Modern Physics

Unit 1: Classical Models and the Birth of Modern Physics
Lecture 1.6: Compton Effect

Ron Reifenberger
Professor of Physics
Purdue University
V. The Compton Effect (1923)

Compton's Apparatus (schematic)

Compton applies conservation of energy and momentum principles to explain puzzling data.

Recall collision theory:

Conservation of momentum:
\[ m_1 v_{f1} + m_2 v_{f2} = m_1 v_{i1} + m_2 v_{i2} \]

Conservation of kinetic energy:
\[ \frac{1}{2} m_1 v_{f1}^2 + \frac{1}{2} m_2 v_{f2}^2 = \frac{1}{2} m_1 v_{i1}^2 + \frac{1}{2} m_2 v_{i2}^2 \]
What's the momentum of a photon?

\[ p = ??? \]

Classical mechanics:

\[ p = mv \]

but….photon mass = 0

\[ \lambda \]

\[ E \]

· from Planck

\[ E = hf \]

· from Maxwell, the momentum of an EM Wave is:

\[ p = \frac{U}{c} \]

(see L1.03)

\[ p = \frac{hf}{c} = \frac{hf}{f\lambda} = \frac{h}{\lambda} \]

\[ h = \text{Planck's constant} = 6.626 \times 10^{-34} \text{ m}^2 \text{ kg/s} \]
Compton’s analysis considered an X-ray photon scattering from a single electron inside a target.

Before

After

How is $\lambda_f$ related to $\lambda_i$?
Geometry of Compton Scattering

Compton wavelength \( \lambda_f - \lambda_i = \frac{h}{m_e c} \left( 1 - \cos \Theta \right) \)

Compton wavelength \( \equiv \frac{h}{m_e c} = \frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{(9.109 \times 10^{-31} \text{ kg})(2.998 \times 10^8 \text{ m/s})} \)

\[ = 2.43 \times 10^{-3} \text{ nm} = 2.43 \times 10^{-12} \text{ m} = 2.43 \text{ pm} \]

\[ \text{Note: } 1 \text{ pm} = 1 \times 10^{-12} \text{ m} \]
Summary: Compton Scattering

\[
\lambda_f - \lambda_i = \frac{h}{m_e c} (1 - \cos \Theta)
\]

Scattered Beam

\[\Theta = \pi\]

Elastic Scattering

\[\Theta = 0\]

Inelastic Scattering Increases

number of photons detected
**Example:** EM radiation with **differing wavelengths** is scattered by an electron through an angle of $37^\circ$ (w.r.t. incident beam). What is the wavelength of the scattered radiation?

$$\lambda_f - \lambda_i = \frac{h}{m_e c} (1 - \cos \theta)$$

<table>
<thead>
<tr>
<th>If $\lambda_i$ =</th>
<th>$\frac{h}{m_e c} (1 - \cos \theta)$</th>
<th>then $\lambda_f$ =</th>
<th>Fractional Shift</th>
</tr>
</thead>
<tbody>
<tr>
<td>420 nm</td>
<td>0.489 pm</td>
<td>420.000489 nm</td>
<td>1.16x10^{-6}</td>
</tr>
<tr>
<td>42 nm</td>
<td>0.489 pm</td>
<td>42.000489 nm</td>
<td>1.16x10^{-5}</td>
</tr>
<tr>
<td>4.2 nm</td>
<td>0.489 pm</td>
<td>4.200489 nm</td>
<td>1.16x10^{-4}</td>
</tr>
<tr>
<td>0.042 nm</td>
<td>0.489 pm</td>
<td>0.042489 nm</td>
<td>1.16x10^{-2}</td>
</tr>
<tr>
<td>0.0042 nm (4.20 pm)</td>
<td>0.489 pm</td>
<td>0.004689 nm (4.69 pm)</td>
<td>1.16x10^{-1}</td>
</tr>
</tbody>
</table>
Why assume incident EM radiation is scattered only by an electron?

Compton’s analysis is completely general. Evaluate the Compton wavelength for scattering by different particles.

Compton wavelength \( \equiv \frac{h}{mc} \)

<table>
<thead>
<tr>
<th>Possible Scattering Center</th>
<th>Mass ( m ) (kg)</th>
<th>Compton Wavelength</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carbon atom</td>
<td>( 1.993 \times 10^{-26} )</td>
<td>( 1.11 \times 10^{-16} ) m</td>
</tr>
<tr>
<td>proton</td>
<td>( 1.672 \times 10^{-27} )</td>
<td>( 1.32 \times 10^{-15} ) m</td>
</tr>
<tr>
<td>electron</td>
<td>( 9.109 \times 10^{-31} )</td>
<td>( 2.43 \times 10^{-12} ) m</td>
</tr>
</tbody>
</table>

really small!
Physical Significance of Compton Wavelength?

Compton wavelength \( \equiv \lambda_c = \frac{h}{mc} \)

rewriting: \( mc = h \frac{1}{\lambda_c} \) \( \Rightarrow \) \( mc^2 = hf \frac{1}{\lambda_c} \)

What is smallest wavelength possible for an EM wave?

Photons with a wavelength smaller than \( \lambda_c \) tend to split into electron-positron pairs.

Historically, Compton’s experiment was important because it finally convinced the last skeptics that light can behave as a particle.
We are discussing different wavelengths

- the wavelength of EM radiation
- the de Broglie wavelength of a particle (next set of lectures)
- the Compton wavelength of a particle

Can you associate the correct wavelength with each experiment?

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Which wavelength?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maxwell’s EM radiation</td>
<td></td>
</tr>
<tr>
<td>Young’ 2-slit experiment</td>
<td></td>
</tr>
<tr>
<td>Blackbody radiation</td>
<td></td>
</tr>
<tr>
<td>Photoelectric effect</td>
<td></td>
</tr>
<tr>
<td>Electron diffraction</td>
<td></td>
</tr>
<tr>
<td>Discrete line spectra</td>
<td></td>
</tr>
<tr>
<td>X-ray diffraction</td>
<td></td>
</tr>
<tr>
<td>Compton Scattering</td>
<td></td>
</tr>
</tbody>
</table>