Spring 2019 Purdue University

ECE 255: L17

BJT Transistor Amplifiers

(Sedra and Smith, 7th Ed., Sec. 7.1, 7.2.2)

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Announcements

1) LTspice Project 2 is posted. The due date for it is Wed. Feb. 27th by 5:00PM electronically. You should find your assigned your beta value in your grade book.

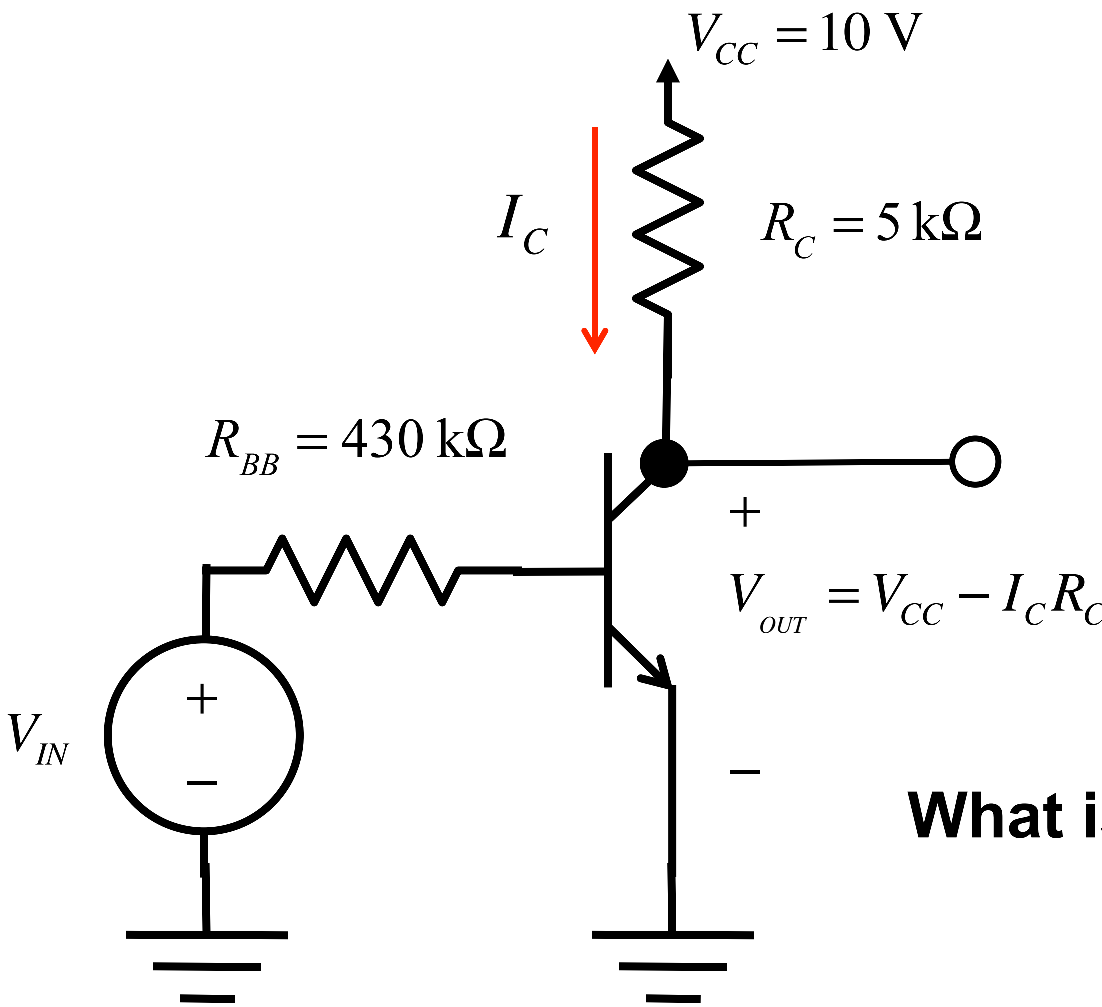
2) HW6 will be posted today

3) Exam 2 in on Tuesday, March 5, 6:30-7:30 PM PHYS 112

Outline

1) Voltage transfer characteristic (VTC)
2) Small signal model for BJT
3) Small signal analysis

Voltage Transfer Characteristic

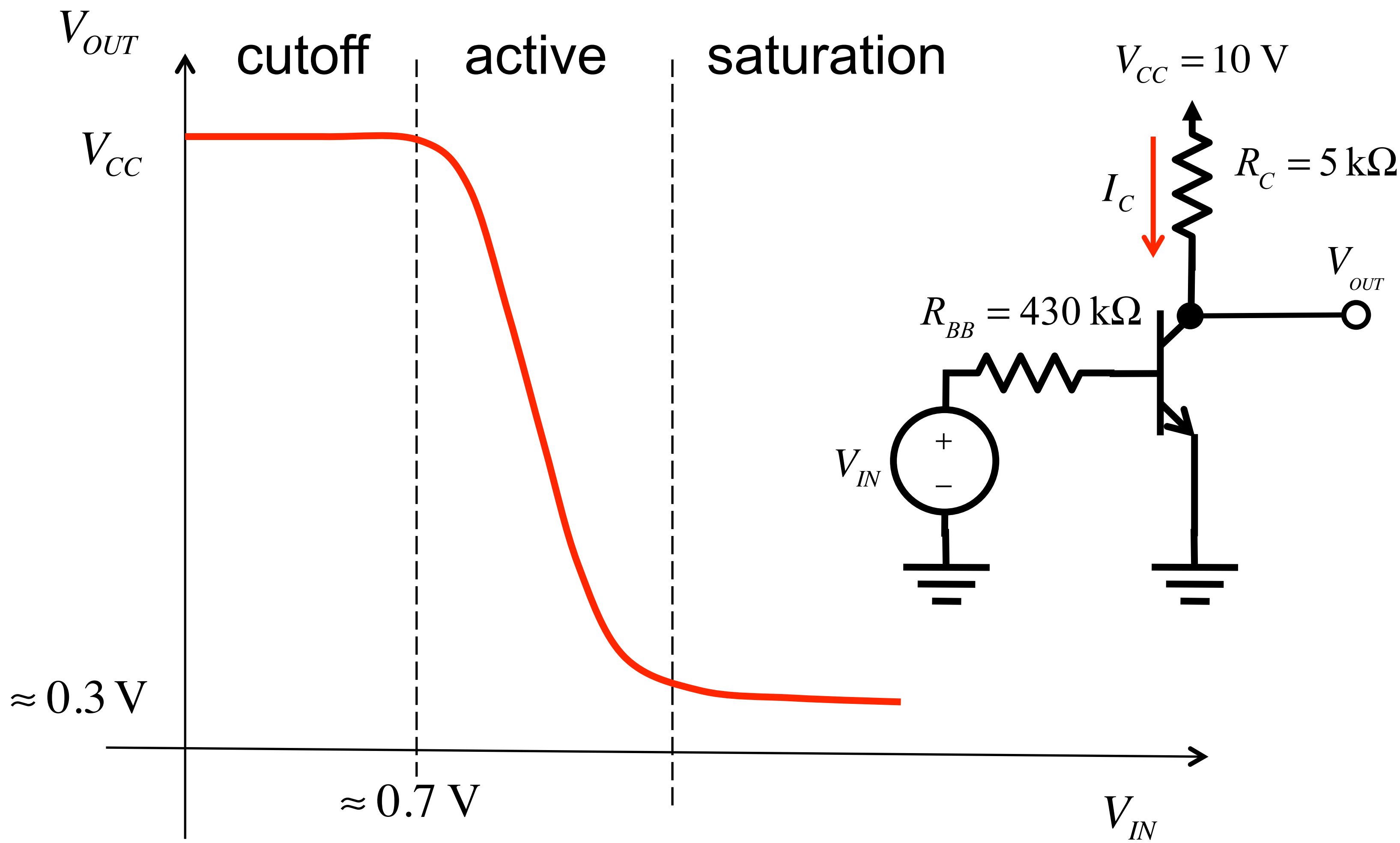


What is V_{OUT} vs. V_{IN}?

