PHYS 342
Fall semester 2014

L12.05: Relativistic time and distance intervals between two events

Ron Reifenberger
Birck Nanotechnology Center
Purdue University

Often, we want to know the **difference** in space and time between a **pair of point events**.

Label the events as 1 and 2.

In the S frame: $\Delta x = x_2 - x_1 \quad and \quad \Delta t = t_2 - t_1$

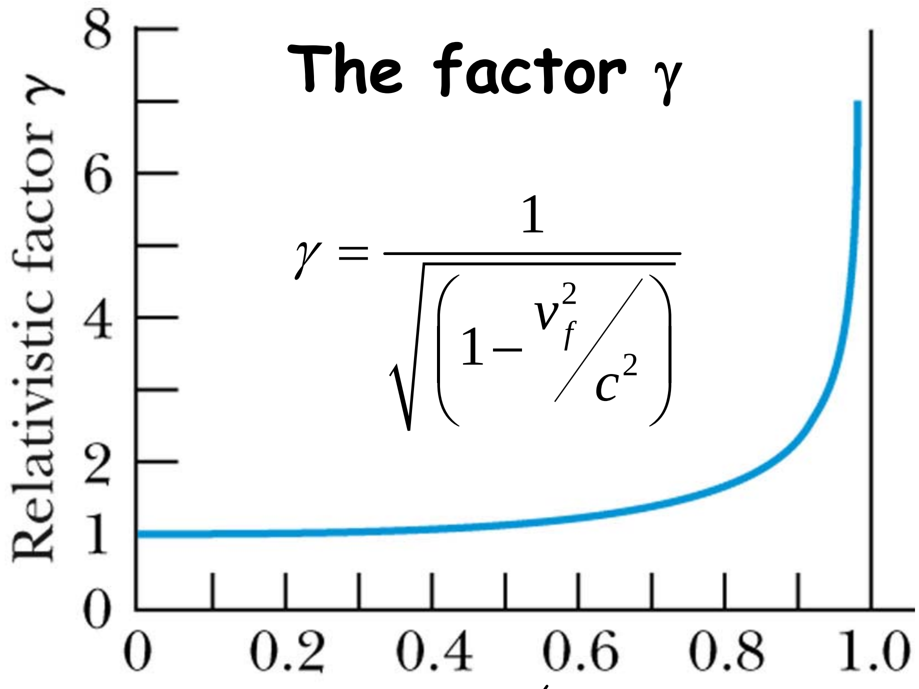
In the S′ frame: $\Delta x' = x'_2 - x'_1 \quad and \quad \Delta t' = t'_2 - t'_1$

$Then$

$$\boxed{\Delta x = \gamma\left(\Delta x' + v_f \Delta t'\right) \; and \; \Delta t = \gamma\left(\Delta t' + \frac{v_f}{c^2}\Delta x'\right)}$$

Be careful:
If events 1 and 2 occur **at the same place** in S′, then Δx′=0

If events 1 and 2 occur **at the same time** in S′, then Δt′=0



$$\gamma = \frac{1}{\sqrt{\left(1-{v_f^2}/{c^2}\right)}}$$

v_f / c

β	β² (exact)	β² (fract.)	γ	γ (exact)
0.5	0.25	1/4	2/√3	1.155
0.6	0.36	9/25	5/4	1.250
0.7	0.49	~ 1/2	~ √2	1.400
0.8	0.64	16/25	5/3	1.667
0.9	0.81	~ 4/5	~ √5	2.294

Often easier to deal with fractional values of β

A few important facts

1. Galilean transform is recovered when c -> ∞
2. The equations predict that when x'=0, t'=0 then x=0, t=0 (origins coincide and clocks are synchronized)
3. The velocity (v_f) of frame S' can never equal c

You can also solve for x' and t' in terms of x and t:

