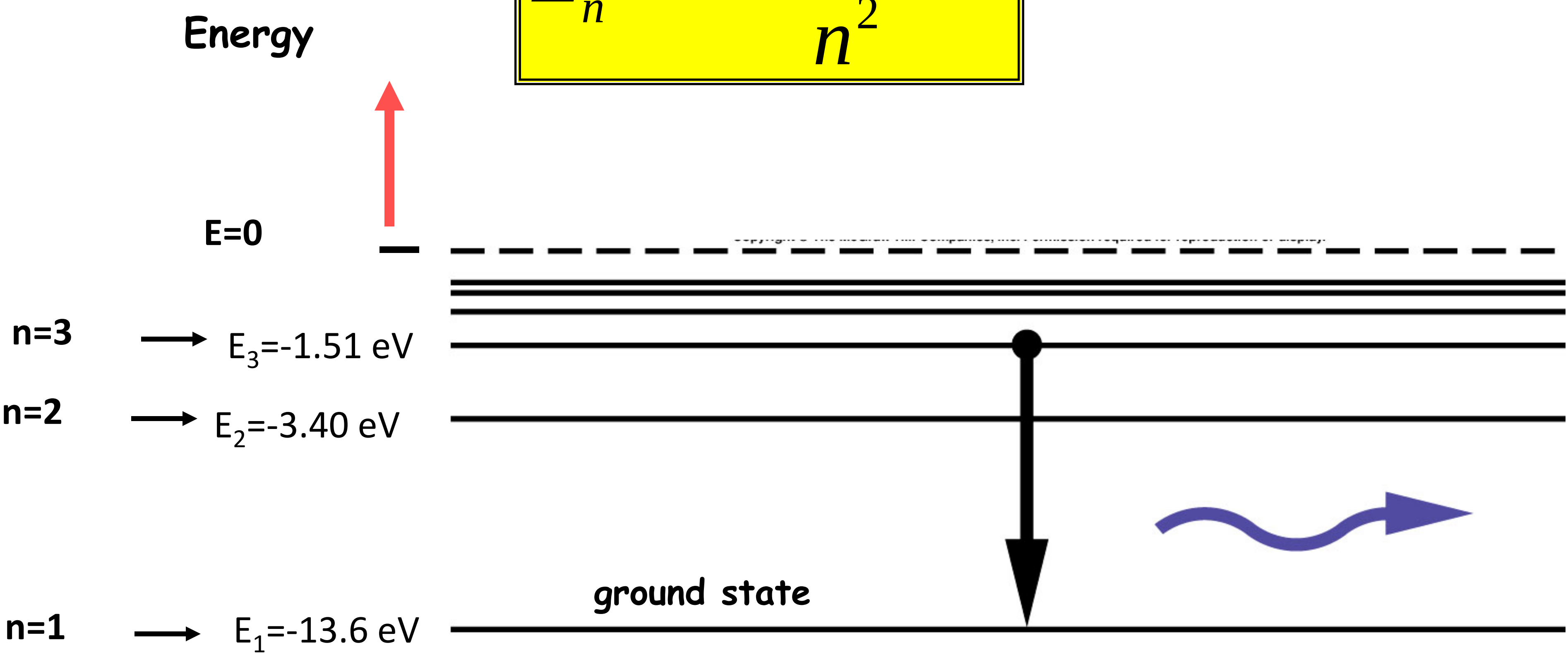
Each orbit gives rise to discrete energy level



$$\Delta E = hf = \frac{hc}{\lambda}$$

Predicting the energy levels for hydrogen

$$E_n = \frac{-13.6\,eV}{n^2}$$



Appendix A: Measuring Energies in Quantum Systems

The energy unit of a Joule is suitable for measuring energies of macroscale objects, but in quantum physics, this energy unit is too big to be of much use. Over the years, different energy units that are more "convenient" have been invented. It is important to know these definitions and be able to convert between them.

We will need values for some fundamental constants:

$|e^-| = 1.602\times10^{-19}$ C, h=6.626x10-34 J-s, $c=2.998\times10^{8}$ m/s

1. The electron volt (eV):

If a particle with charge q is moved through a potential difference of 1.0 V = 1.0 J/C, it acquires an energy

$$\text{Energy} = |q|\Delta V$$

If $q=|e^-|$ (the charge on an electron) the energy gained is

$$E = |e^-|\,\Delta V = 1.602\times10^{-19}\ \text{J} = 1\ \text{eV}$$

2(a). The energy of a photon:
EM radiation of **frequency** f (or v) has an energy of E=hf=hv.

2(b). The energy of a photon:
If the **wavelength** λ of the EM radiation is specified, then E=hc/λ.

3. Wavenumbers

Photons with a wavelength λ in the infrared (IR) are divided into the near, mid, and far IR. This EM radiation is often described by a wavenumber $\overline{\nu}$ which is defined by the number of wavelengths λ (measured in cm) that fit into a 1.0 cm length

$$\overline{\nu}\,\lambda = 1$$

$$\overline{\nu} = \frac{1}{\lambda}\left[cm^{-1}\right]$$

$$E = hc\,\overline{\nu}$$

It follows that $f = c\,\overline{\nu}$

Example: Suppose the frequency f of EM radiation is specified to be $f=3\times10^{14}$ Hz. You should be able to perform the following conversions.

$$If\ \ f=3\times10^{14}\,Hz,\quad \lambda=\frac{c}{f}=\frac{2.998\times10^{8}\,m/s}{3\times10^{14}\,s^{-1}}=1.0\times10^{-6}\,m=1\,\mu m=1.0\times10^{-4}\,cm=1000\ nm$$

$$E=hf=\left(6.626\times10^{-34}\,J\,s\right)\bullet\left(3\times10^{14}\,s^{-1}\right)=1.98\times10^{-19}\,J\bullet\frac{1eV}{1.602\times10^{-19}\,J}=1.24eV$$

$$E=\frac{hc}{\lambda}=\frac{\left(6.626\times10^{-34}\,J\,s\right)\bullet\left(2.998\times10^{8}\,ms^{-1}\right)}{1000\times10^{-9}\,m}=\frac{19.8}{1000}\times10^{-17}\,J\bullet\frac{1eV}{1.602\times10^{-19}\,J}=1.24\,eV$$

$$Convenient\ approx.:\quad E(in\ eV)\simeq\frac{1234}{\lambda(in\ nm)}$$

$$\overline{\nu}=\frac{1}{\lambda}=\frac{1}{1\times10^{-4}\,cm}=10,000\,cm^{-1}\ \ or\ 10,000\ \ wavenumbers$$

$$E=hc\overline{\nu}=\left(6.626\times10^{-34}\,J\,s\right)\bullet\left(2.998\times10^{8}\,ms^{-1}\right)\left(10,000\,cm^{-1}\right)=\left(19.8\times10^{-25}\,J\right)\left(10,000\,cm^{-1}\right)\frac{100\,cm}{1m}$$

$$=1.98\times10^{-19}\,J\bullet\frac{1eV}{1.602\times10^{-19}\,J}=1.24eV$$

$$Convenient\ approx.:\quad E(in\ eV)\simeq\frac{\overline{\nu}\,(in\ cm^{-1})}{8000}$$

$$E(in\ meV)\simeq\frac{\overline{\nu}\,(in\ cm^{-1})}{8}$$