ECE 255: L17

BJT Transistor Amplifiers
(Sedra and Smith, 7th Ed., Sec. 7.1, 7.2.2)

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Lundstrom: 2019
Announcements

1) LTspice Project 2 is posted. The due date for it is Wed. Feb. 27th by 5:00PM electronically. You should find your assigned your beta value in your grade book.

2) HW6 will be posted today

3) Exam 2 in on Tuesday, March 5, 6:30-7:30 PM PHYS 112
1) Voltage transfer characteristic (VTC)
2) Small signal model for BJT
3) Small signal analysis
Voltage Transfer Characteristic

\[ V_{OUT} = V_{CC} - I_C R_C \]

\[ I_C \]

\[ R_C = 5 \, \text{k}\Omega \]

\[ R_{BB} = 430 \, \text{k}\Omega \]

\[ V_{IN} \]

\[ V_{BE\,(ON)} = 0.7 \, \text{V} \]

\[ \beta = 100 \]

What is \( V_{OUT} \) vs. \( V_{IN} \)?

\( V_{CC} = 10 \, \text{V} \)

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\[ V_{out} \]

\[ V_{CC} \]

\[ V_{CC} \approx 0.3 \text{ V} \]

\[ V_{CC} \approx 0.7 \text{ V} \]

\[ V_{IN} \]

\[ V_{out} = 10 \text{ V} \]

\[ I_C \]

\[ R_C = 5 \text{ k}\Omega \]

\[ R_{BB} = 430 \text{ k}\Omega \]

Lundstrom: 2019
Biasing in the active region

$V_{OUT} = V_{CC} - I_C R_C$

$V_{OUT} = V_{CC} - \left( I_S e^{V_{IN}/V_T} \right) R_C$

$Lundstrom: 2019$
Bias Circuit

\[ \beta = 100 \]

\[ V_{BE\,(ON)} = 0.7 \, V \]

\[ V_{BB} = 5 \, V \]

\[ R_{BB} = 430 \, k\Omega \]

\[ I_C = ? \, mA \]

\[ I_B = ? \, mA \]

\[ V_{CE} = ? \, V \]

\[ V_{CC} = 10 \, V \]

\[ R_C = 5 \, k\Omega \]
Bias Circuit

\[ \beta = 100 \]

\[ V_{BE} \text{ (ON)} = 0.7 \text{ V} \]

\[ V_{BB} = 5 \text{ V} \]

\[ R_{BB} = 430 \text{ k}\Omega \]

\[ R_C = 5 \text{ k}\Omega \]

\[ I_C = 1.0 \text{ mA} \]

\[ I_B = 0.01 \text{ mA} \]

\[ V_{CE} = 5.0 \text{ V} \]

\[ V_{CC} = 10 \text{ V} \]
Biasing

$V_{OUT}$

$V_{CC} = 10\, V$

$V_o = 5\, V$

$\approx 0.3\, V$

$cutoff$  $active$  $saturation$

$V_{BB} = 5.0\, V$

$Lundstrom: 2019$

$Q: \left( I_C = 1\, mA, V_{CE} = 5.0\, V \right)$
Add ac small signal

“Common emitter amplifier”

\[ i_C = I_C + i_c \]

\[ V_{CC} = 10 \text{ V} \]

\[ R_C = 5 \text{ k}\Omega \]

\[ V_i \sin \omega t \]

\[ R_{BB} = 430 \text{ k}\Omega \]

\[ V_{BB} = 5 \text{ V} \]

\[ \beta = 100 \]

\[ V_{BE \ (ON)} = 0.7 \text{ V} \]

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\[ A_v = \frac{\delta V_o}{\delta V_i} \approx \frac{dV_o}{dV_i} \]

voltage gain

\[ A_v < 0 \]