$\beta = 100$

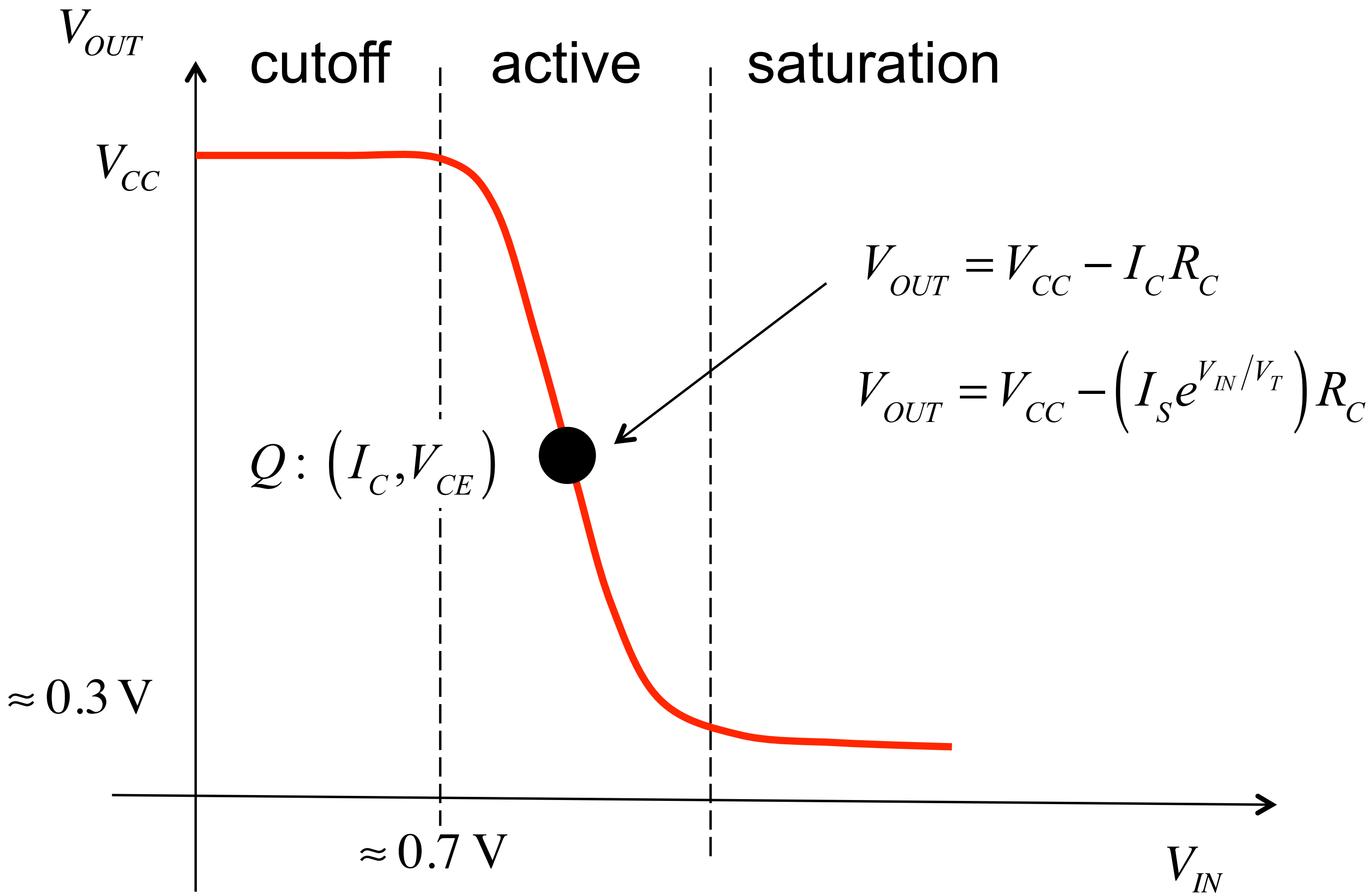
$V_{BE}(\text{ON}) = 0.7\text{ V}$

VTC

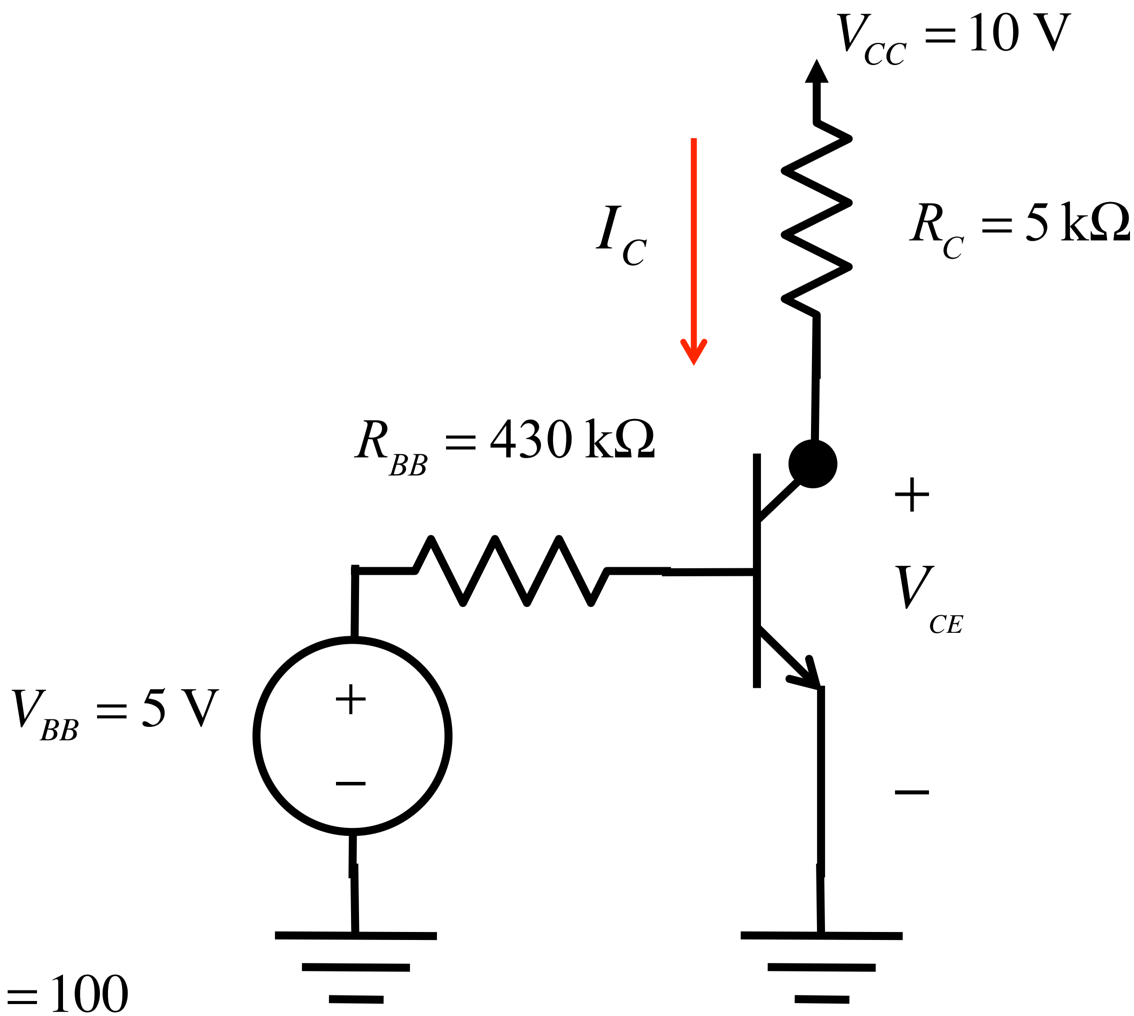


V_{OUT}
cutoff
active
saturation
V_{CC}
≈ 0.3 V
≈ 0.7 V
V_{IN}
$V_{CC} = 10$ V
I_C
$R_C = 5$ kΩ
V_{OUT}
$R_{BB} = 430$ kΩ
V_{IN}
+
–

Biasing in the active region



Bias Circuit



$I_B = ?\text{ mA}$

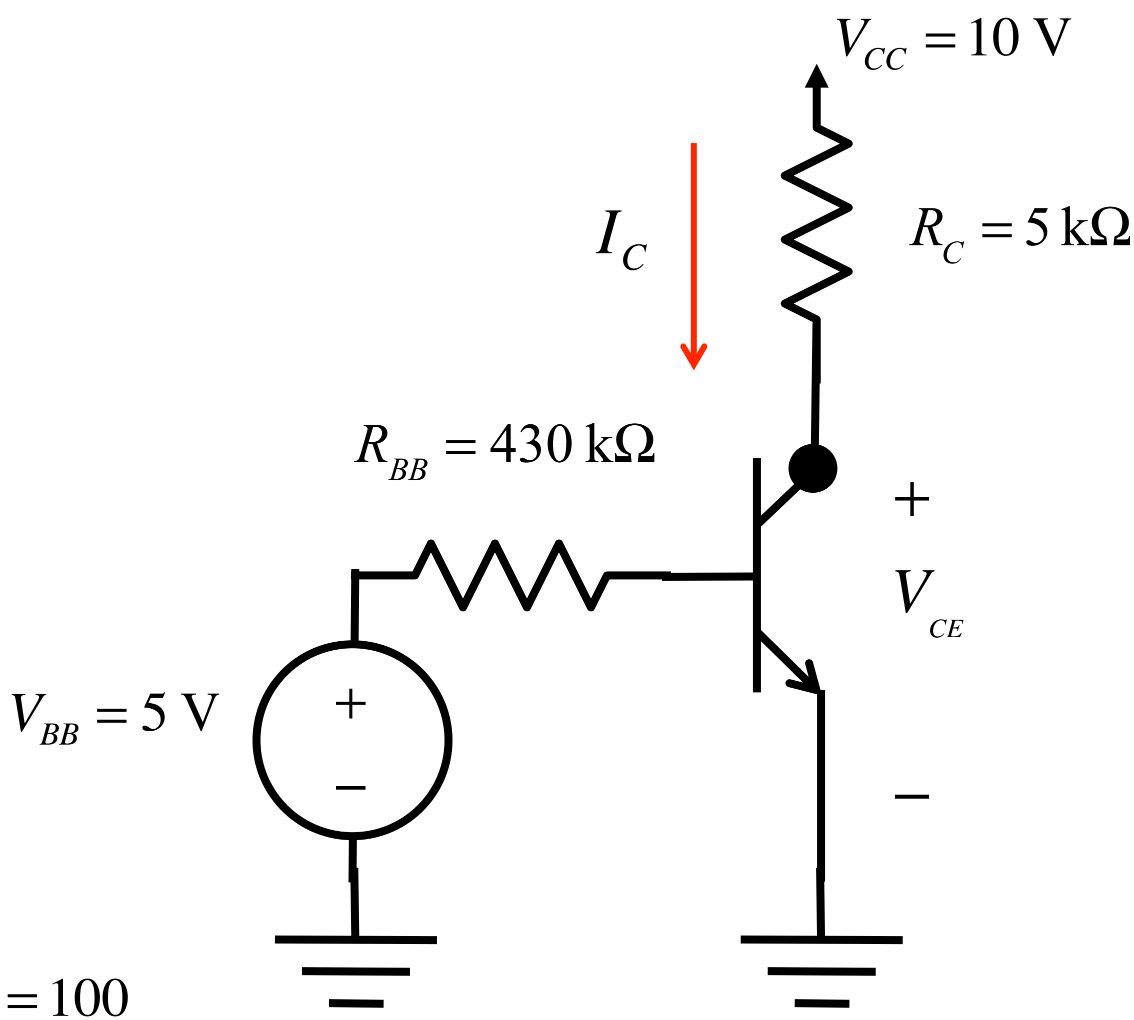
$I_C = ?\text{ mA}$

$V_{CE} = ?\text{ V}$

$\beta = 100$

$V_{BE}(\text{ON}) = 0.7\text{ V}$

Bias Circuit



$I_B = 0.01$ mA

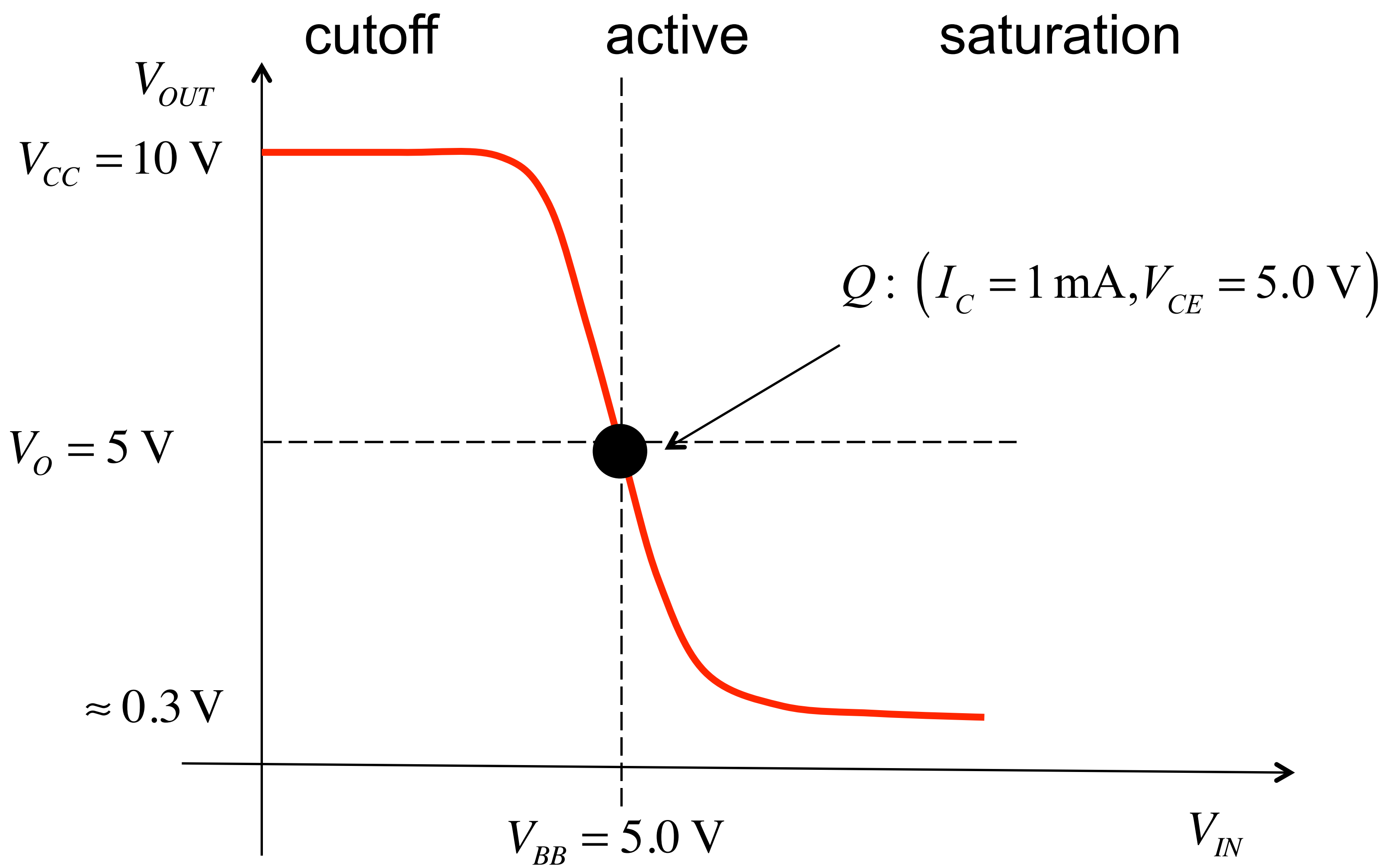
$I_C = 1.0$ mA

$V_{CE} = 5.0$ V

$\beta = 100$

$V_{BE}(\mathrm{ON}) = 0.7$ V

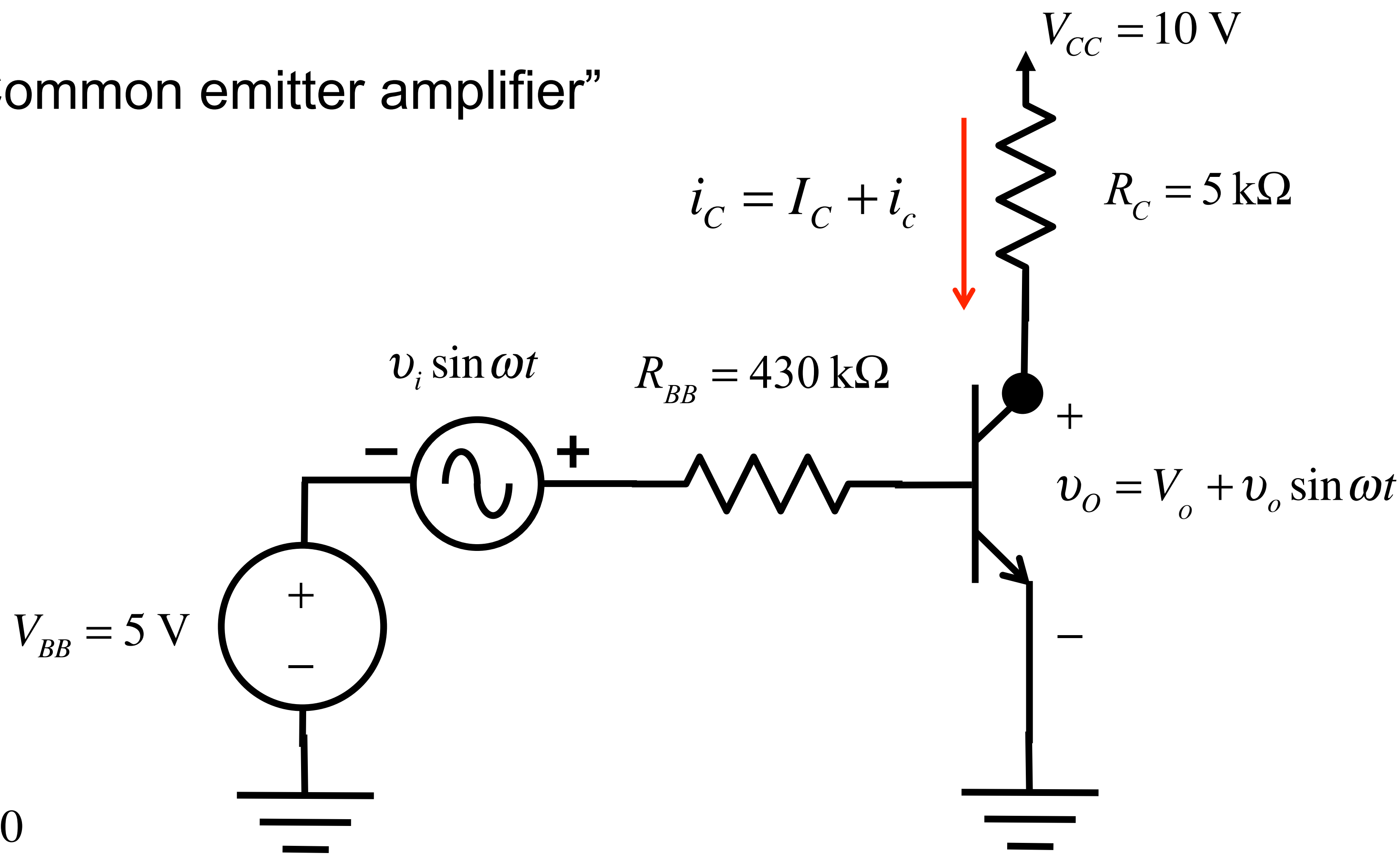
Biasing



cutoff
active
saturation
V_{OUT}
$V_{CC} = 10\ \mathrm{V}$
$Q: \left(I_C = 1\ \mathrm{mA}, V_{CE} = 5.0\ \mathrm{V}\right)$
$V_O = 5\ \mathrm{V}$
$\approx 0.3\ \mathrm{V}$
$V_{BB} = 5.0\ \mathrm{V}$
V_{IN}

