PHYS 342
Fall semester 2014

L12.06: Time Dilation

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In previous lecture, we considered unrelated (and somewhat arbitrary) "point events" separated in time and space.

What happens when the separation in space is correlated with the length of an object? Or when a time interval is correlated to successive ticks of a clock?

Length contraction and **time dilation** are two of the most surprising implications of the Lorentz-Einstein transformations of special relativity.

The True Nature of a Measurement?

What is Space?

Space is what we measure with a measuring rod

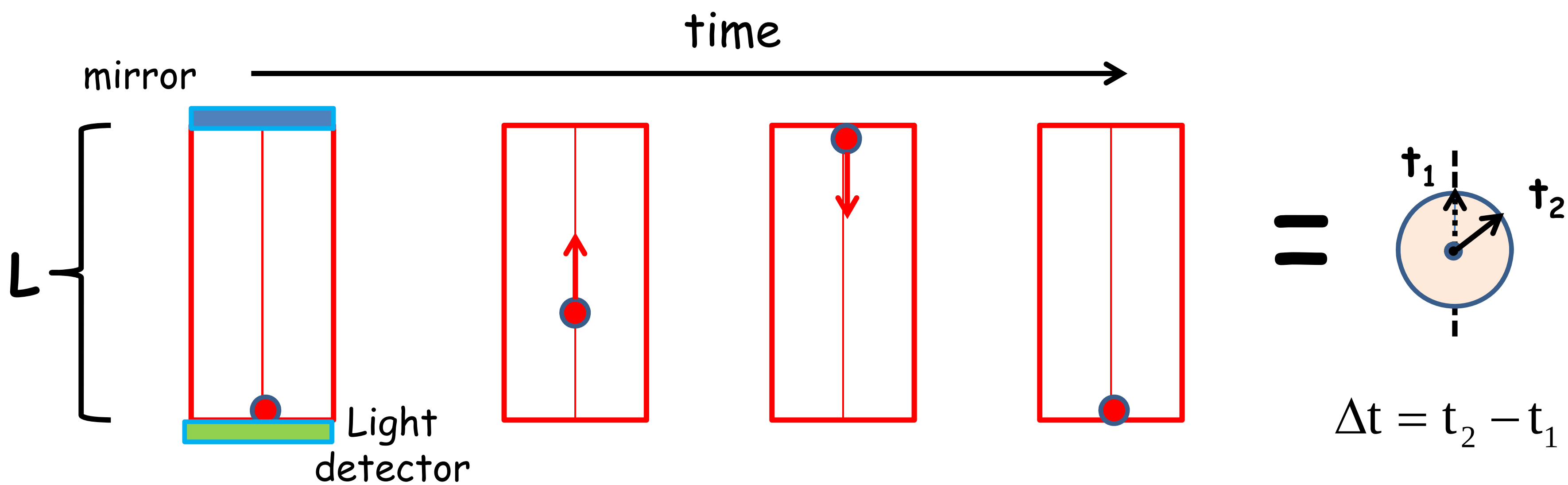
What is Time?

Time is what we measure with a clock

NOTE
In some of the examples that follow, we will require the **difference** in coordinates between a pair of point events. In other examples, the coordinate of some object (like a clock) will remain the same. You **must** be able to distinguish between these two cases.
(see last lecture.)

How do we tell time?

Light Clock (no mechanical parts)



