Modern Physics

Unit 12: Special Relativity - Introduction

Lecture 12.2: Measuring the Speed of Light

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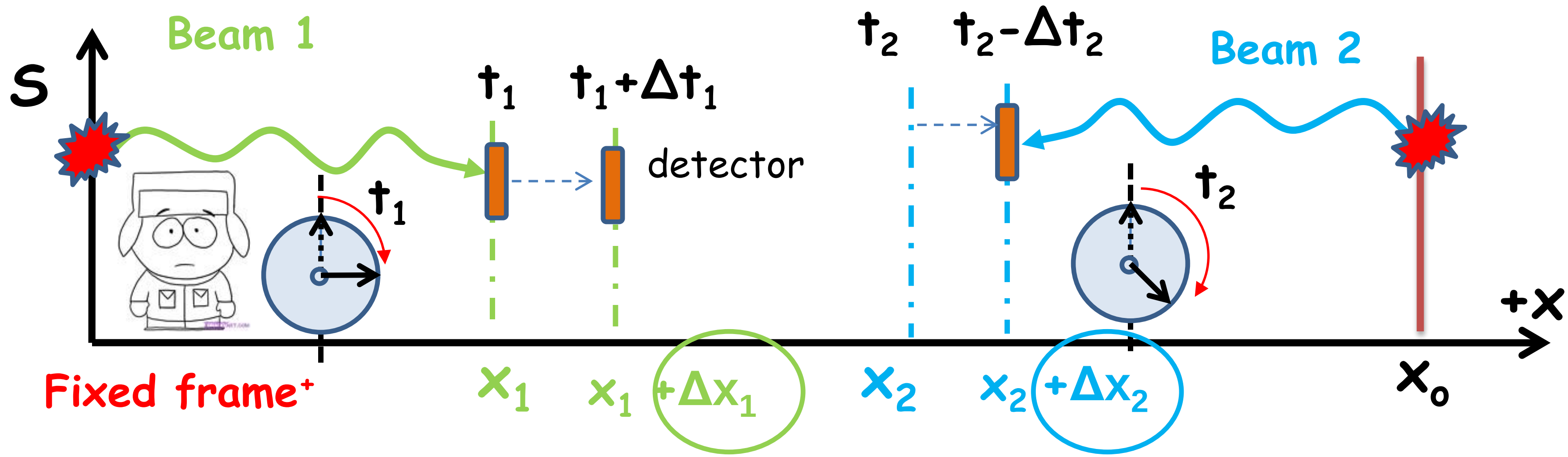
Consider two light beams in a fixed frame moving in opposite directions

Light travels a distance Δx_1 in a time Δt_1

Light travels a distance Δx_2 in a time Δt_2



Precision displacement!

$$EXP\ I: \quad x_1 = ct_1$$

$$EXP\ II: \quad x_1 + \Delta x_1 = c(t_1 + \Delta t_1)$$

$$but \quad x_1 = ct_1 \quad from\ EXP\ I$$

$$\cancel{x_1} + \Delta x_1 = \cancel{ct_1} + c\Delta t_1$$

$$c = \frac{\Delta x_1}{\Delta t_1}$$

$$EXP\ I: \quad x_o - x_2 = ct_2$$

$$EXP\ II: \quad x_o - (x_2 + \Delta x_2) = c(t_2 - \Delta t_2)$$

$$but \quad x_o - x_2 = ct_2 \quad from\ EXP\ I$$

$$\cancel{(x_o - x_2)} - \Delta x_2 = \cancel{ct_2} - c\Delta t_2$$

$$c = \frac{\Delta x_2}{\Delta t_2}$$

⁺ Fixed with respect to what??