Add ac small signal

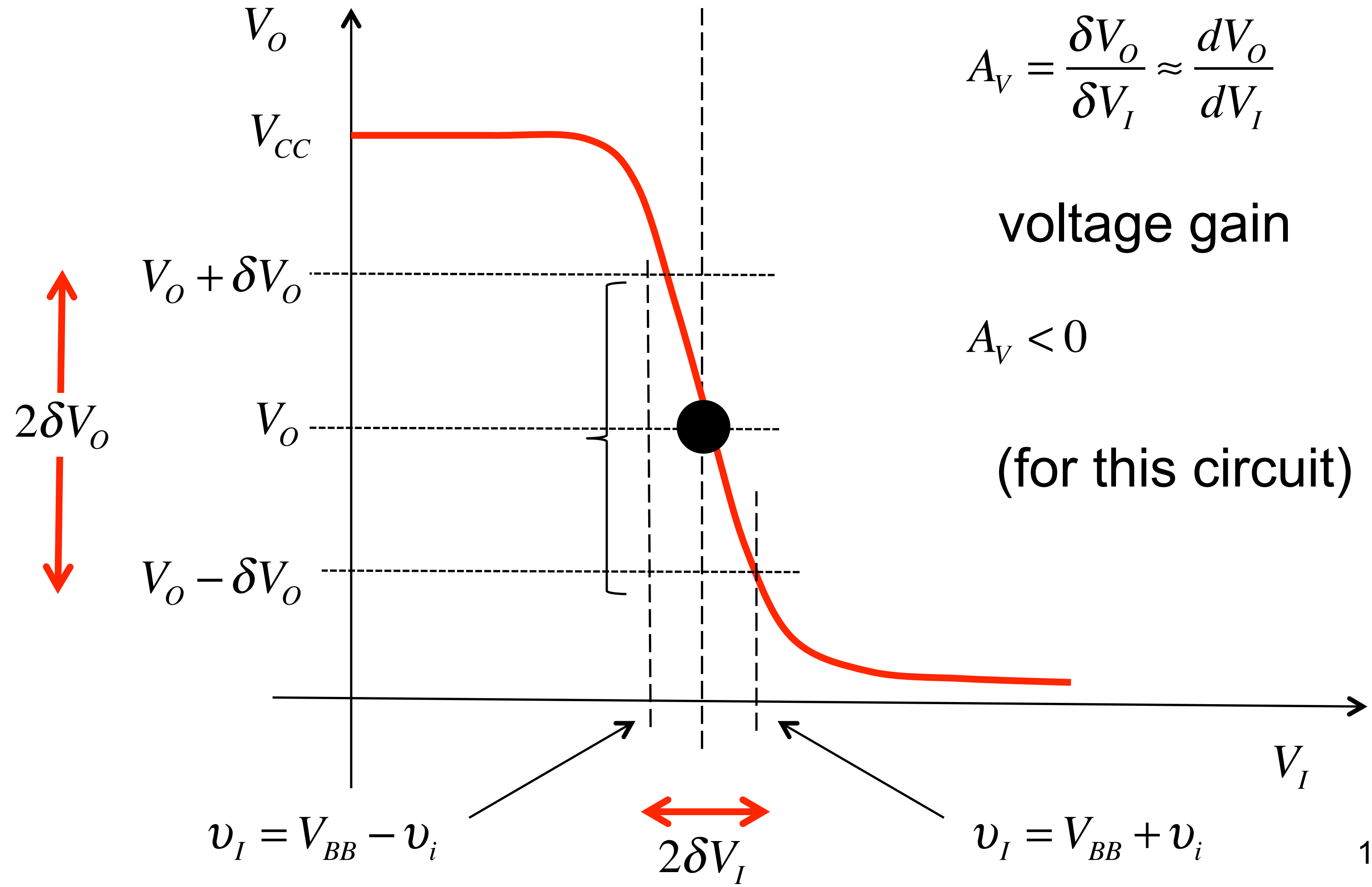
"Common emitter amplifier"



$\beta = 100$

$V_{BE}(\text{ON}) = 0.7\text{ V}$

Qualitative



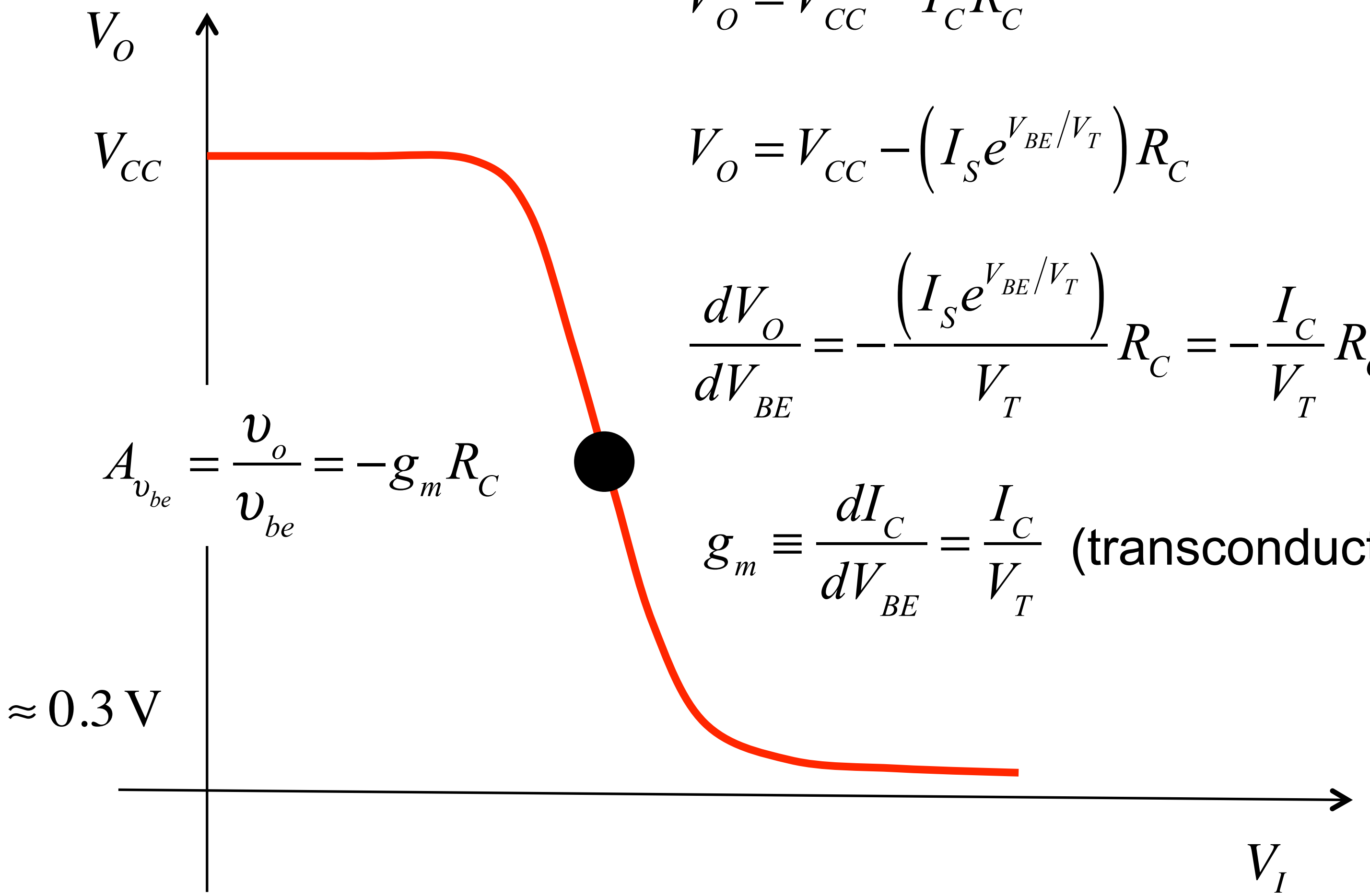
$$A_V = \frac{\delta V_O}{\delta V_I} \approx \frac{dV_O}{dV_I}$$

voltage gain

$$A_V < 0$$

(for this circuit)

Qualitative



$$A_{\upsilon_{be}} = \frac{\upsilon_o}{\upsilon_{be}} = -g_m R_C$$

$$V_O = V_{CC} - I_C R_C$$

$$V_O = V_{CC} - \left(I_S e^{V_{BE}/V_T}\right) R_C$$

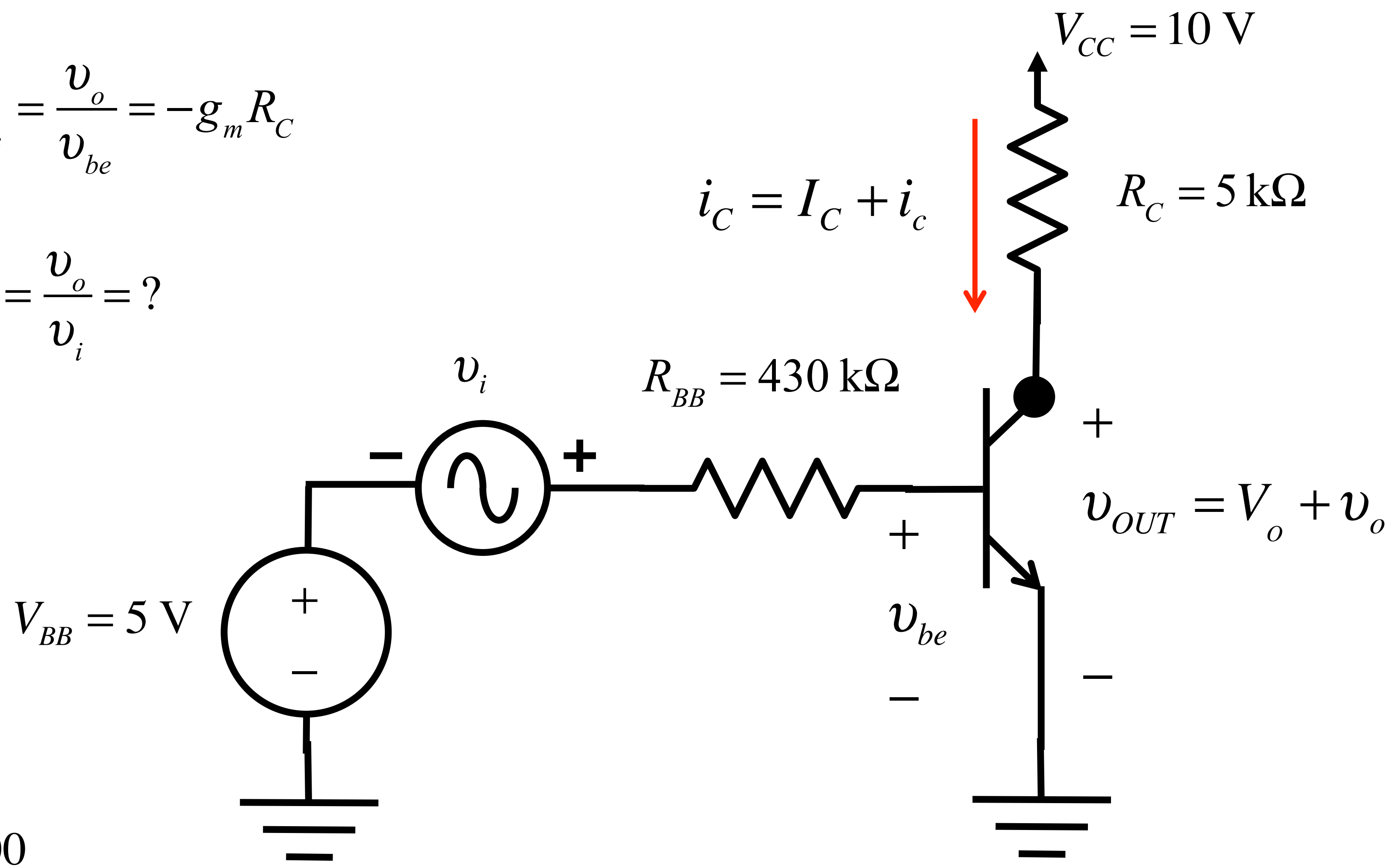
$$\frac{dV_O}{dV_{BE}} = -\frac{\left(I_S e^{V_{BE}/V_T}\right)}{V_T} R_C = -\frac{I_C}{V_T} R_C$$

$$g_m \equiv \frac{dI_C}{dV_{BE}} = \frac{I_C}{V_T}$$ (transconductance)