$$\Delta t = t_2 - t_1$$

t=0 emit light flash

$$\Delta t = \frac{2L}{c}$$

one "tick" later

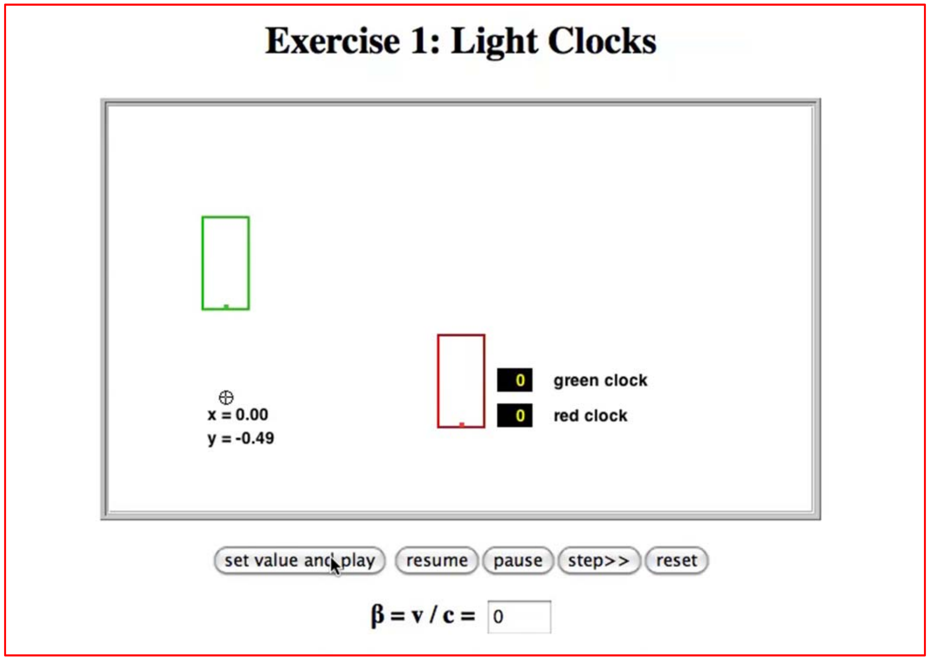
if L=1.5 m, then

$$\Delta t = \frac{2L}{c} = \frac{2(1.5\,\text{m})}{3.0\times10^{8}\ \text{m/s}} = 1.0\times10^{-8}\,\text{s} = 10\ \text{ns}$$

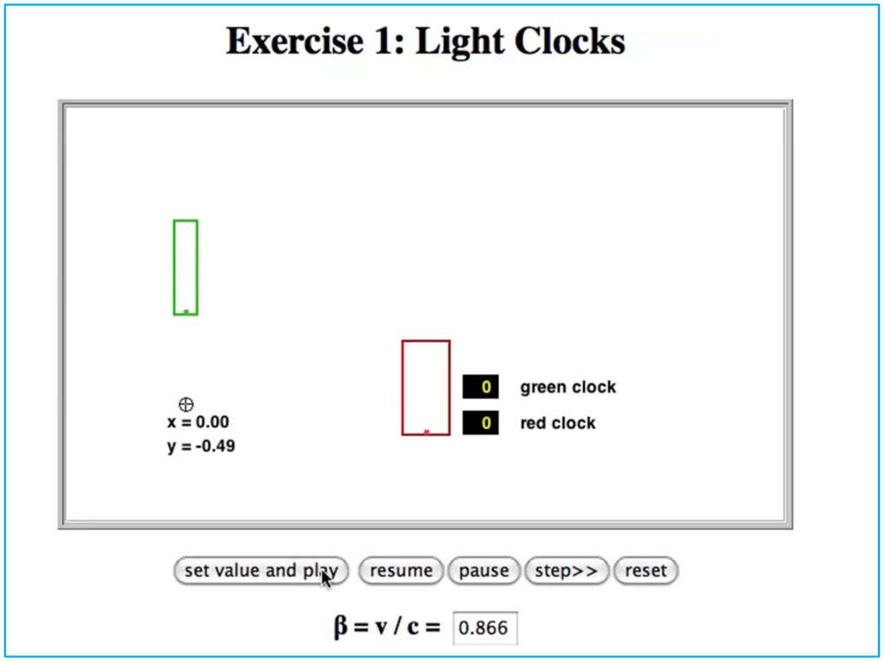
Measuring time?

Light Clock (no mechanical parts)

