Modern Physics

Unit 2: Schrödinger Equation in 1 Dimension
Lecture 2.1: Wave-Particle Duality

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I. Interference Effect for Light

Light behaves as a wave.

Monochromatic Light Source

Not to scale!
Young’s Double Slit Geometry

c) as distance to screen increases, \( \alpha \to 0 \)

\( (\alpha = 0 \) is the limiting condition for parallel rays)
Path Difference = $ABC - DE = AB + BC - (DE)$

but, $DE = BC$

Path Difference = $AB = d \sin \theta$

phase change $\equiv 2\pi \times \frac{\text{path difference}}{\lambda}$

$$= \left( \frac{2\pi}{\lambda} \right) d \sin \theta$$

constructive interference when phase change $= 2\pi n$; $n = \text{integer}$

$\therefore n\lambda = d \sin \theta$
Young’s double slit effect was discovered in 1803. Any similar phenomena reported since then?

1. Thin film interference – late 1800’s:

2. X-ray reflection experiments
   - von Laue – Germany, 1912; Noble Prize in 1914
X-rays reflected from crystalline solids show strong reflections in well-defined directions, similar to interference effect in two slit experiment.

Expected result, since an X-ray is an EM wave?
What's the physics?

Partial reflection of X-ray beam from two planes of atoms

Condition for constructive interference:

Path length difference = n\lambda

Many parallel crystallographic planes possible?

Expect many directions for possible interference.
Working it out:

Path Difference = \( ABC - AD = AB + BC - (AR + RD) \)

but, \( AR = BC \)

\[
\text{Path Difference} = AB - RD = \frac{d}{\sin \theta} - \frac{d}{\sin \theta} \cos 2\theta
\]

\[
= \frac{d}{\sin \theta} (1 - \cos 2\theta)
\]

\[
= \frac{d}{\sin \theta} (1 - (1 - 2 \sin^2 \theta)) = 2d \sin \theta
\]

phase change = \( \frac{2\pi \times \text{path difference}}{\lambda} \)

\[
= \left(\frac{2\pi}{\lambda}\right) 2d \sin \theta = 2\pi n \quad \text{for constructive interference}
\]

\[\therefore \text{constructive interference when } \lambda = \frac{2d}{n} \sin \theta \quad \text{Bragg condition}\]
Note 1: The physical model of diffraction used to derive Bragg's law is oversimplified, but the law itself is correct.

Note 2: For interference to occur, require
\[ \lambda \cong d \cong 0.3 \text{ nm } \cong 300 \text{ pm}^* \]

Note 3: Since \( c = f\lambda \rightarrow f \cong 1 \times 10^{18} \text{ Hz} \rightarrow \text{X-ray photons} \)

Summary: Wave-Particle Duality of Light

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<th>Light behaves as a wave</th>
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\[ * \text{Note: } 1 \text{ pm} = 1 \times 10^{-12} m \]
III. de Broglie matter-waves (1923, Noble Prize 1929)

Is it possible that wave-like behavior could describe a particle like an electron?

de Broglie thought so and hypothesized that the well-established relation between wavelength (\(\lambda\)) and momentum (\(p\)) for photons also applies to particles!

\[ p = \frac{U}{c} = \frac{hf}{c} = \frac{h}{\lambda} = \frac{h}{2\pi \lambda} = \frac{\hbar k}{p} \]

also applies to particles!

This idea implies that a particle can have a wavelength (or wavenumber) given by

\[ \lambda = \frac{h}{p} \quad (k = \frac{p}{\hbar}) \]
Electron Diffraction

Bell Telephone Laboratories

Davisson-Germer Experiment - 1927
Noble Prize 1937

$\lambda \approx 0.3 \text{nm}$

d $\approx 0.3 \text{nm}$

Reflected electron beam travels in preferred directions!

Electron Diffraction
Modern Implementation

Now known as Low Energy Electron Diffraction (LEED)

KEY IDEA: Unable to determine in advance how any given electron will be reflected. The overall pattern can be predicted, and the probability that an electron will be reflected to a specific region on a screen can be calculated, but we cannot state in advance that, say, the tenth electron will land in any particular spot.
Check out the electron diffraction simulation

Two-slit Interference of Electrons

Monoenergetic Electron Source
(~1000 e’s per second)


More recently (March 2013):
IMPORTANT: The de Broglie wavelength $\lambda$ roughly marks a “confinement” boundary between particle and wavelike behavior in quantum physics.

If $L_1 \gg \lambda$, electron behaves as a particle

If $\lambda \geq L_2$, electron behaves as a wave

$\lambda = \frac{h}{p}$ \hspace{1cm} \left( k = \frac{p}{\hbar} \right)$
Summary: Dual Character of Light and Electrons

Properties seem to change?

**LIGHT**

- Ray-like behavior
- Fermat's Principle
- \( d \gg \lambda \)

**ELECTRON**

- Wave-like behavior
- Particle Nature
- Box, \( d \gg \lambda_{\text{deBroglie}} \)
- Wave Nature
- Newton's Laws
- \( d \sim \lambda_{\text{deBroglie}} \)

\( p = mv \)

more later in semester