Voltage gain of CE amplifier

$$A_{v_{be}} = \frac{v_o}{v_{be}} = -g_m R_C$$

$$A_{v_i} = \frac{v_o}{v_i} = ?$$

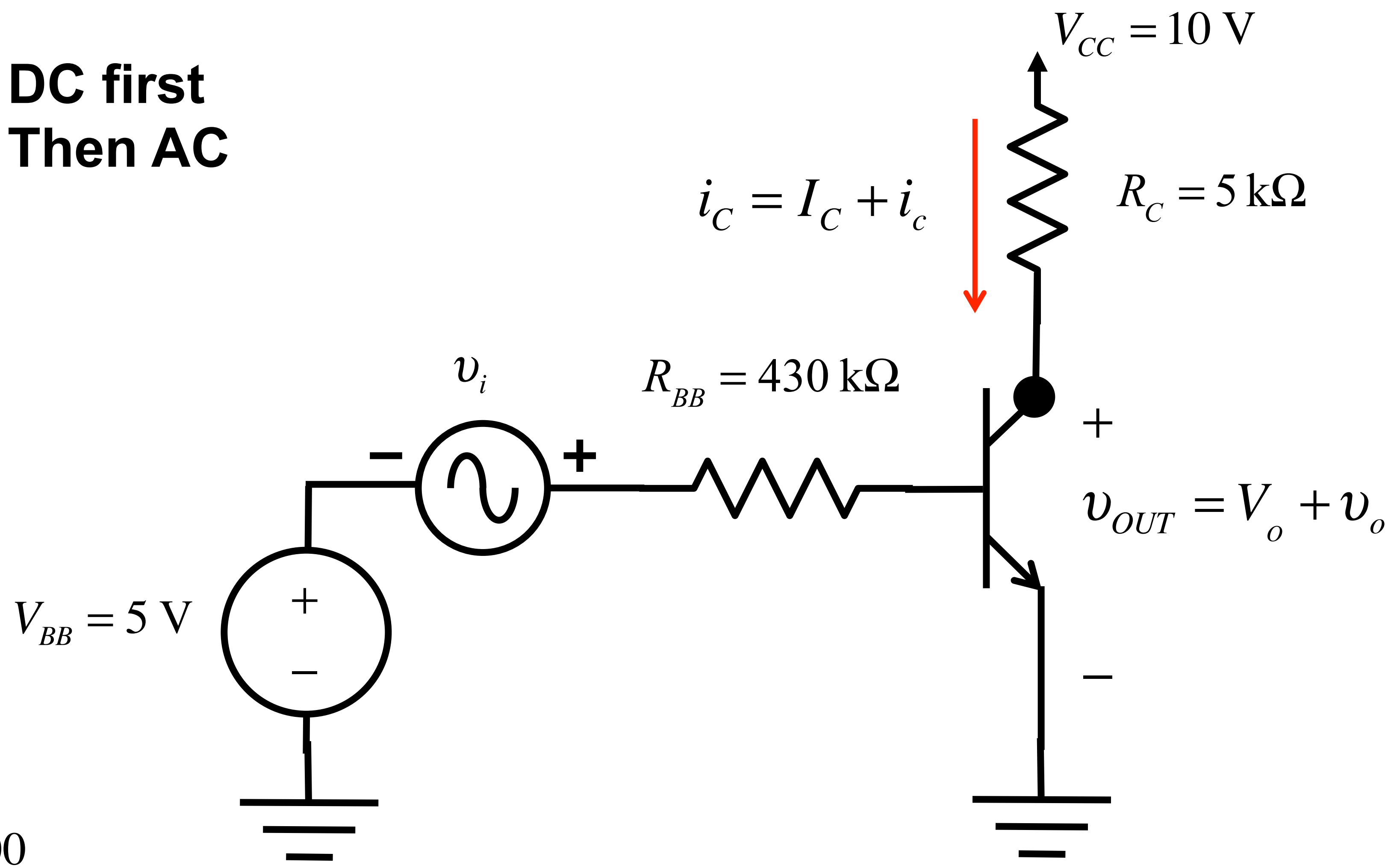


$\beta = 100$

$V_{BE}(\text{ON}) = 0.7\text{ V}$

Analysis by superposition

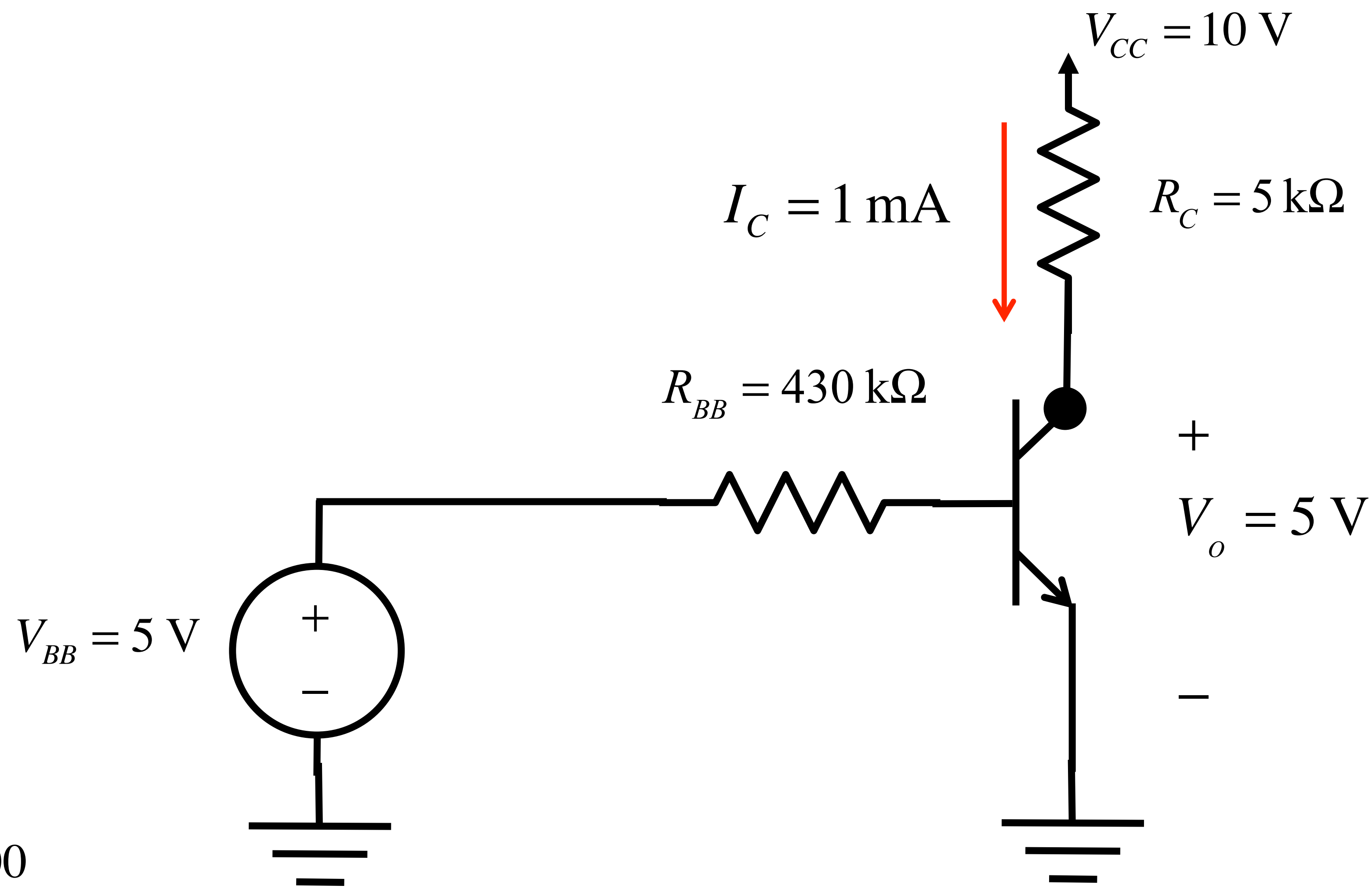
1) DC first
2) Then AC



$\beta = 100$

$V_{BE}(\text{ON}) = 0.7\ \text{V}$

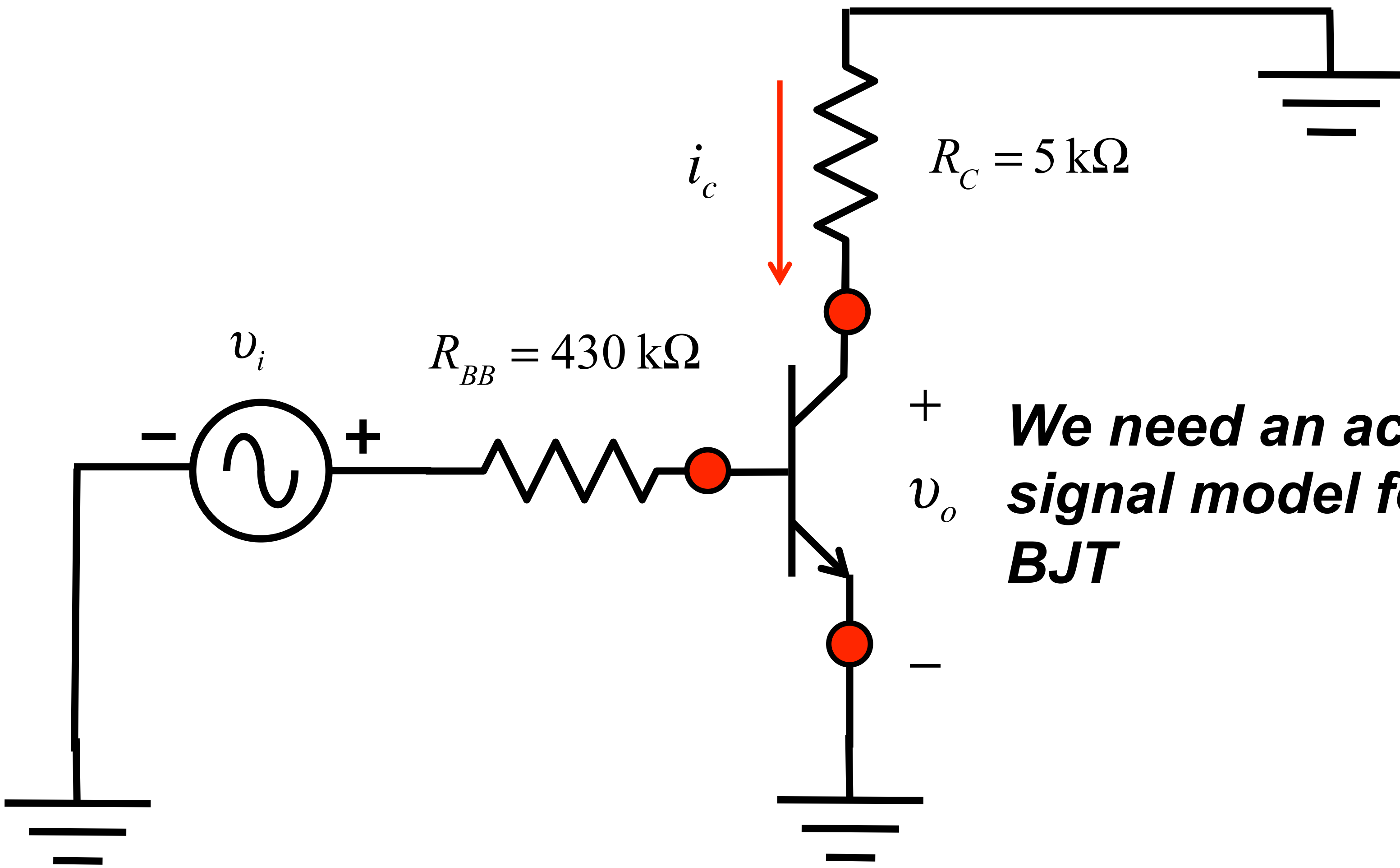
1) DC analysis



$\beta = 100$

$V_{BE}(\text{ON}) = 0.7\ \text{V}$

2) AC analysis

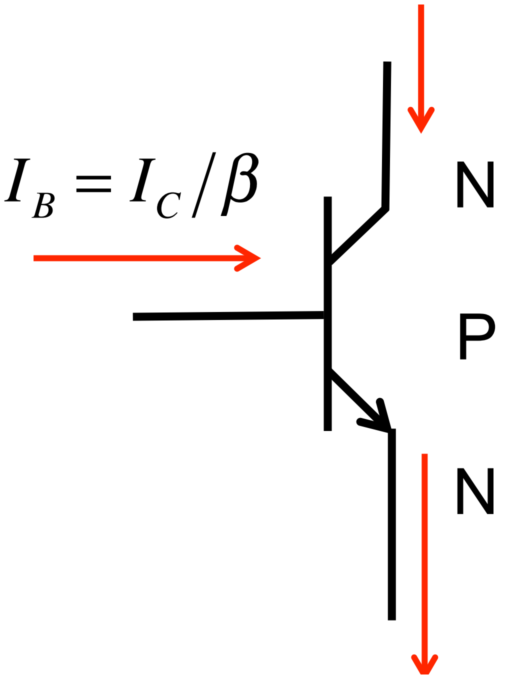


We need an ac small signal model for the BJT

Recall: DC currents

$$I_C = \beta I_B = \alpha I_E$$

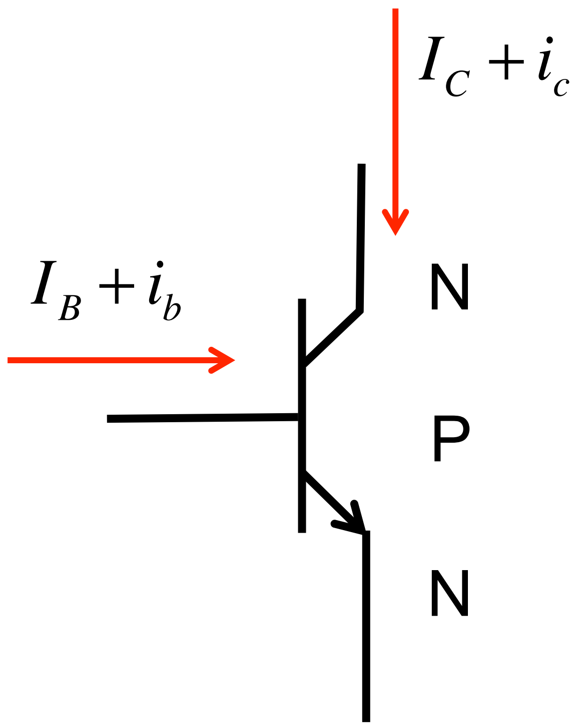
$$I_B = I_C/\beta$$



$$I_E = \frac{\beta+1}{\beta} I_C = \frac{I_C}{\alpha}$$

What is the a.c. small signal model?