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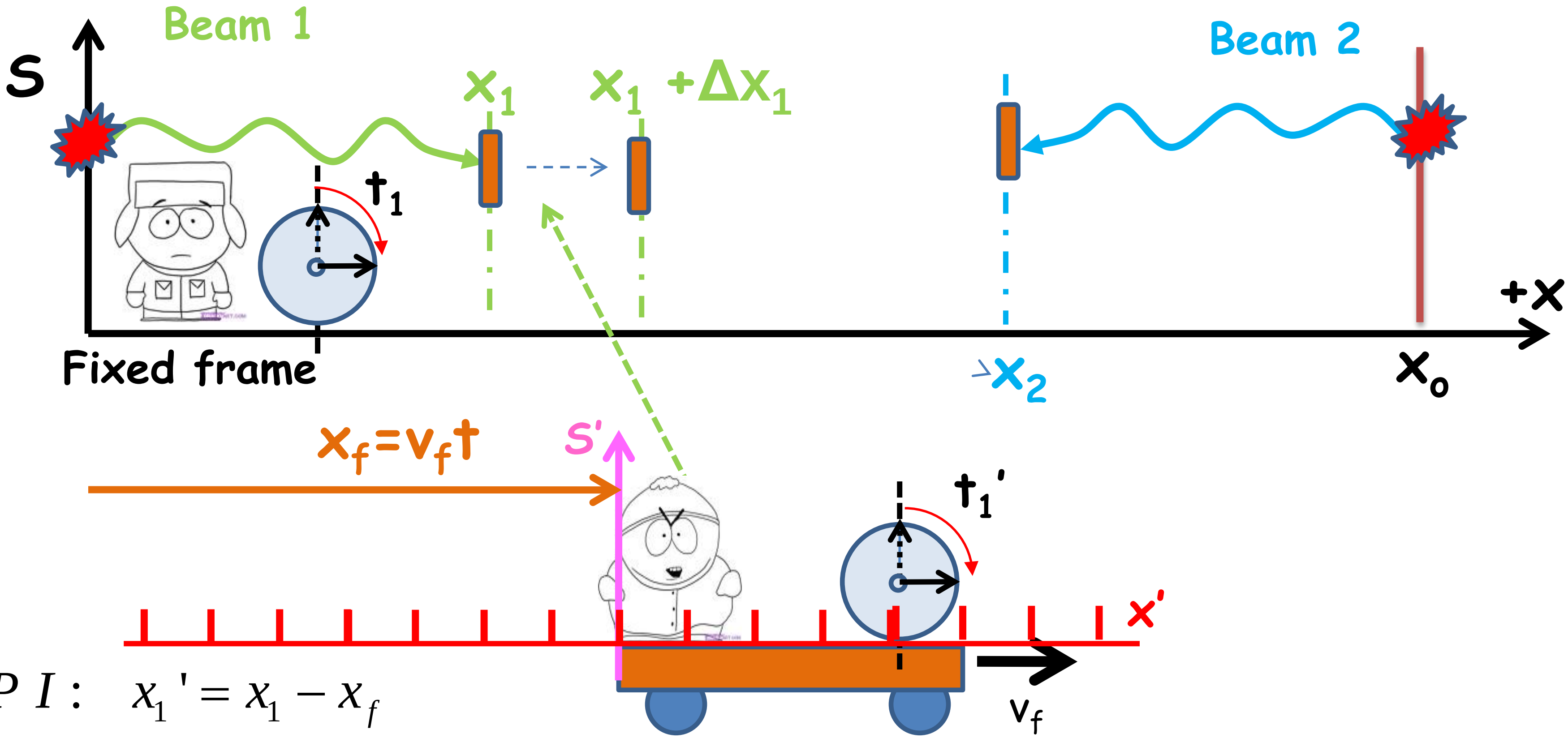
A. What does S' measure for light beam from L to R?



$$EXP\ I: \quad x_1' = x_1 - x_f$$

$$EXP\ II: \quad x_1' + \Delta x_1' = c\left(t_1 + \Delta t\right) - v_f\left(t + \Delta t\right)$$

$$but \quad x_1' = x_1 - x_f \quad from\ EXP\ I$$

$$\cancel{x_1} - \cancel{x_f} + \Delta x_1' = c\left(t_1 + \Delta t\right) - v_f\left(t + \Delta t\right)$$