inverse transform

$$x' = \gamma\left[x - v_f t\right]$$

$$y' = y$$

$$z' = z$$

$$t' = \gamma\left[t - \frac{v_f x}{c^2}\right]$$

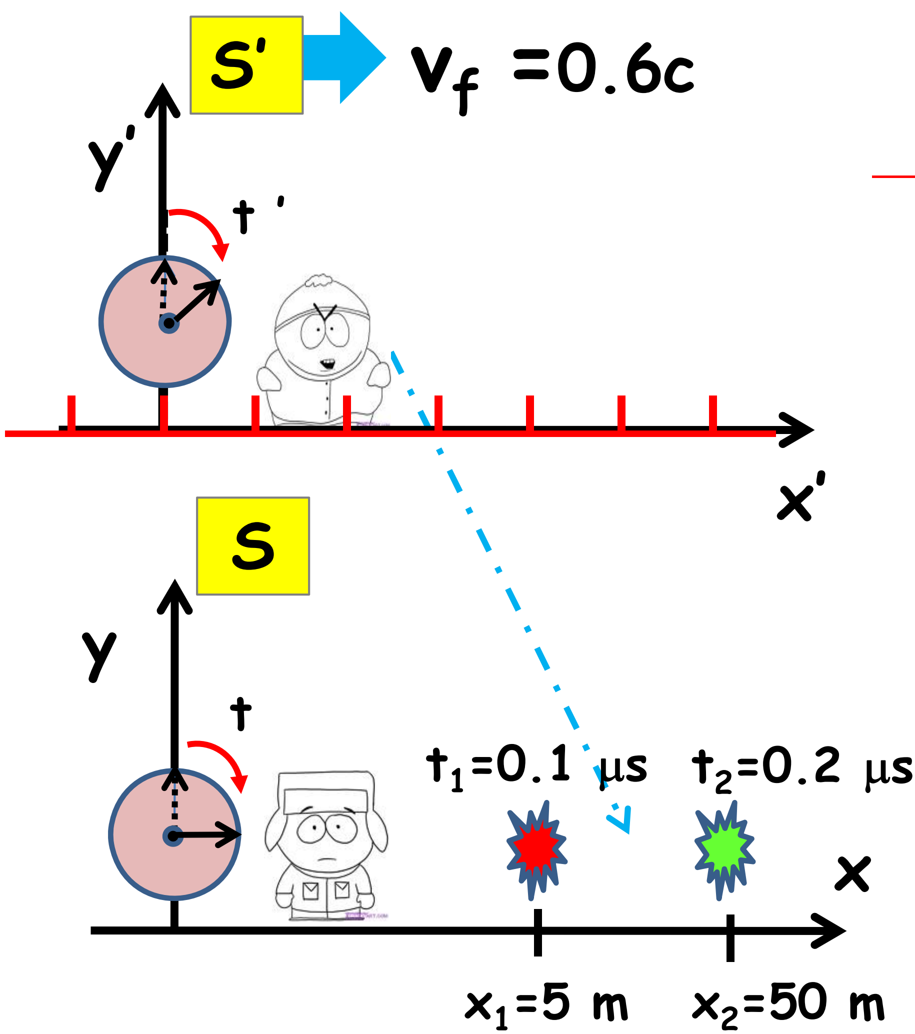
$$u' = \frac{u - v_f}{1 - \left(v_f / c^2\right) u}$$

where

$$\gamma = \frac{1}{\sqrt{\left(1 - v_f^2 / c^2\right)}} = \frac{1}{\sqrt{1-\beta^2}}; \quad \beta \equiv \frac{v_f}{c}$$

"Switch primes and change the sign of v_f"

Example: Separation of two Point Events in space. What would S' measure?



Lorentz-Einstein Transformation

$$\beta^2 = v_f^2 / c^2 = (0.6)^2 = \frac{9}{25}$$

$$\gamma = \frac{1}{\sqrt{1 - v_f^2 / c^2}} = \frac{1}{\sqrt{1 - 9/25}} = \sqrt{\frac{25}{16}} = \frac{5}{4}$$

Measure separation of two events in "S". You need " S' " coordinates on the LHS of equations:

$$\left.\begin{aligned} x_1' &= \gamma\left[x_1 - v_f t_1\right] \\ x_2' &= \gamma\left[x_2 - v_f t_2\right] \end{aligned}\right\} \quad x_2' - x_1' = \gamma\left[(x_2 - x_1) - v_f (t_2 - t_1)\right]$$