ac collector current



$$I_C = I_S e^{V_{BE}/V_T} \rightarrow I_C + i_c = I_S e^{(V_{BE}+\upsilon_{be})/V_T}$$

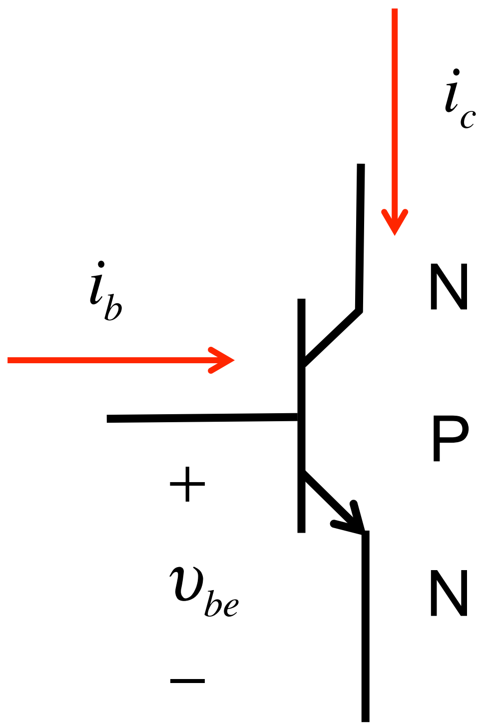
$$I_C + i_c = I_S e^{V_{BE}/V_T} e^{\upsilon_{be}/V_T}$$

$$I_C + i_c = I_S e^{V_{BE}/V_T} \left(1 + \upsilon_{be}/V_T\right)$$

$$i_c = I_S e^{V_{BE}/V_T} \left(\upsilon_{be}/V_T\right) = I_C \left(\upsilon_{be}/V_T\right)$$

$$i_c = \frac{I_C}{V_T} \upsilon_{be} = g_m \upsilon_{be}$$ Ohm's Law

Summary: s.s. collector current



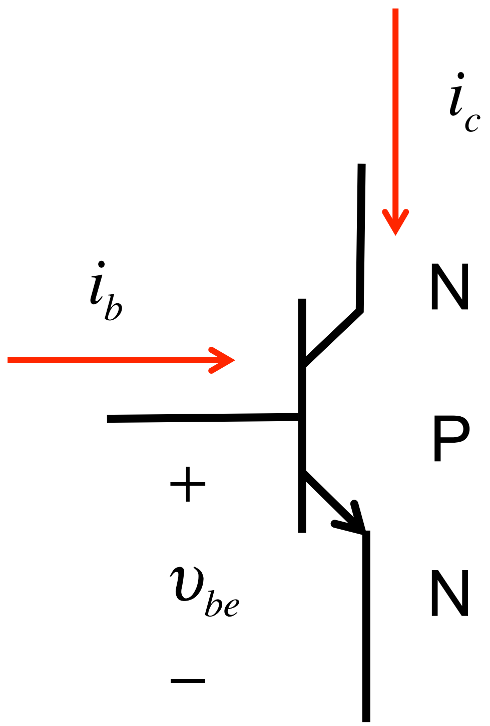
$$i_c = g_m \upsilon_{be}$$

$$g_m = \frac{I_C}{V_T}$$

"transconductance"

Note that the ac model parameter, g_m, depends on the dc bias current, I_C.

s.s. base current



$$i_c = g_m \upsilon_{be}$$

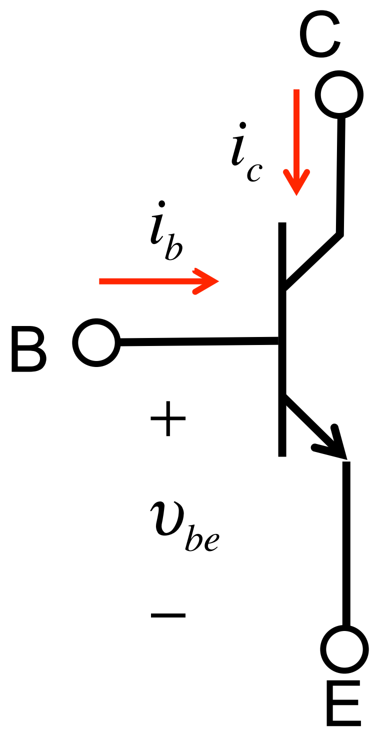
$$g_m = \frac{I_C}{V_T}$$

$$i_b = \frac{i_c}{\beta} = \frac{g_m}{\beta}\upsilon_{be} = \frac{\upsilon_{be}}{\beta / g_m} = \frac{\upsilon_{be}}{r_\pi}$$

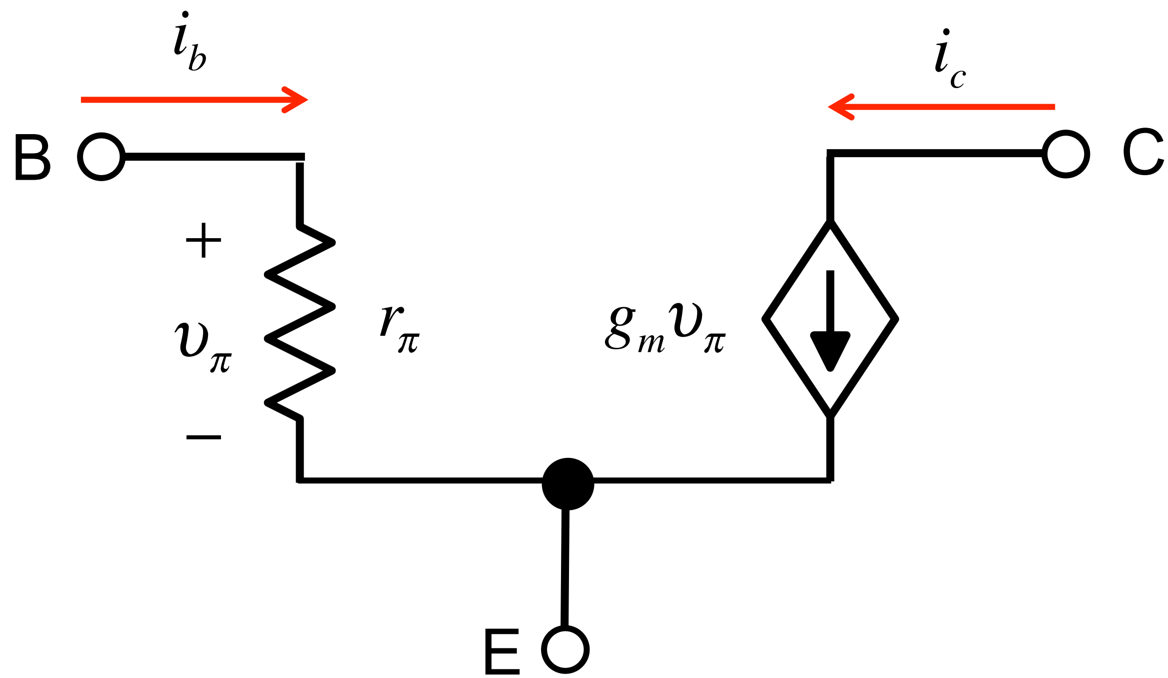
$$i_b = \frac{\upsilon_{be}}{r_\pi}$$

$$g_m r_\pi = \beta$$

s.s. eqv. circuit model: g_m form

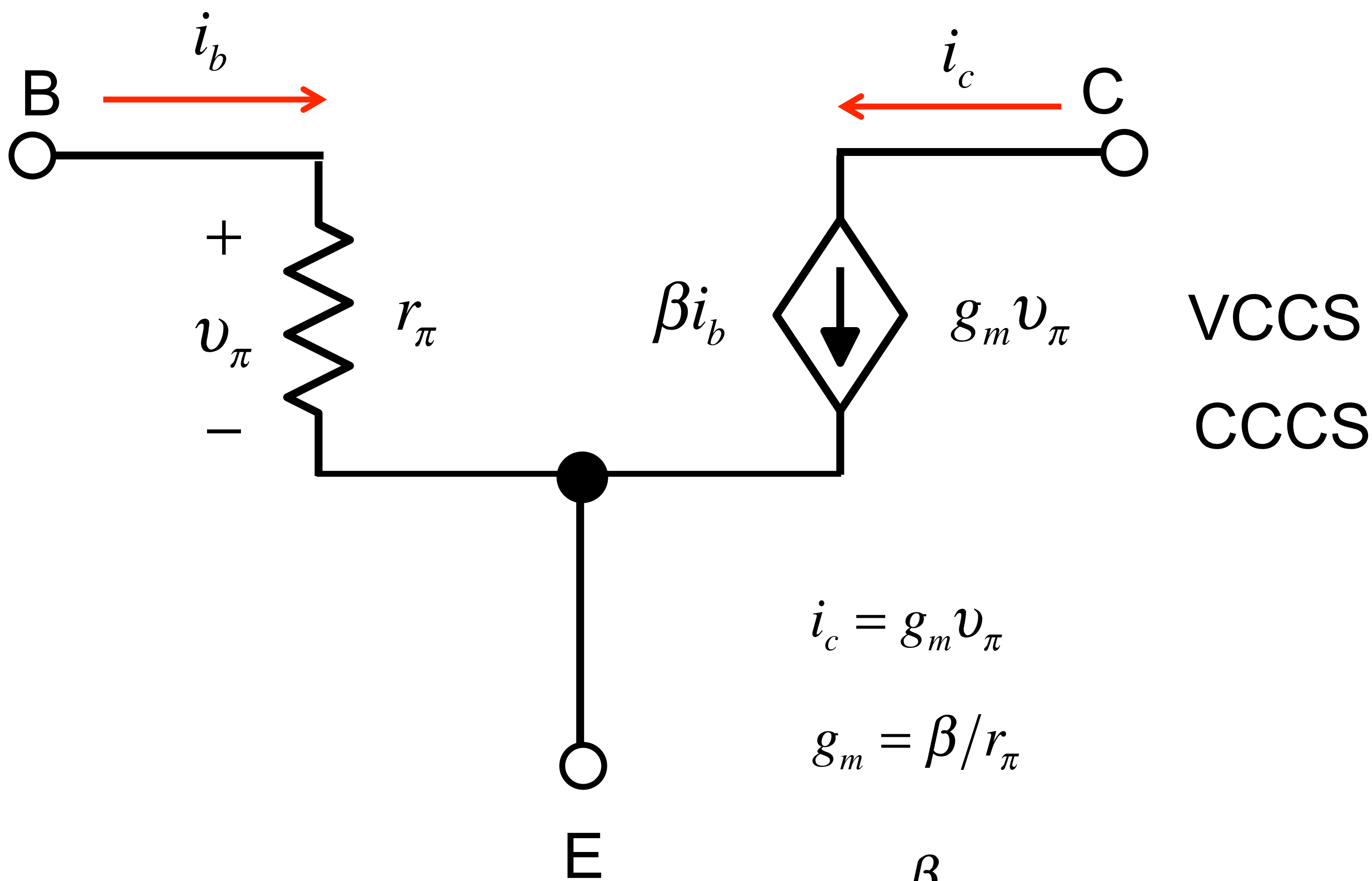


"hybrid pi model"