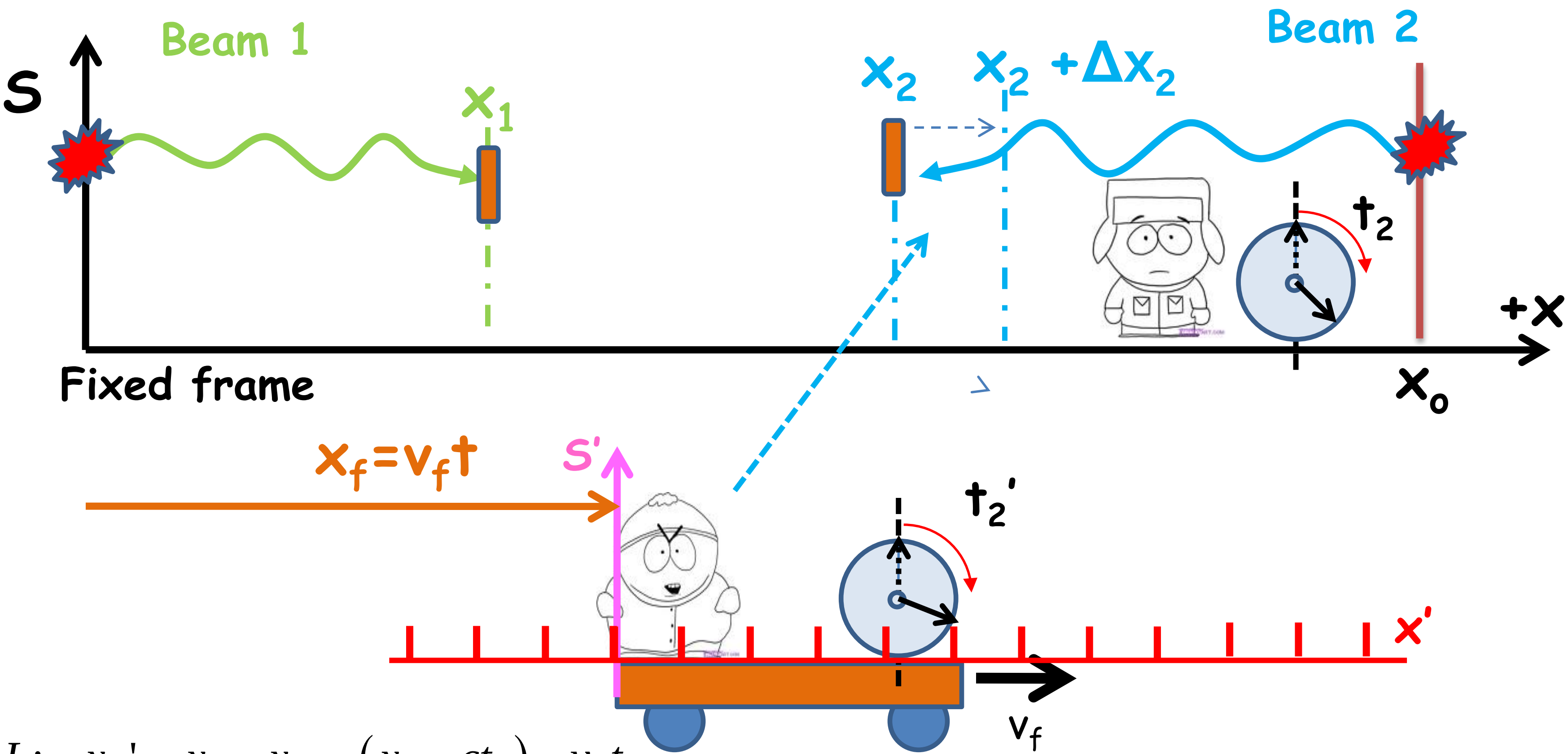
$$= \cancel{ct_1} + c\Delta t - \cancel{v_f t} - v_f \Delta t$$

$$\Delta x_1' = c\Delta t - v_f \Delta t$$

$$\boxed{c' = \frac{\Delta x_1'}{\Delta t} = c - v_f}$$

Note two times: t_1 measures transit of light beam while t measures displacement of frame S'

B. What does S' measure for light beam from R to L?



$$EXP\ I: \quad x_2' = x_2 - x_f = (x_o - ct_2) - v_f t$$

$$EXP\ II: \quad x_2' + \Delta x_2' = (x_o - c(t_2 - \Delta t)) - v_f (t - \Delta t)$$

$$but \quad x_2' = x_2 - x_f \quad from\ EXP\ I$$

$$\cancel{x_2} - \cancel{x_f} + \Delta x_2' = x_o - ct_2 + c\Delta t - v_f t + v_f \Delta t$$

$$= (\cancel{x_o - ct_2}) - (\cancel{v_f t}) + (c + v_f)\Delta t$$

$$\Delta x_2' = (c + v_f)\Delta t$$

$$\boxed{c' = \frac{\Delta x_2'}{\Delta t} = (c + v_f)}$$

Note two times: t_2 measures transit of light beam while t measures displacement of frame S'

Summary

	Speed of light measured from S	Speed of light measured from S'
Light beam from L to R	c	$c - v_f$
Light beam from R to L	c	$c + v_f$

Forced to conclude that the speed of light depends on observer's relative motion.