Both clocks stationary



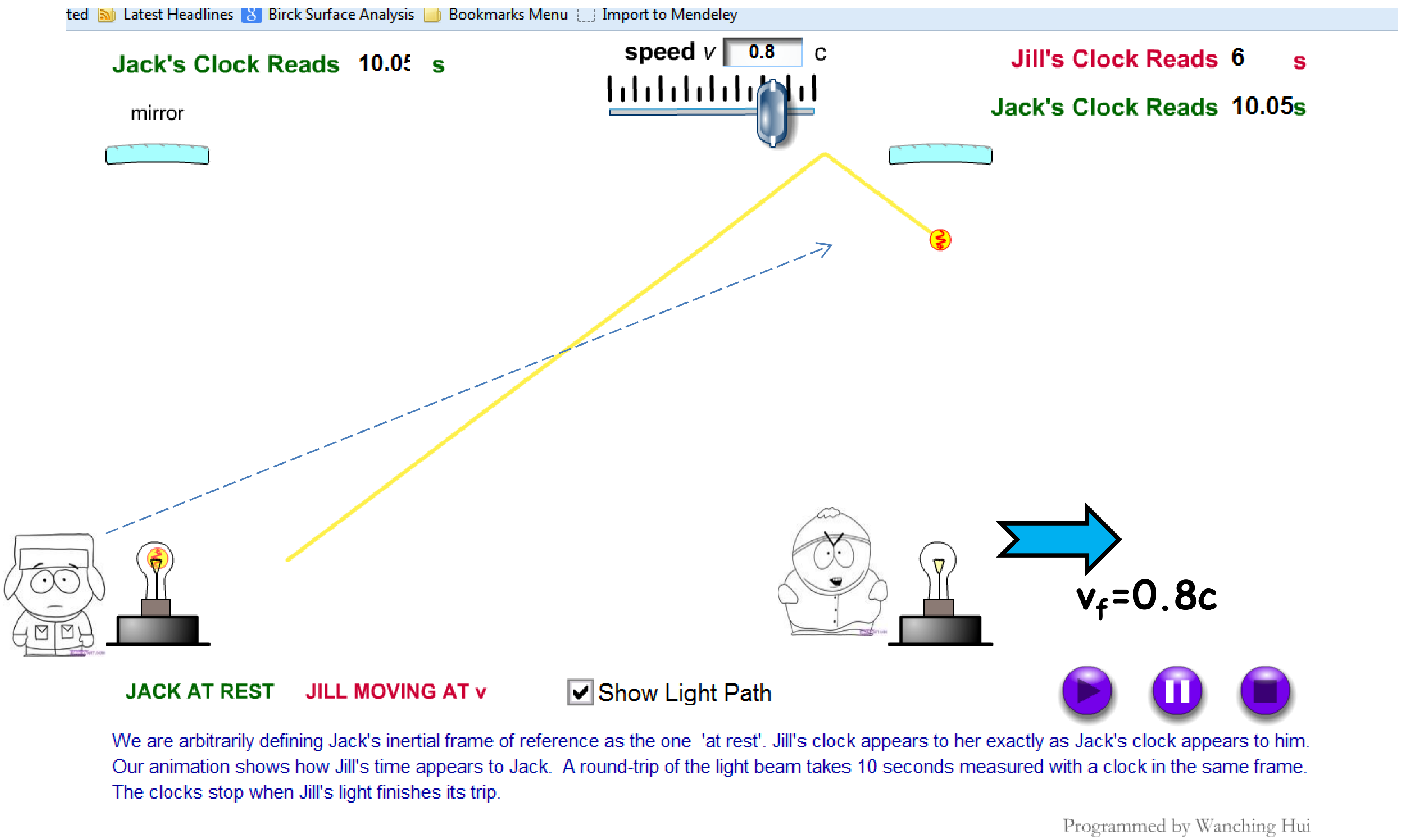
Green clock moving



Time depends on velocity!

http://webphysics.davidson.edu/course_material/py230l/relativity/relativity-ex1.htm

