Spring 2019 Purdue University

ECE 255: L18

MOS Transistor Amplifiers

(Sedra and Smith, 7th Ed., Sec. 7.1, 7.2.1)

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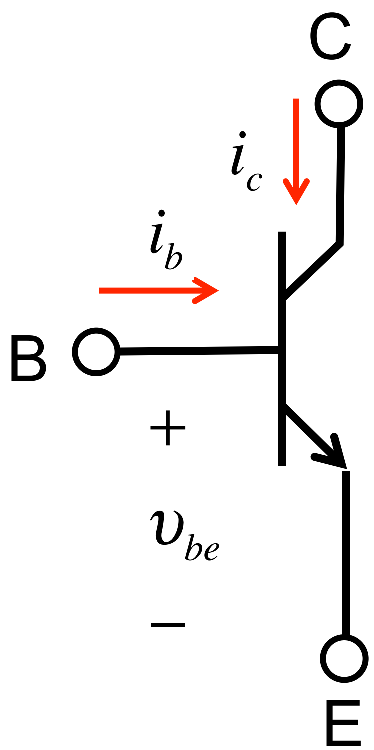
Announcements

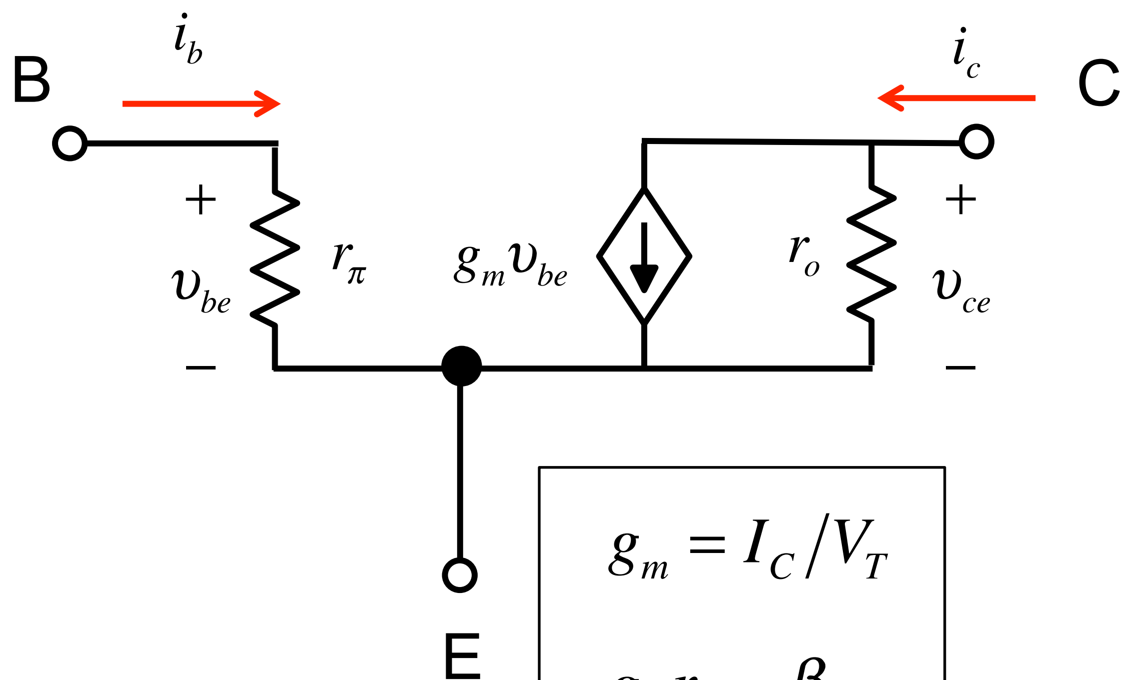
1) LTspice Project 2 is posted. The due date for it is Wed. Feb. 27th by 5:00PM electronically. You should find your assigned your beta value in your grade book.