But there is a subtle problem…..Maxwell's Equations predict the speed of light in vacuum.

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} = 4\pi k \rho \qquad \oint \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$$

$$\nabla \cdot \vec{B} = 0 \qquad \oint \vec{B} \cdot d\vec{A} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

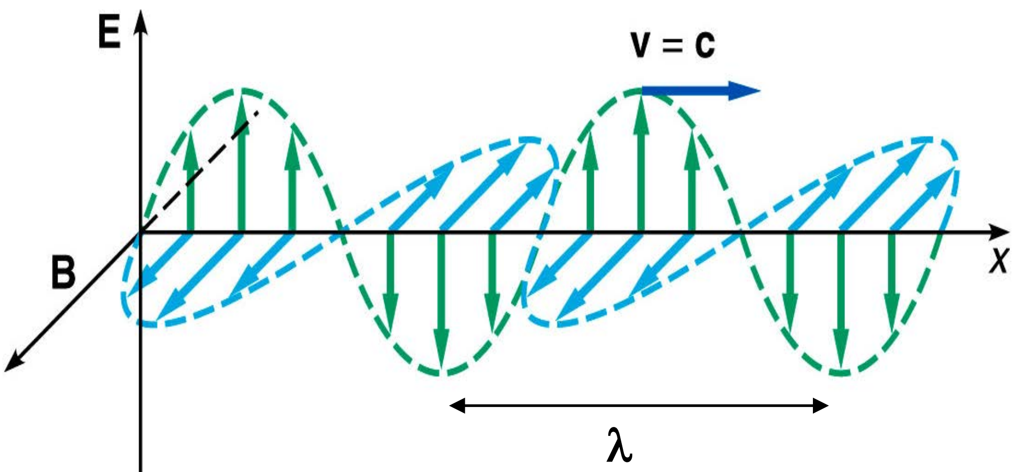
$$\nabla \times \vec{B} = \frac{\vec{J}}{\varepsilon_0 c^2} + \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t} \qquad \oint \vec{B} \cdot d\vec{s} = \mu_0 i + \frac{1}{c^2}\frac{\partial}{\partial t}\int \vec{E} \cdot d\vec{A}$$

In free space, ρ=0 and J=0:

$$\frac{\partial^2 E}{\partial x^2} = \varepsilon_o \mu_o \frac{\partial^2 E}{\partial t^2}$$

$$\frac{\partial^2 B}{\partial y^2} = \varepsilon_o \mu_o \frac{\partial^2 B}{\partial t^2}$$