(for this circuit)

\[ \nu_i = V_{BB} - \nu_i \]

\[ 2\delta V_i \]

\[ \nu_i = V_{BB} + \nu_i \]
\[ V_O = V_{CC} - I_C R_C \]

\[ V_O = V_{CC} - \left( I_S e^{V_{BE}/V_T} \right) R_C \]

\[ \frac{dV_O}{dV_{BE}} = -\left( \frac{I_S e^{V_{BE}/V_T}}{V_T} \right) R_C = -\frac{I_C}{V_T} R_C \]

\[ g_m = \frac{dI_C}{dV_{BE}} = \frac{I_C}{V_T} \quad \text{(transconductance)} \]
Voltage gain of CE amplifier

\[ A_{v_{be}} = \frac{v_o}{v_{be}} = -g_m R_C \]

\[ A_{v_i} = \frac{v_o}{v_i} = ? \]

\[ V_{BB} = 5 \text{ V} \]

\[ \beta = 100 \]

\[ V_{BE\,(ON)} = 0.7 \text{ V} \]

\[ V_{CC} = 10 \text{ V} \]

\[ R_C = 5 \text{ k}\Omega \]

\[ R_{BB} = 430 \text{ k}\Omega \]

\[ i_C = I_C + i_c \]

\[ v_{OUT} = V_o + v_o \]
Analysis by superposition

1) DC first
2) Then AC

\[ i_C = I_C + i_c \]

\[ V_{CC} = 10 \text{ V} \]

\[ R_C = 5 \text{ k}\Omega \]

\[ V_{BB} = 5 \text{ V} \]

\[ R_{BB} = 430 \text{ k}\Omega \]

\[ V_{OUT} = V_o + v_o \]

\[ \beta = 100 \]

\[ V_{BE\ (ON)} = 0.7 \text{ V} \]
1) DC analysis

\[ V_O = 5 \text{ V} \]
\[ R_C = 5 \text{ k}\Omega \]
\[ R_{BB} = 430 \text{ k}\Omega \]
\[ I_C = 1 \text{ mA} \]

\[ V_{BB} = 5 \text{ V} \]
\[ \beta = 100 \]
\[ V_{BE (ON)} = 0.7 \text{ V} \]

Lundstrom: 2019
2) AC analysis

We need an ac small signal model for the BJT

Lundstrom: 2019
Recall: DC currents

\[ I_C = \beta I_B = \alpha I_E \]

\[ I_B = I_C / \beta \]

\[ I_E = \frac{\beta + 1}{\beta} I_C = \frac{I_C}{\alpha} \]

What is the a.c. small signal model?

Lundstrom: 2019
ac collector current

\[ I_C + i_c = I_S e^{\frac{V_{BE}}{V_T}} \rightarrow I_C + i_c = I_S e^{\left(\frac{V_{BE} + \nu_{be}}{V_T}\right)} \]

\[ I_C + i_c = I_S e^{\frac{V_{BE}}{V_T}} e^{\nu_{be}/V_T} \]

\[ I_C + i_c = I_S e^{\frac{V_{BE}}{V_T}} \left(1 + \frac{\nu_{be}}{V_T}\right) \]

\[ i_c = I_S e^{\frac{V_{BE}}{V_T}} \left(\frac{\nu_{be}}{V_T}\right) = I_C \left(\frac{\nu_{be}}{V_T}\right) \]

\[ i_c = \frac{I_C}{V_T} \nu_{be} = g_m \nu_{be} \quad \text{Ohm’s Law} \]
Summary: s.s. collector current

Note that the ac model parameter, $g_m$, depends on the dc bias current, $I_C$. 

\[ i_c = g_m v_{be} \]

\[ g_m = \frac{I_C}{V_T} \]

"transconductance"
s.s. base current

\[ i_b = \frac{i_c}{\beta} = \frac{g_m \nu_{be}}{\beta} = \frac{\nu_{be}}{\beta / g_m} = \frac{\nu_{be}}{r_\pi} \]

\[ i_b = \frac{\nu_{be}}{r_\pi} \]

\[ g_m r_\pi = \beta \]

\[ i_c = g_m \nu_{be} \]

\[ g_m = \frac{I_C}{V_T} \]
s.s. eqv. circuit model: \(g_m\) form

\[
\begin{align*}
g_m &= I_C / V_T \quad g_m r_\pi = \beta
\end{align*}
\]
s.s. eqv. circuit model: beta form

\[ g_m = \frac{I_c}{V_T} \]

\[ g_m r_\pi = \beta \]

\[ i_c = g_m v_\pi \]

\[ g_m = \frac{\beta}{r_\pi} \]

\[ i_c = \frac{\beta}{r_\pi} v_\pi = \beta i_b \]