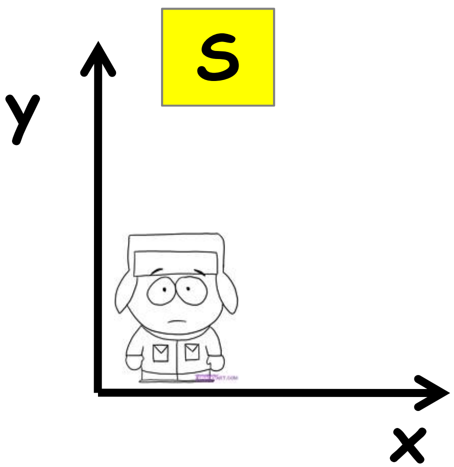
Another Simulation

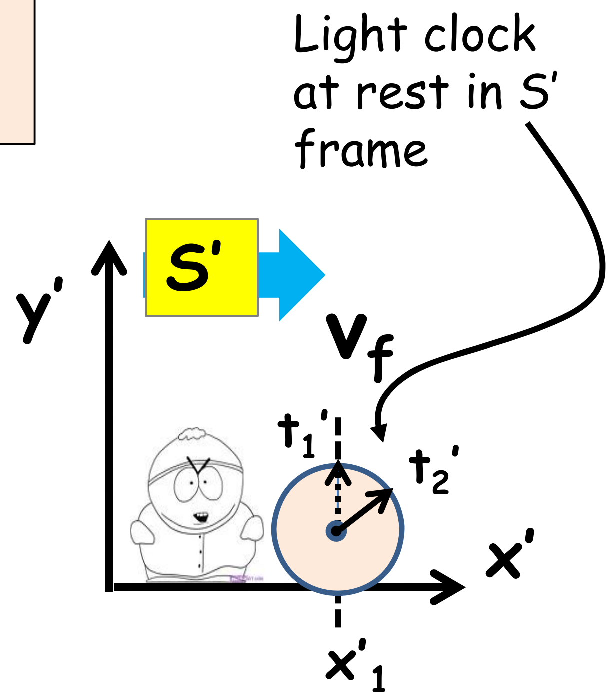


http://galileoandeinstein.physics.virginia.edu/more_stuff/flashlets/lightclock.swf

Time Dilation

Key Idea: Measure time interval in "S' ". You need "S" coordinates on the LHS of Lorentz equations.





Takes you from S ' to S

$$x = \gamma\left[x' + v_f t'\right]$$

$$y = y'$$

$$z = z'$$

$$t = \gamma\left[t' + \frac{v_f x'}{c^2}\right]$$

$$u = \frac{u' + v_f}{1 + \left(v_f / c^2\right)u'}$$

Trust the equations:

$$t_1 = \gamma\left[t_1' + \frac{v_f x_1'}{c^2}\right] \qquad t_2 = \gamma\left[t_2' + \frac{v_f x_1'}{c^2}\right]$$

$$t_2 - t_1 = \gamma\left(t_2' - t_1'\right)$$

$$\Delta t = \gamma\,\Delta t'; \quad \gamma > 1$$

A correct statement:
"Kyle in S measures a longer time between ticks on Cartman's clock in S' than does Cartman."

What about other clocks?

All clocks are based on a periodic event:

- water spilling from a hole in a bucket (water clock),
- the swing of a pendulum (grandfather clock),
- periodic spring compression (spring clock; wristwatch from 1970s),
- vibration of a quartz crystal (all modern clocks),
- biological clocks

Each periodic event depends on a complicated combination of phenomena such as the forces between atoms and molecules and on Newton's laws.

If the time a stationary clock registers did not differ from the time a moving clock registers by a factor of γ, then we would conclude that the laws of mechanics, electromagnetism, etc. are different between the two frames. This is contrary to the fundamental principle of relativity.

So yes, time dilation does affect all clocks, including biological clocks as well.

Tabulating Some Numbers

$$\gamma = \frac{1}{\sqrt{\left(1 - \frac{v_f^2}{c^2}\right)}} = \frac{1}{\sqrt{1-\beta^2}}; \quad \beta \equiv \frac{v_f}{c}$$