$$g_m = I_C / V_T \qquad g_m r_\pi = \beta$$

s.s. eqv. circuit model: beta form



$$g_m = \frac{I_C}{V_T}$$

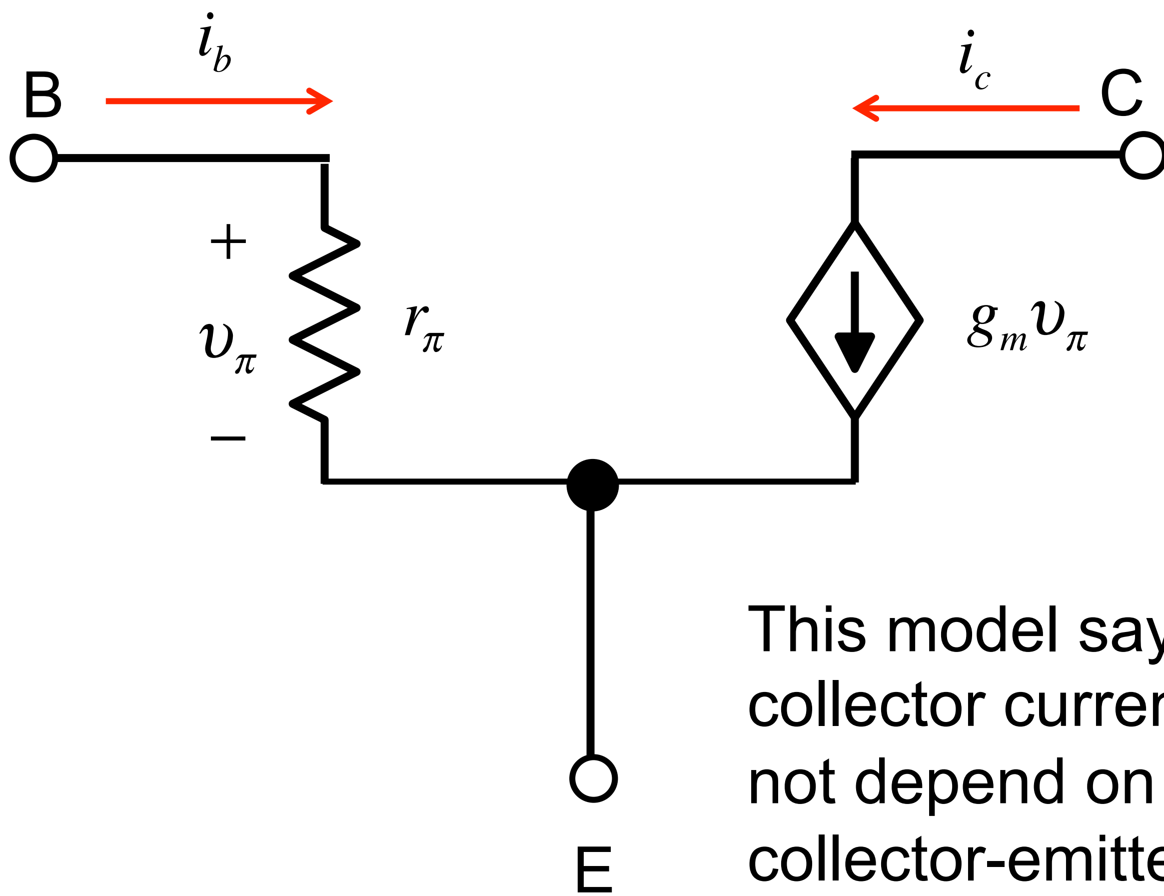
$$g_m r_\pi = \beta$$

$$i_c = g_m \upsilon_\pi$$

$$g_m = \beta / r_\pi$$

$$i_c = \frac{\beta}{r_\pi} \upsilon_\pi = \beta i_b$$

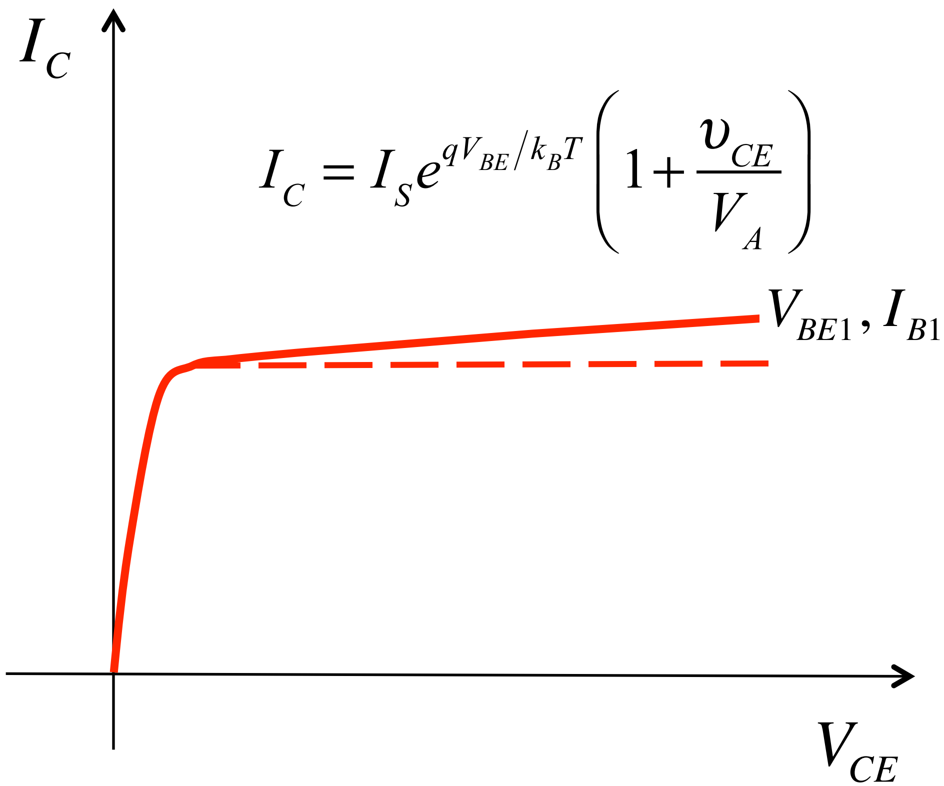
s.s. eqv. circuit model: v_{ce} dependence



This model says that the collector current does not depend on the collector-emitter voltage.

Output resistance

$$I_C = I_S e^{qV_{BE}/k_BT}\left(1+\frac{\upsilon_{CE}}{V_A}\right)$$



$$\frac{dI_C}{dV_{CE}} = I_S e^{qV_{BE}/k_BT}\left(\frac{1}{V_A}\right) = \frac{I'_C}{V_A}$$

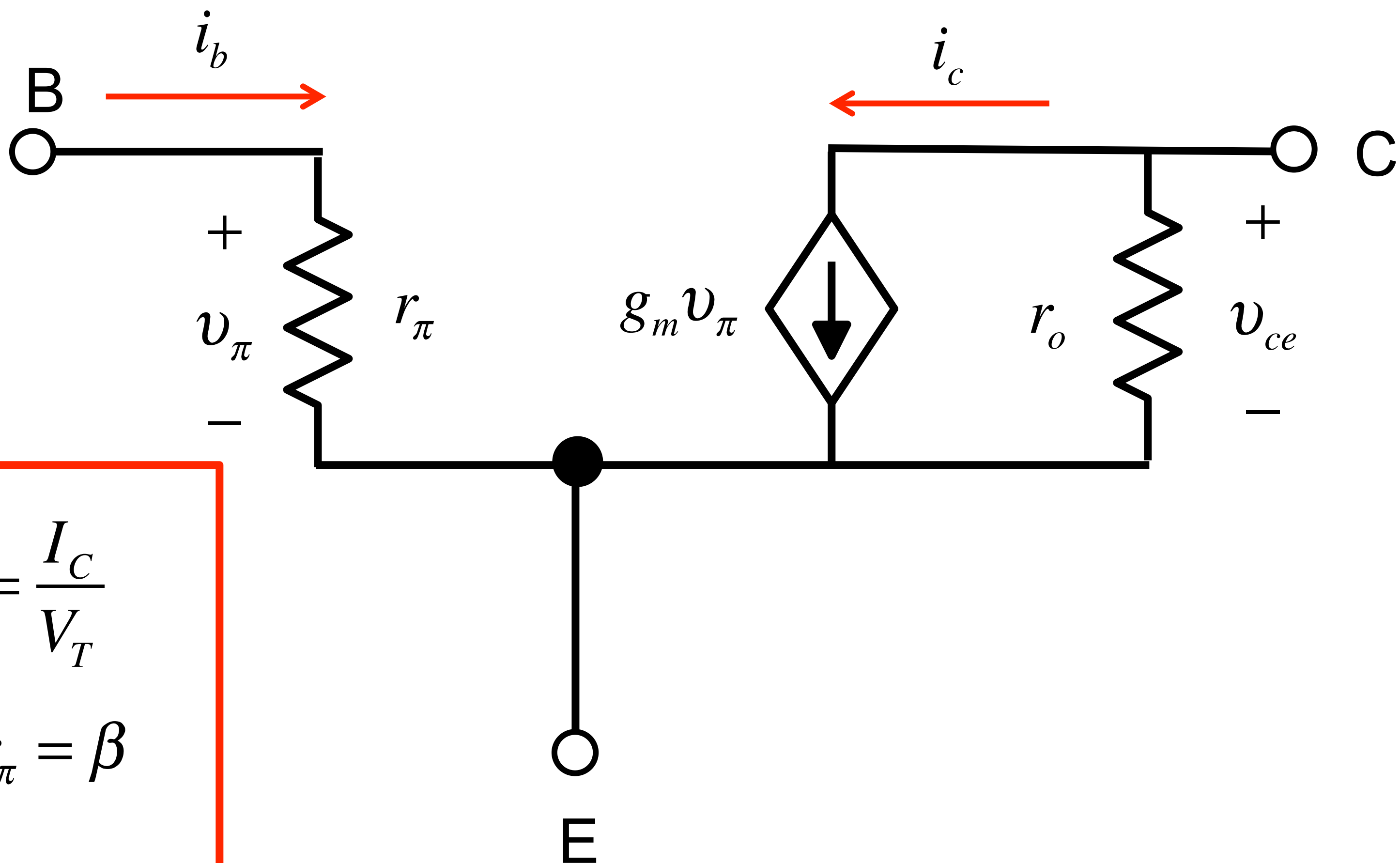
$$\frac{dI_C}{dV_{CE}} \approx \frac{i_c}{\upsilon_{ce}}$$

$$i_c = \frac{\upsilon_{ce}}{r_0}$$

$$r_o \approx \frac{V_A}{I_C}$$

V_A = “Early Voltage”