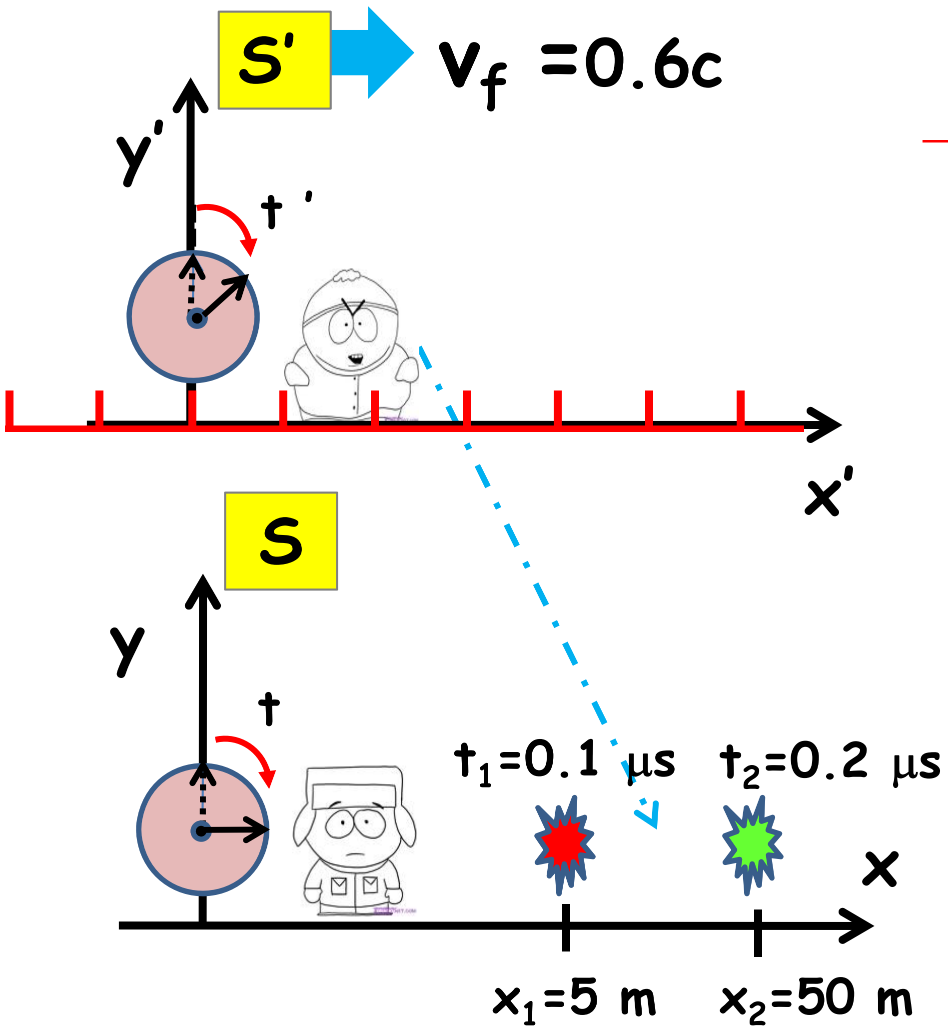
$$x_2' - x_1' = \gamma\left[(x_2 - x_1) - v_f (t_2 - t_1)\right]$$

$$= \frac{5}{4}\left[(50 - 5) - 0.6c\left(2\times10^{-7} - 1\times10^{-7}\right)\right]$$

$$= \frac{5}{4}\left[(45\ m) - 0.6\bullet3\times10^{8}\left(1\times10^{-7}\right)\right]$$

$$= \frac{5}{4}\left[(45\ m) - (18\ m)\right] = 33.75\,m$$

Example: Separation of Two Point Events in Time



Lorentz-Einstein Transformation

$$\beta^2 = v_f^2/c^2 = (0.6)^2 = \frac{9}{25}$$

$$\gamma = \frac{1}{\sqrt{1-v_f^2/c^2}} = \frac{1}{\sqrt{1-9/25}} = \sqrt{\frac{25}{16}} = \frac{5}{4}$$

Measure separation of two events in "S". You need " S' " coordinates on the LHS of equations:

$$\left.\begin{aligned} t_1' &= \gamma\left[t_1 - \frac{v_f\, x_1}{c^2}\right] \\ t_2' &= \gamma\left[t_2 - \frac{v_f\, x_2}{c^2}\right] \end{aligned}\right\} \quad t_2' - t_1' = \gamma\left[(t_2 - t_1) - \frac{v_f}{c^2}(x_2 - x_1)\right]$$