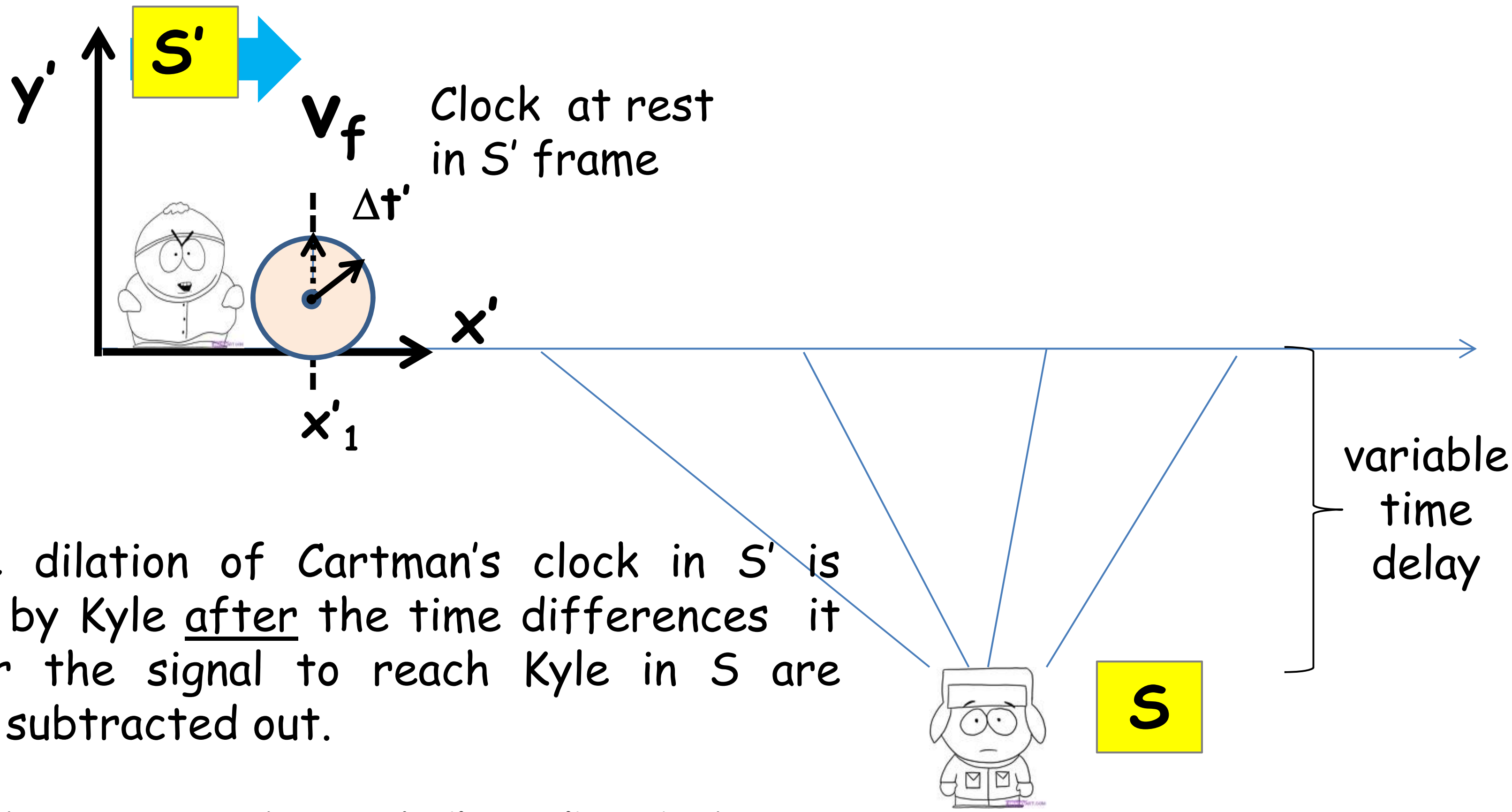
$$t_2 - t_1 = \gamma\left(t_2' - t_1'\right)$$

Let $\left(t'_2 - t'_1\right)$ = **1 s**

$\beta = v_f/c$	0.100	0.300	0.600	0.800	0.900	0.950	0.990
$\sqrt{1-\beta^2}$	0.995	0.954	0.800	0.600	0.436	0.312	0.141
$\gamma = 1/\sqrt{1-\beta^2}$	1.005	1.048	1.250	1.667	2.294	3.205	7.092
$t_2 - t_1$	1.005 s	1.048 s	1.250 s	1.667 s	2.294 s	3.205 s	7.092 s

A given time interval in a moving frame is LONGER when measured from a rest frame. It appears that time in the moving frame is "running slow".