Hybrid pi model

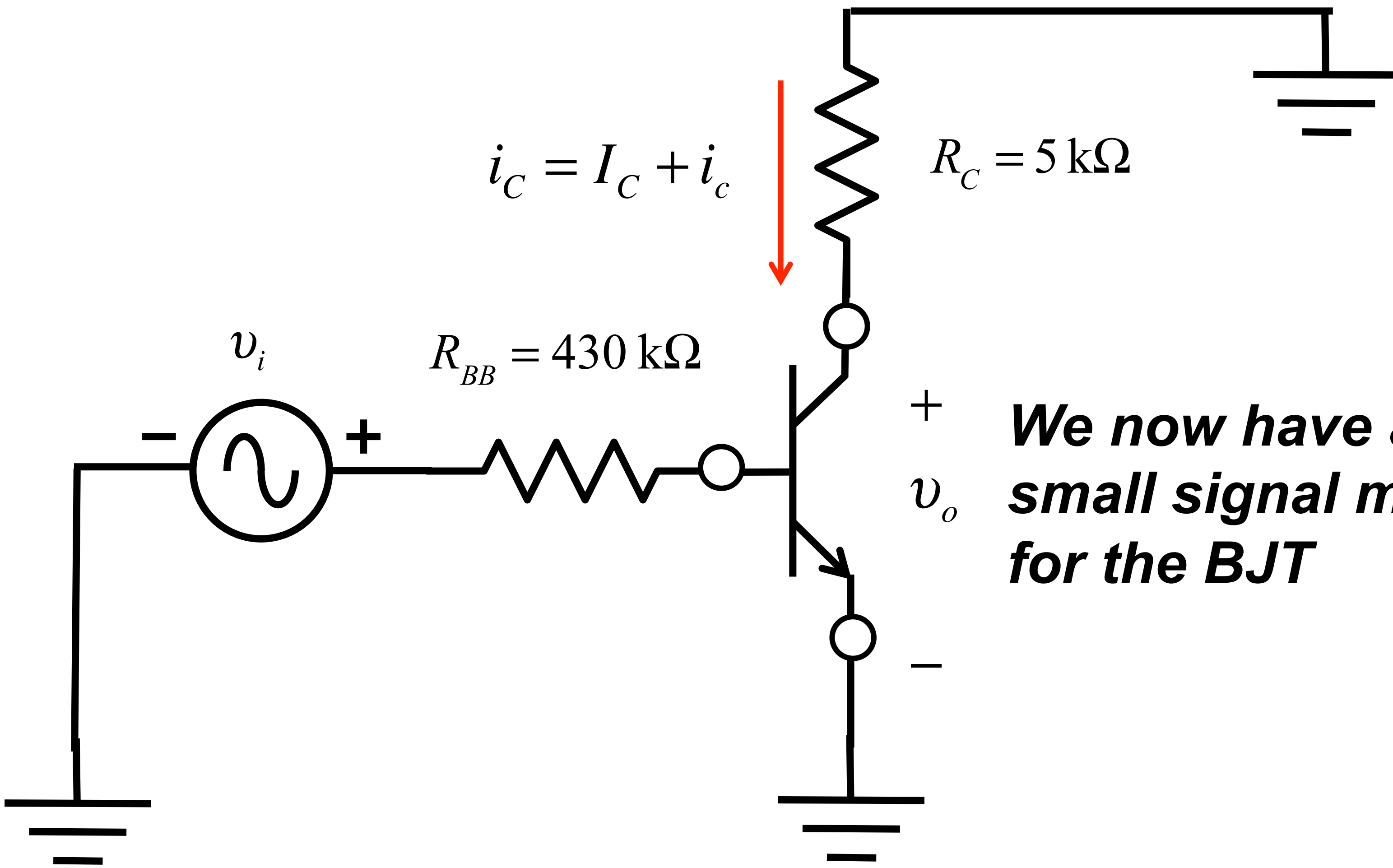


$$g_m = \frac{I_C}{V_T}$$

$$g_m r_\pi = \beta$$

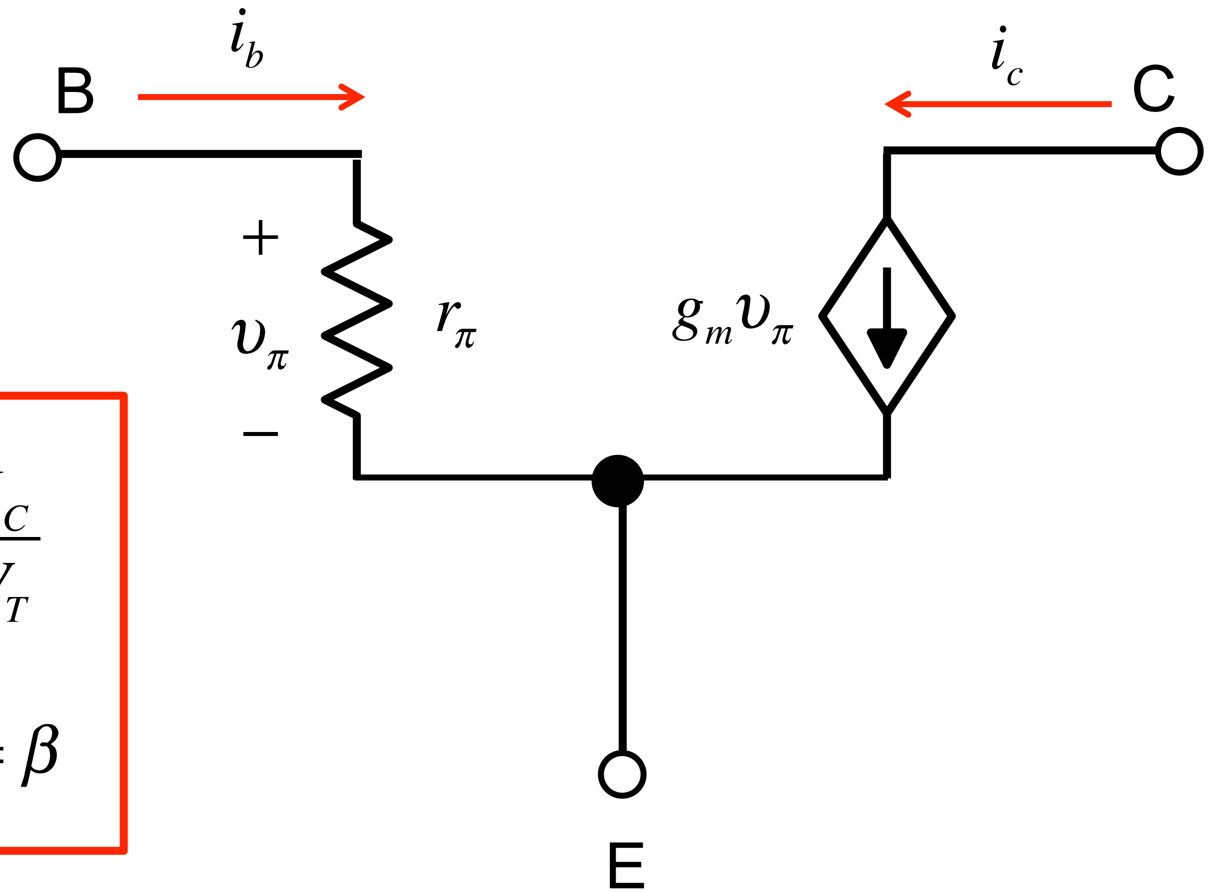
$$r_o = V_A / I_C$$

Small signal circuit



We now have an ac small signal model for the BJT

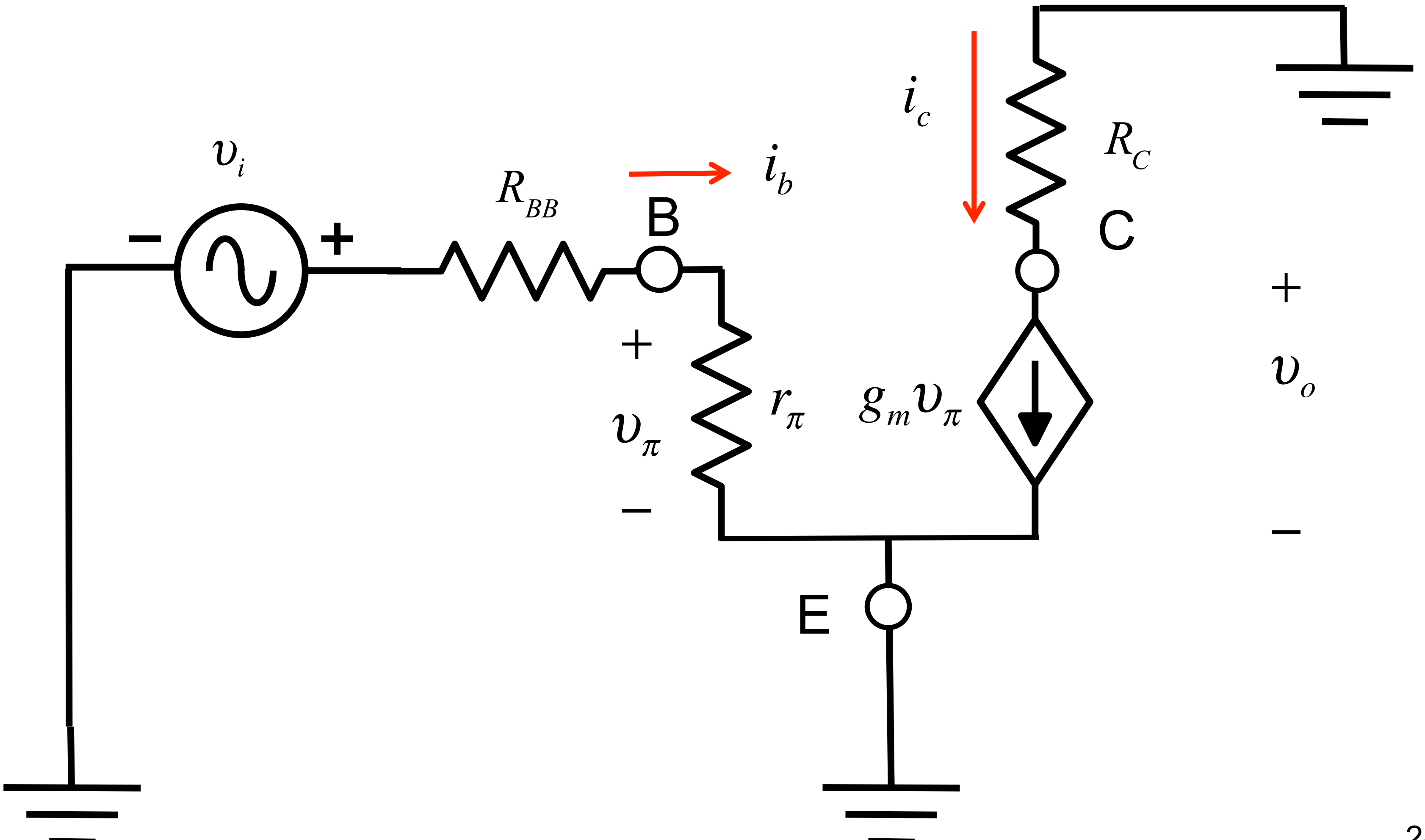
Simple s.s. model



$$g_m = \frac{I_C}{V_T}$$

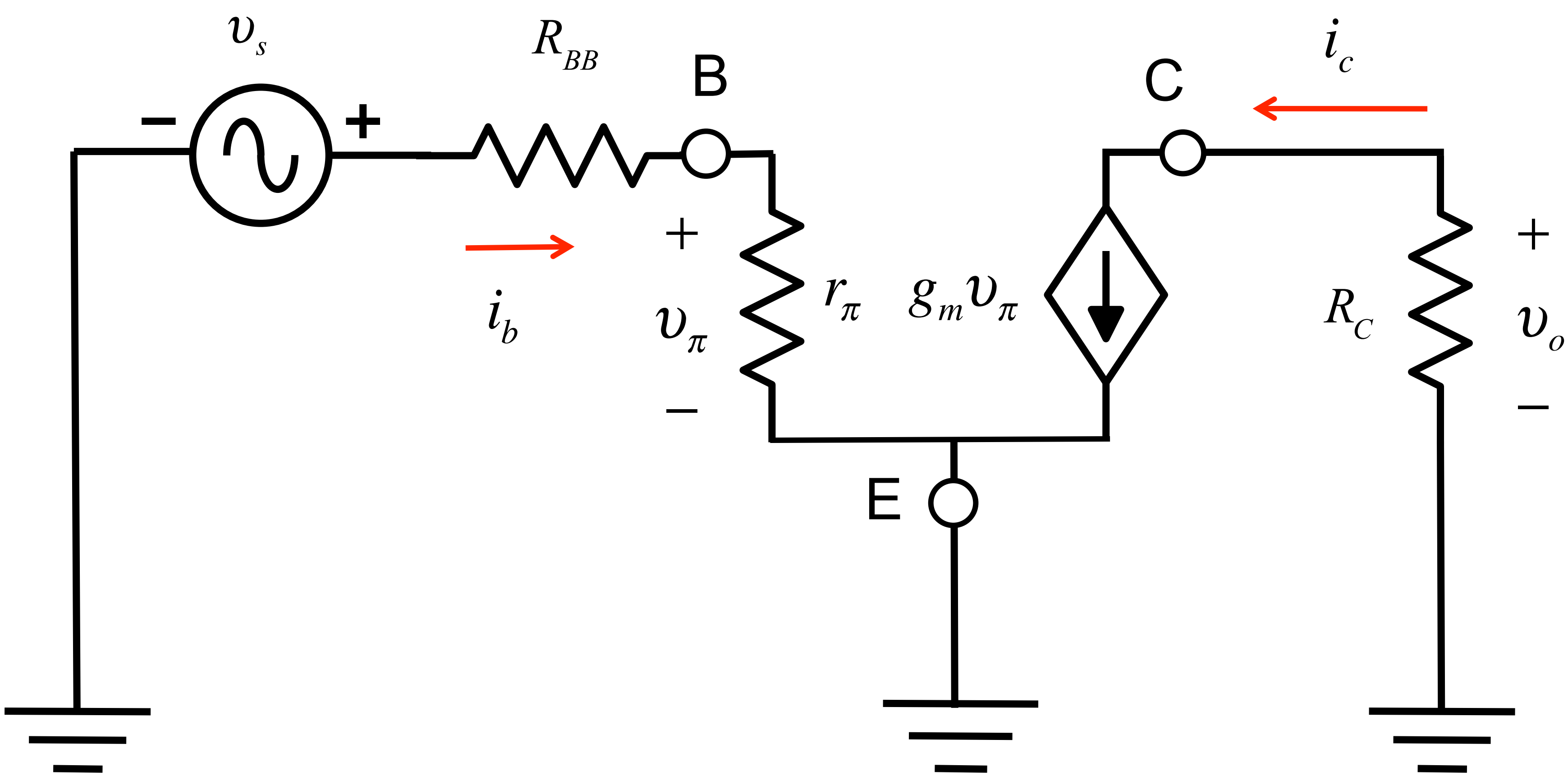
$$g_m r_\pi = \beta$$

ac analysis



υ_i
- +
R_{BB}
B
i_b
i_c
R_C
C
+
υ_π
−
r_π
$g_m\upsilon_\pi$
+
υ_o
−
E

Same circuit



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ac analysis

$$\upsilon_o = -g_m \upsilon_\pi R_C$$

$$\upsilon_\pi = \frac{r_\pi}{r_\pi + R_{BB}} \upsilon_s$$

$$\upsilon_o = -\left(\frac{r_\pi}{r_\pi + R_{BB}} \right) g_m R_C \upsilon_s$$

$$\frac{\upsilon_o}{\upsilon_s} = A_{\upsilon_s} = -\left(\frac{r_\pi}{r_\pi + R_{BB}} \right) g_m R_C$$