$$\boxed{c = \frac{1}{\sqrt{\varepsilon_o \mu_o}}}$$

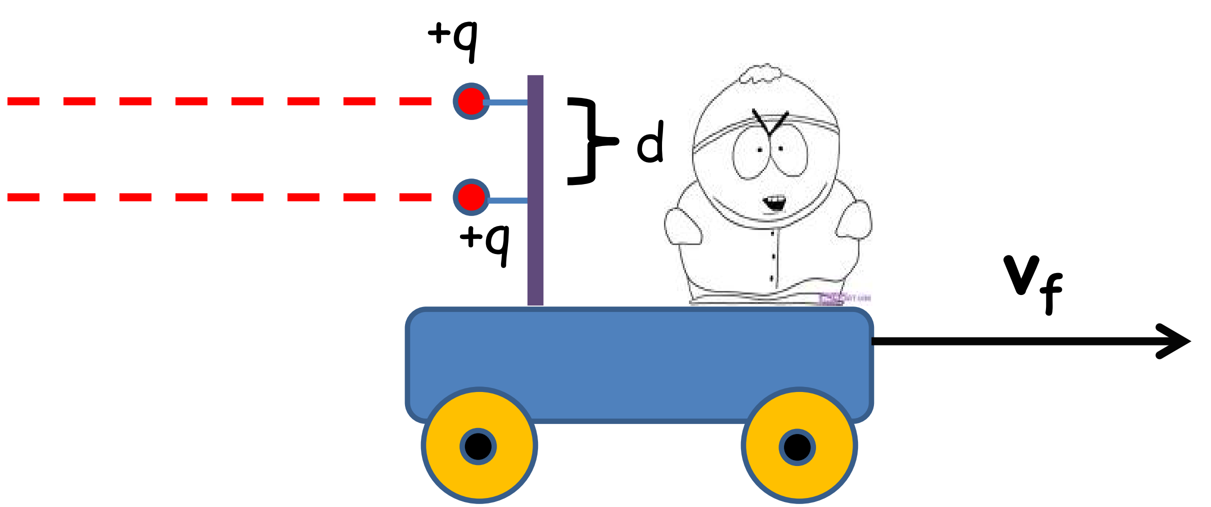


Maxwell's equations give just **one** universal number for c …… Is this the velocity of an EM wave in some special frame of reference?

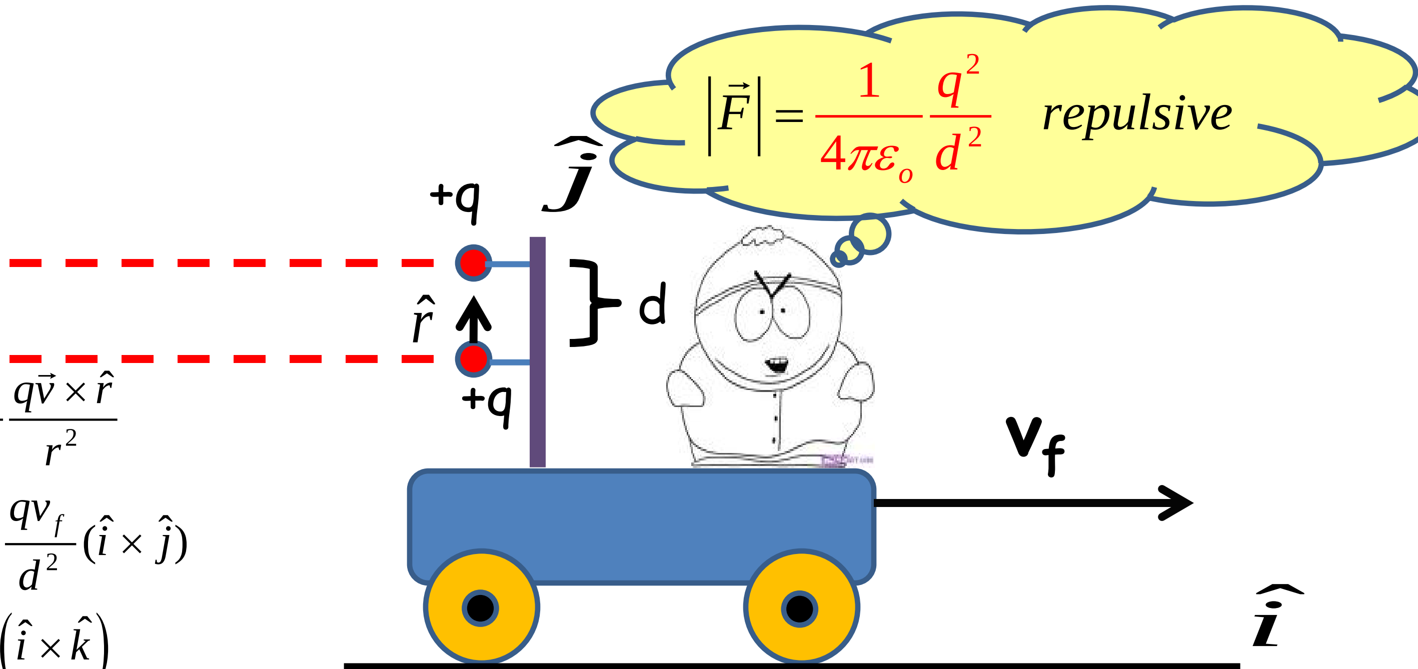
The speed of light should depend on the reference frame (see previous slides).