A subtle point

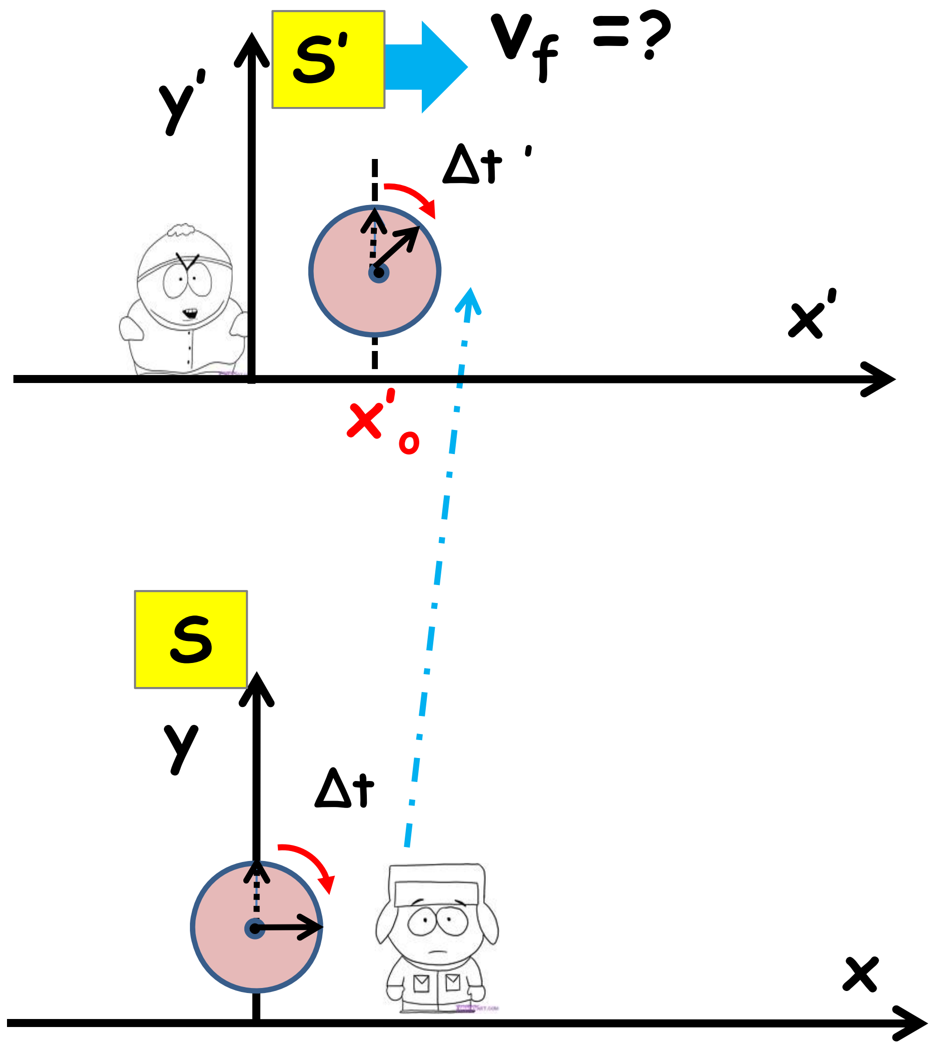


The time dilation of Cartman's clock in S' is **observed** by Kyle <u>after</u> the time differences it takes for the signal to reach Kyle in S are correctly subtracted out.

Distinguish between what Kyle "**sees**" and what he "**observes**" or "**measures**".

Measurements or Observations become possible in a **Reference Frame** which is a well-equipped lab designed for such measurements. More on this later.

A clock in a spaceship runs at 3/5 the time of an identical clock on earth. How fast is the spaceship moving?



$$t = \gamma\left[t' + \frac{v_f x_o'}{c^2}\right]$$

calculate time difference

$$\therefore \quad \Delta t = \gamma\, \Delta t'$$

$$\Delta t' = \frac{3}{5}\Delta t \quad (given)$$

$$\Rightarrow \gamma = \frac{5}{3} = \frac{1}{\sqrt{1 - v_f^2 / c^2}}$$

$$v_f = 0.8c$$

Another Example – Time to traverse a fixed distance

S frame	S′ frame
$t_1=?$	$t_1'=?$
$t_2=?$	$t_2'=?$
$\Delta t = t_2-t_1 = ?$	$\Delta t' = t_2'-t_1' = ?$

a) $t_1=t_1'=0$ – clocks synchronized
b) $\Delta t=t_2$ and $\Delta t'=t_2'$
c) $t_2=x_o/v_f$
d) what is t_2'?

$$t = \gamma\left[t' + \frac{v_f x_o'}{c^2}\right] \qquad \textit{but } t' \textit{ is required ?}$$