$\beta = 100$ $R_{BB} = 430\ \text{k}\Omega$

$I_C = 1\ \text{mA}$ $R_C = 5\ \text{k}\Omega$

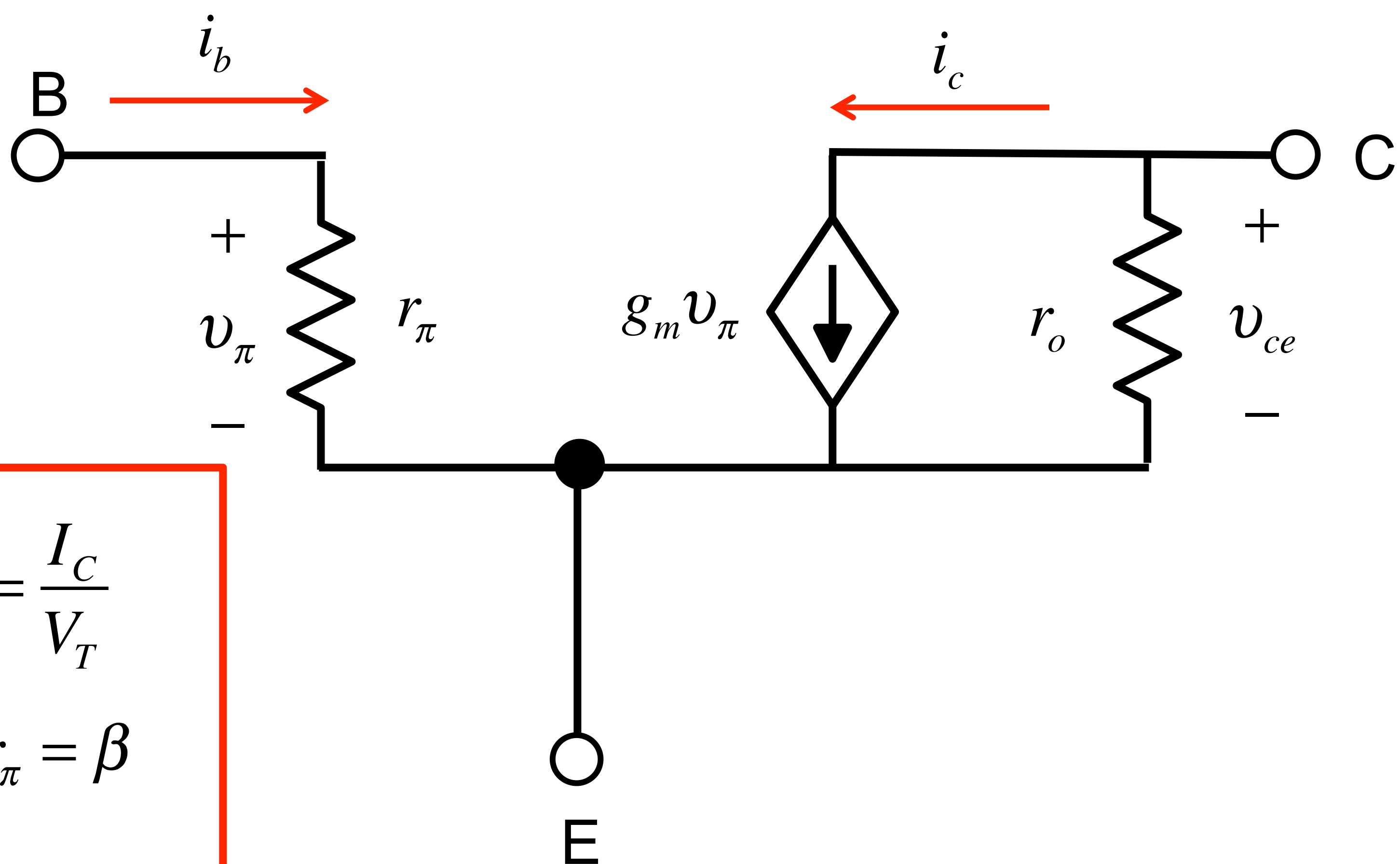
$$g_m = \frac{I_C}{V_T} = \frac{1\ \text{mA}}{0.026\ \text{V}} = 38.5\ \text{mS}$$

$$g_m r_\pi = \beta$$

$$r_\pi = \beta / g_m = 100/0.039 = 2.6\ \text{k}\Omega$$

$$A_{\upsilon_s} = -1.2$$

Hybrid pi model with output resistance



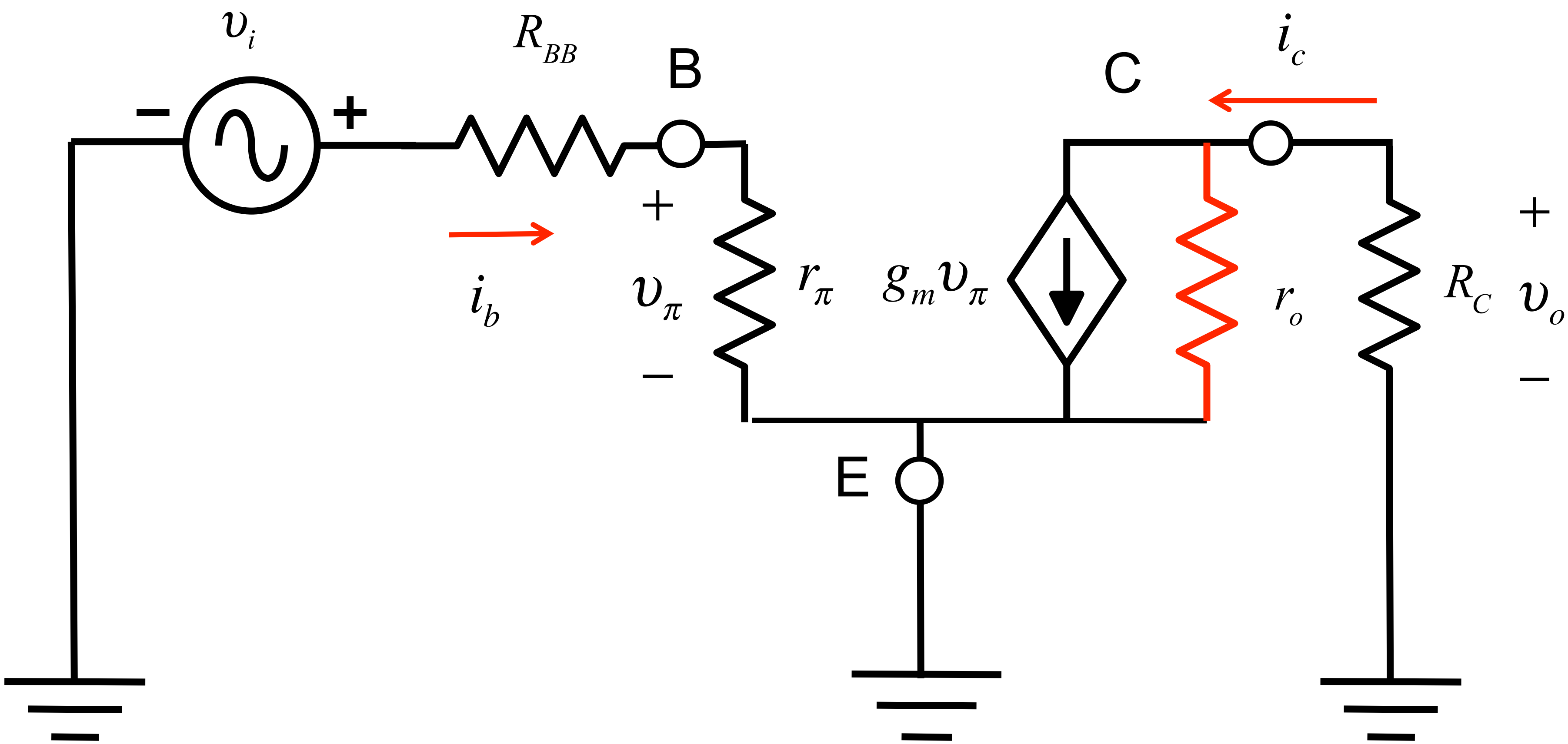
$$g_m = \frac{I_C}{V_T}$$

$$g_m r_\pi = \beta$$

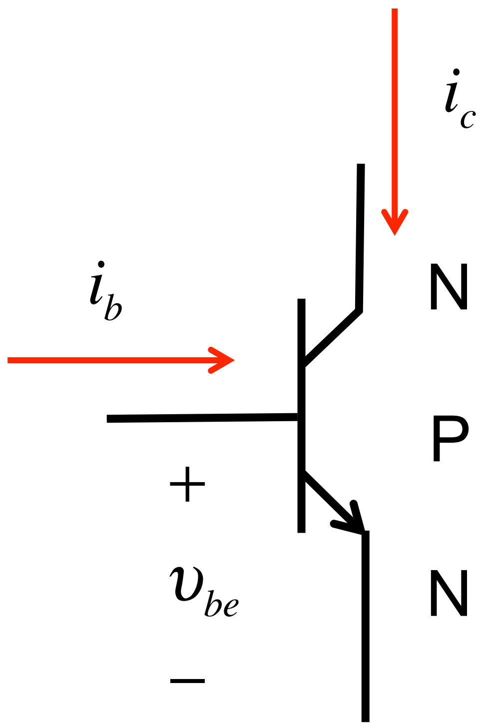
$$r_o = V_A / I_C$$

Lundstrom: 2019

Circuit with output resistance



Transistor parameters



$$\beta = 100$$

$$V_{BE}(\mathrm{ON}) = 0.7\ \mathrm{V}$$

$$V_A = 100\ \mathrm{V}$$

$$r_o = \frac{V_A}{I_C} = \frac{100\ \mathrm{V}}{1\ \mathrm{mA}} = 100\ \mathrm{k}\Omega$$

2) AC analysis with output resistance

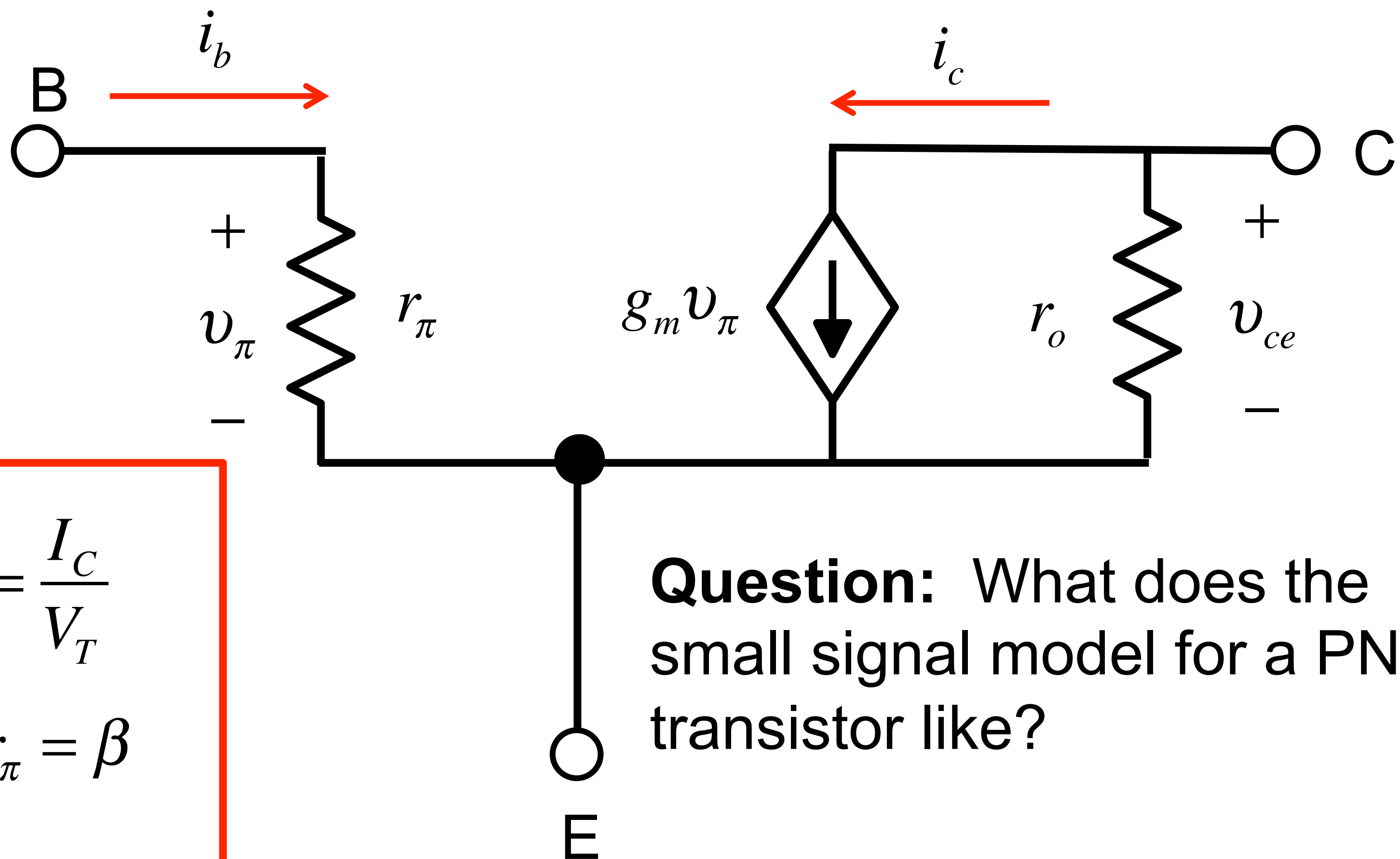
$$\frac{\upsilon_o}{\upsilon_i} = A = -\left(\frac{r_\pi}{r_\pi + R_{BB}}\right) g_m R_C$$ (without r_o)

$$\frac{\upsilon_o}{\upsilon_i} = A = -\left(\frac{r_\pi}{r_\pi + R_{BB}}\right) g_m R_C \parallel r_0$$ (with r_o)

$$5\ \text{k}\Omega \parallel 100\ \text{k}\Omega = 4.76\ \text{k}\Omega$$

$$A_v = -1.2 \rightarrow A_v = -1.1$$

Hybrid pi model of BJT



$$g_m = \frac{I_C}{V_T}$$

$$g_m r_\pi = \beta$$

$$r_o = V_A / I_C$$

Question: What does the small signal model for a PNP transistor like?

Summary

The DC bias circuit places the operating point in the portion of the Voltage Transfer Characteristics where the output voltage changes rapidly with input voltage.

The small signal model of a BJT consists of two resistors and one voltage-controlled current course. The values of the ac model parameters are determined by the dc bias current.

Circuit analysis consists of two steps: 1) dc analysis to determine the OP, and 2) ac small signal analysis using the ac circuit model.

Transistor Amplifiers

1) Voltage transfer characteristic (VTC)
2) Small signal model for BJT
3) Small signal analysis