Lundstrom: 2019
This model says that the collector current does not depend on the collector-emitter voltage.
Output resistance

\[ I_C = I_S e^{qV_{BE}/k_B T} \left( 1 + \frac{\nu_{CE}}{V_A} \right) \]

\[ \frac{dI_C}{dV_{CE}} = I_S e^{qV_{BE}/k_B T} \left( \frac{1}{V_A} \right) = \frac{I_C'}{V_A} \]

\[ \frac{dI_C}{dV_{CE}} \approx \frac{i_c}{\nu_{ce}} \]

\[ i_c = \frac{\nu_{ce}}{r_0} \]

\[ r_o \approx \frac{V_A}{I_C} \]

\[ V_A = \text{“Early Voltage”} \]
Hybrid pi model

\[
g_m = \frac{I_C}{V_T}
\]

\[
g_m r_\pi = \beta
\]

\[
r_o = \frac{V_A}{I_C}
\]

Lundstrom: 2019
Small signal circuit

\[ i_C = I_C + i_c \]

\[ R_C = 5 \text{k}\Omega \]

\[ R_{BB} = 430 \text{k}\Omega \]

We now have an ac small signal model for the BJT

Lundstrom: 2019


Simple s.s. model

\[ g_m = \frac{I_C}{V_T} \]

\[ g_m r_\pi = \beta \]

Lundstrom: 2019
ac analysis

\[ v_o = v_i - R_{BB} i_b + \pi v_\pi + g_m v_\pi - R_C i_c \]
\[ v_o = -g_m v_\pi R_C \]

\[ v_\pi = \frac{r}{r_\pi + R_{BB}} v_s \]

\[ v_o = -\left( \frac{r}{r_\pi + R_{BB}} \right) g_m R_C v_s \]

\[ \frac{v_o}{v_s} = A_{v_s} = -\left( \frac{r}{r_\pi + R_{BB}} \right) g_m R_C \]

- \[ \beta = 100 \]
- \[ R_{BB} = 430 \text{ k}\Omega \]
- \[ I_C = 1 \text{ mA} \]
- \[ R_C = 5 \text{ k}\Omega \]
- \[ g_m = \frac{I_C}{V_T} = \frac{1 \text{ mA}}{0.026 \text{ V}} = 38.5 \text{ mS} \]
- \[ g_m r_\pi = \beta \]
- \[ r_\pi = \beta / g_m = 100 / 0.039 = 2.6 \text{ k}\Omega \]
- \[ A_{v_s} = -1.2 \]
Hybrid pi model with output resistance

\[ g_m = \frac{I_C}{V_T} \]

\[ g_m r_\pi = \beta \]

\[ r_o = \frac{V_A}{I_C} \]

Lundstrom: 2019
Circuit with output resistance

Lundstrom: 2019
Transistor parameters

\[ \beta = 100 \]

\[ V_{BE} \text{(ON)} = 0.7 \text{ V} \]

\[ V_A = 100 \text{ V} \]

\[ r_o = \frac{V_A}{I_C} = \frac{100 \text{ V}}{1 \text{ mA}} = 100 \text{ kΩ} \]

\[ \]
2) AC analysis with output resistance

\[ \frac{v_o}{v_i} = A = -\left( \frac{r}{r + R_{BB}} \right) g_m R_C \] (without \( r_o \))

\[ \frac{v_o}{v_i} = A = -\left( \frac{r}{r + R_{BB}} \right) g_m R_C \parallel r_o \] (with \( r_o \))

5 kΩ \parallel 100 kΩ = 4.76 kΩ

\[ A_v = -1.2 \rightarrow A_v = -1.1 \]
Hybrid pi model of BJT

\[ g_m = \frac{I_C}{V_T} \]

\[ g_m r_\pi = \beta \]

\[ r_o = \frac{V_A}{I_C} \]

**Question:** What does the small signal model for a PNP transistor like?
The DC bias circuit places the operating point in the portion of the Voltage Transfer Characteristics where the output voltage changes rapidly with input voltage.

The small signal model of a BJT consists of two resistors and one voltage-controlled current course. The values of the ac model parameters are determined by the dc bias current.

Circuit analysis consists of two steps: 1) dc analysis to determine the OP, and 2) ac small signal analysis using the ac circuit model.
Transistor Amplifiers

1) Voltage transfer characteristic (VTC)
2) Small signal model for BJT
3) Small signal analysis