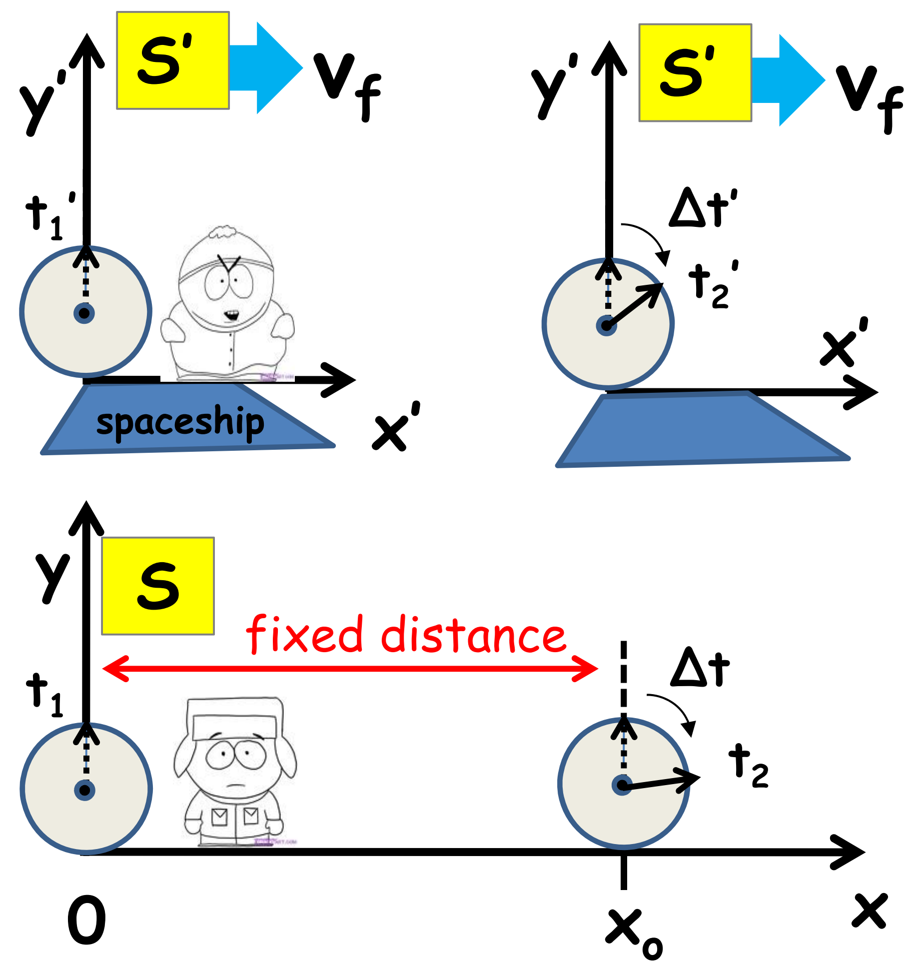
switch the primes, $v_f \rightarrow -v_f$