This implies you should be able to determine the speed of the reference frame you are in by performing some experiment with things that Maxwell's Equations govern - like magnets, currents, charges, etc. The results will tell you when you are in a stationary reference frame (contradicting Galileo!) .

Consider this example:



The Answer Depends on the Reference Frame

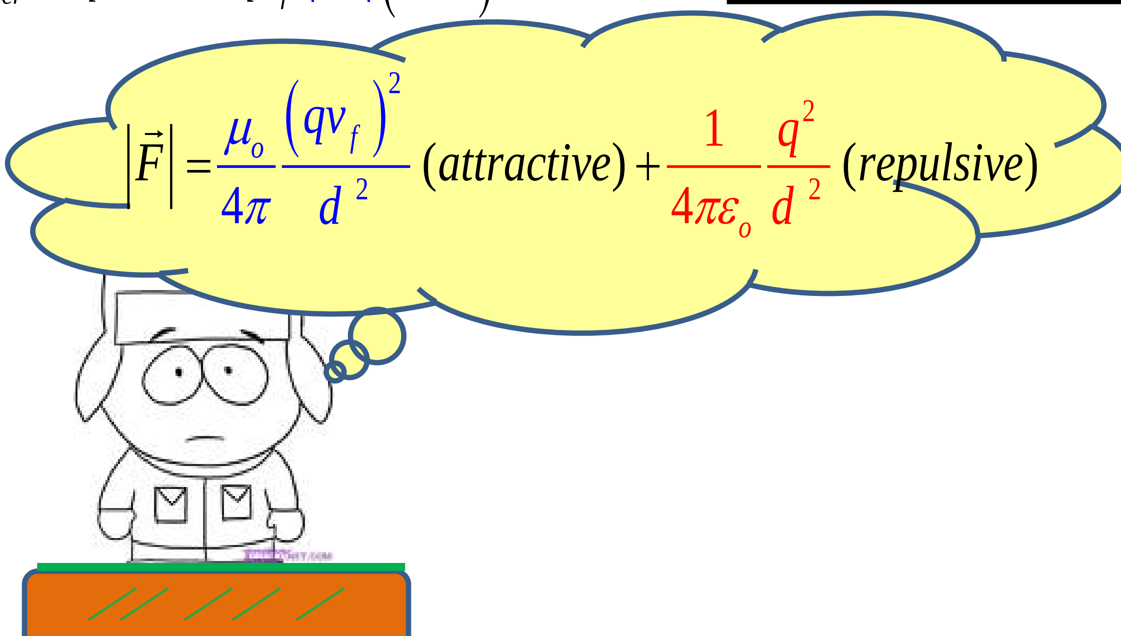


Hmmmm........

$$\vec{B}(at\ upper\ charge) = \frac{\mu_o}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}$$

$$= \frac{\mu_o}{4\pi}\frac{qv_f}{d^2}(\hat{i}\times\hat{j})$$

$$\vec{F}_{upper} = q\vec{v}\times\vec{B} = qv_f\ |B|\left(\hat{i}\times\hat{k}\right)$$



This is an experiment that allows you to predict the motion of a frame. Does this contradict Galileo's ideas about an **inertial frame of reference**?

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In contrast to Newtonian mechanics, E&M seems to give different answers depending on the frame of the observer?

How to reconcile this issue?

The Idea of the Aether (*circa* 1880)

• Light is a wave (Maxwell); all waves travel through a medium

•All of space must be filled with a medium called the **Lumeniferous Aether**. Light travels through the aether.

• What are the physical properties of the aether?
Kind of a jelly-like substance that must be very stiff, transparent, having zero density, and no viscosity, fills all of space…. It must be really hard to detect because no one has ever seen it!

• Light is a disturbance of this aether, just as water waves are a disturbance on the surface of water and sound waves are a disturbance in the density of air.

• The aether explains Maxwell's prediction for the speed of light! Evidently, Maxwell's calculation for c **MUST** be relative to the stationary aether. The stationary aether must therefore be a "privileged" frame of reference.

What is the rest frame for the aether?

It cannot be the earth because we now know the earth moves around the sun.

Maybe the ether is at rest wrt the sun? or maybe it is at rest wrt the center of the galaxy?

How to test? Carefully measure the speed of light on earth as the earth orbits the sun.

Up Next – the Michelson-Morley Experiment