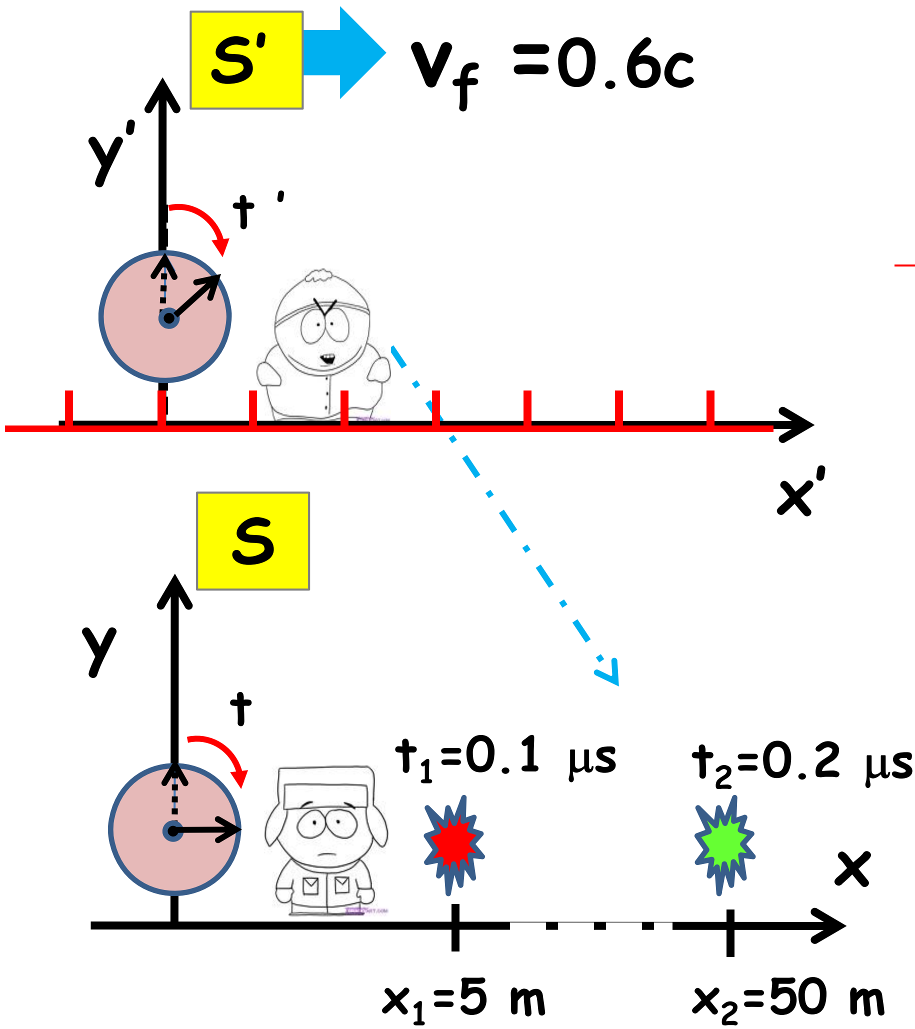
$$t_2' - t_1' = \gamma\left[(t_2 - t_1) - \frac{v_f}{c^2}(x_2 - x_1)\right]$$

$$= \frac{5}{4}\left[\left(2\times10^{-7} - 1\times10^{-7}\right) - \frac{0.6c}{c^2}(50-5)\right]$$

$$= \frac{5}{4}\left[\left(1\times10^{-7}\right) - \frac{0.6}{3\times10^{8}}(45)\right]$$

$$= \frac{5}{4}\left[1.0\times10^{-8}\right]s = 1.25\times10^{-8}\,s = 0.0125\,\mu s$$

Example: Separation of Two Point Events in Space and Time



INCORRECT Results using Galilean transforms

$$x' = x - x_f = x - v_f t = x - 0.6ct$$

$$t' = t$$

$$t_1' = t_1 = 0.1\ \mu s$$
$$t_2' = t_2 = 0.2\ \mu s$$
$$t_2' - t_1' = 0.1\ \mu s$$

$$\left.\begin{array}{l} x_1' = x_1 - 0.6ct_1 \\ x_2' = x_2 - 0.6ct_2 \end{array}\right\} \quad x_2' - x_1' = x_2 - 0.6ct_2 - [x_1 - 0.6ct_1]$$

$$\begin{aligned} x_2' - x_1' &= (x_2 - x_1) - 0.6c(t_2 - t_1) \\ &= (50 - 5) - 0.6 \bullet 3 \times 10^8 (0.1 \times 10^{-6} s) \\ &= 45 - 18 = 27\ m \end{aligned}$$

Note: In 0.1 μs, S' travels 18 m.

Special relativity is not difficult mathematically.

But it becomes difficult because you must be very careful about who measures what about an event and how the measurement is made.

In addition, the results contradict everyday experience.

Up Next – time dilation