$$t' = \gamma\left[t - \frac{v_f x_o}{c^2}\right]$$

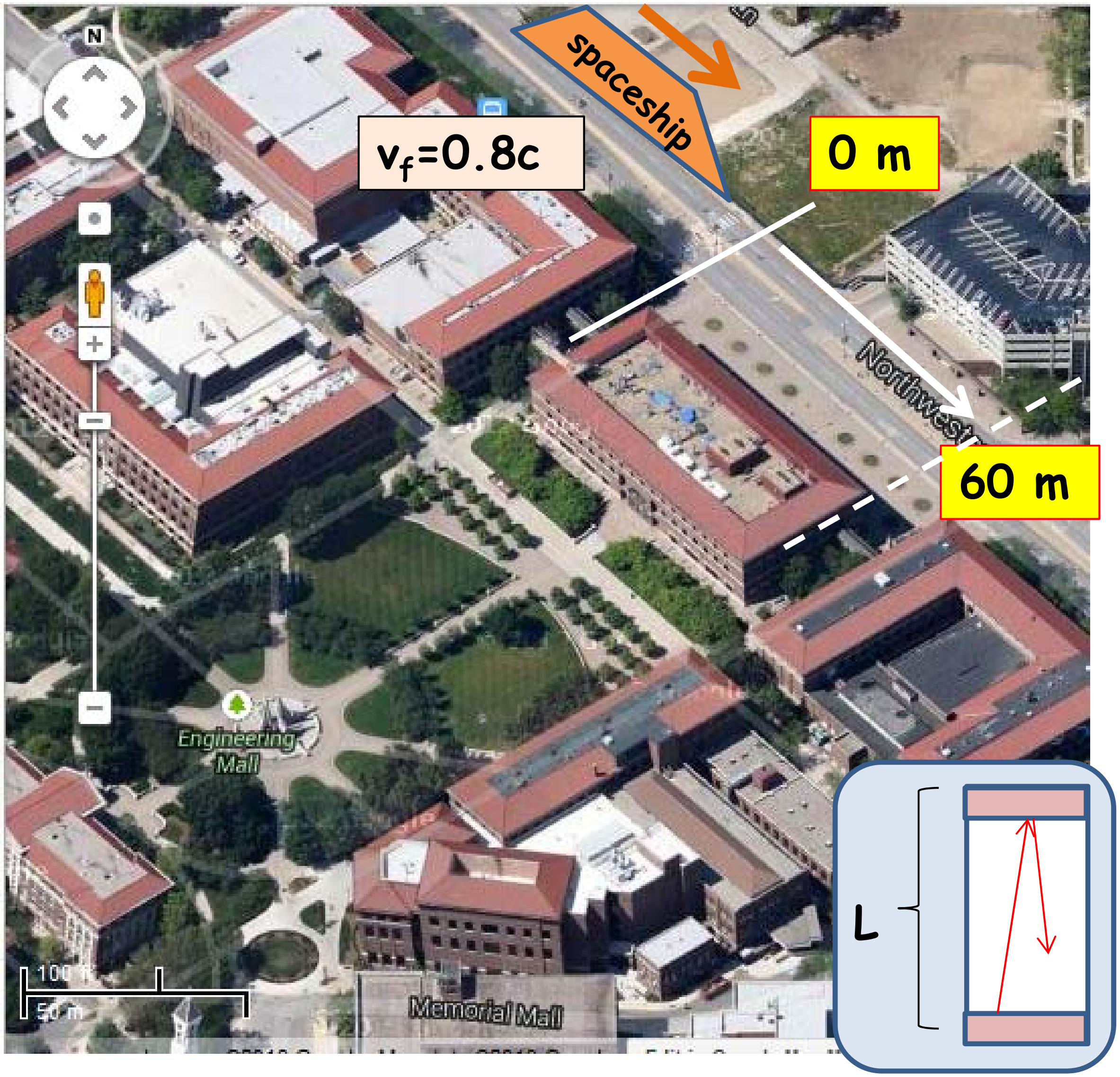
$$t_2' = \gamma\left[t_2 - \frac{v_f x_o}{c^2}\right] = \gamma\left[\frac{x_o}{v_f} - \frac{v_f x_o}{c^2}\right]$$

$$= \gamma\frac{x_o}{v_f}\left[\frac{c^2 - v_f^2}{c^2}\right] = \frac{\gamma x_o}{v_f}\left[\frac{1}{\gamma^2}\right] = \frac{1}{\gamma}\frac{x_o}{v_f}$$

$$= \frac{1}{\gamma}t_2 \qquad \Rightarrow \qquad \Delta t' = \frac{1}{\gamma}\Delta t$$



Time to Fly by MSEE



$$v_f = 0.8c \quad \Rightarrow \quad \gamma = 1.667$$

on Purdue mall :

$$t_2 = \frac{x_o}{v_f} = \frac{60\ m}{0.8c} = 25\mu s$$

on space ship :

$$t_2' = \gamma\left[t_2 - \frac{v_f x_o}{c^2}\right] = \gamma\left[\frac{x_o}{v_f} - \frac{v_f x_o}{c^2}\right]$$

$$= \gamma \frac{x_o}{v_f}\left[1 - \frac{v_f^2}{c^2}\right] \qquad \gamma = \frac{1}{\sqrt{1 - \frac{v_f^2}{c^2}}}$$

$$= \frac{1}{\gamma}\frac{x_o}{v_f} = \frac{25\mu s}{1.667} = 15\mu s$$

Problem suggests natural time for 1 tick (Δt) is 1 μs.

$$\Delta t = \frac{2L}{c} \quad \Rightarrow \quad L = \frac{c\,\Delta t}{2}$$

$$L = \frac{\left(2.998\times10^8\ {}^{m}\!/\!_{s}\right)\left(1\times10^{-6}\ s\right)}{2}$$

$$= 149.9m$$

Temporal duration depends on the observer!

Up Next – Length Contraction