2) HW6 will be posted today

3) Exam 2 in on Tuesday, March 5, 6:30-7:30 PM PHYS 112

Hybrid pi model for NPN



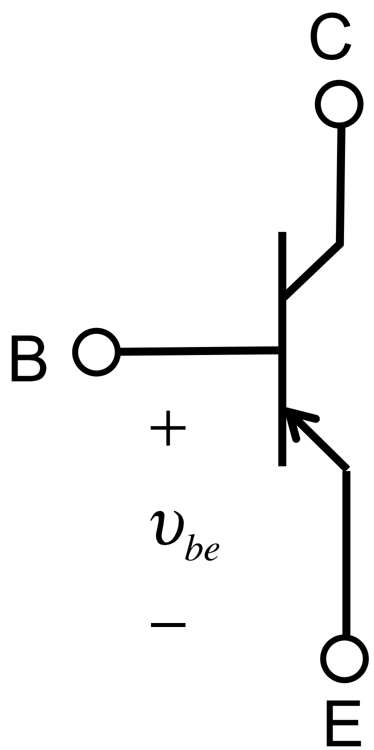


$$g_m = I_C / V_T$$

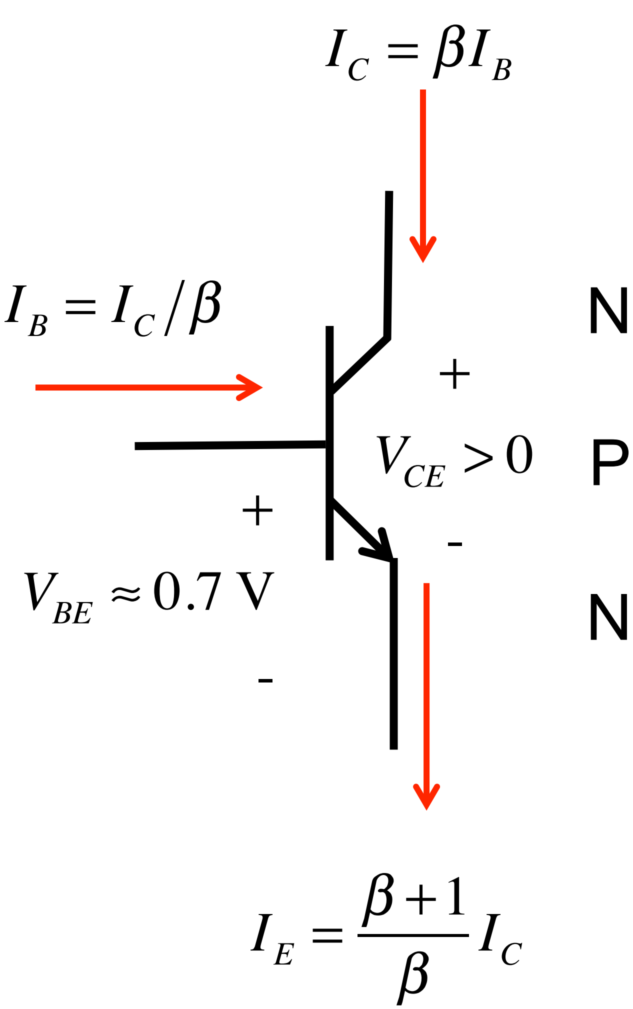
$$g_m r_\pi = \beta$$

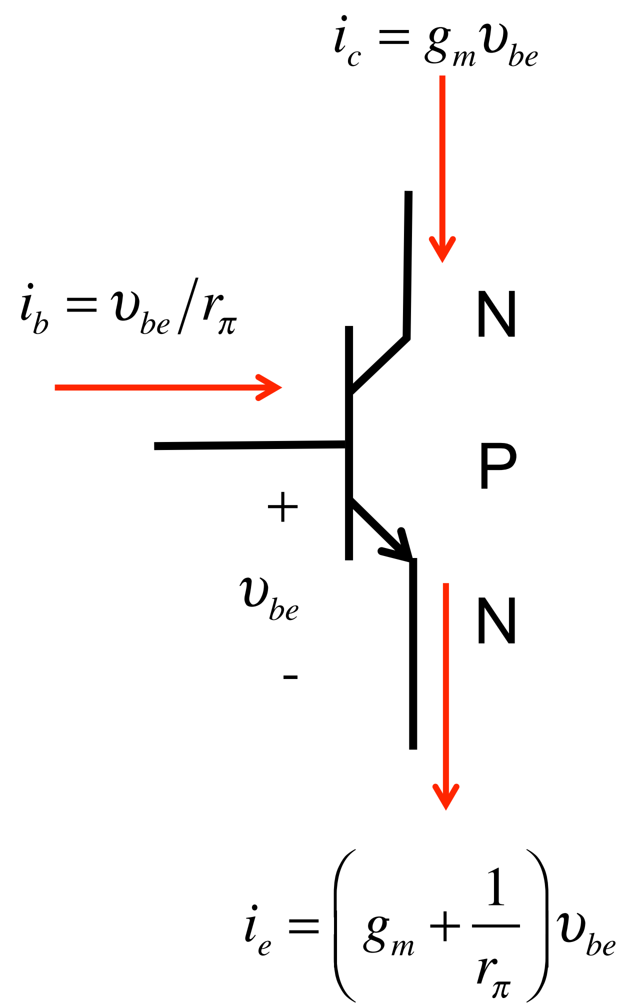
$$r_o = V_A / I_C$$

Hybrid pi model for PNP?

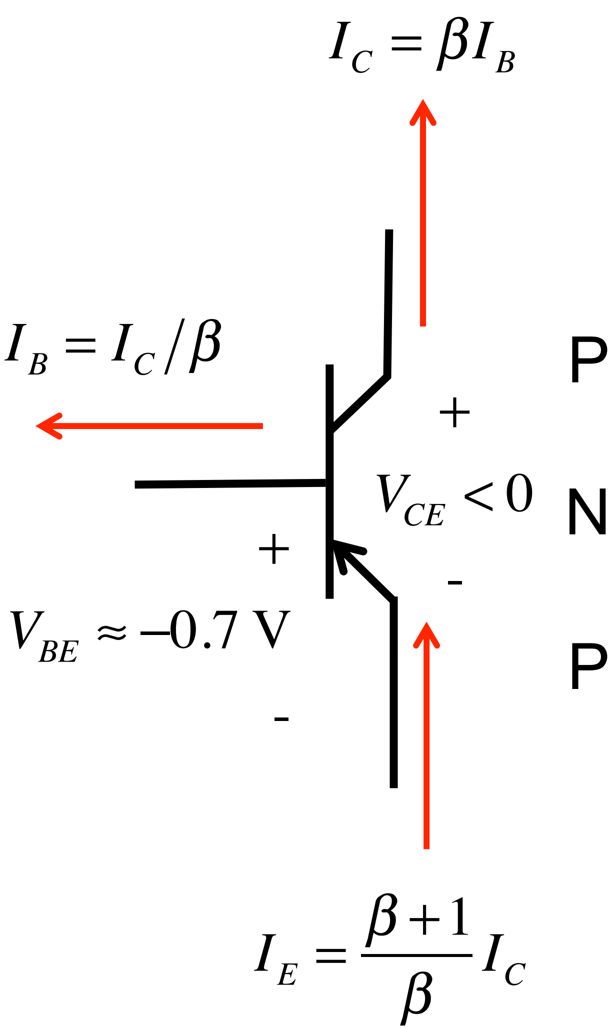


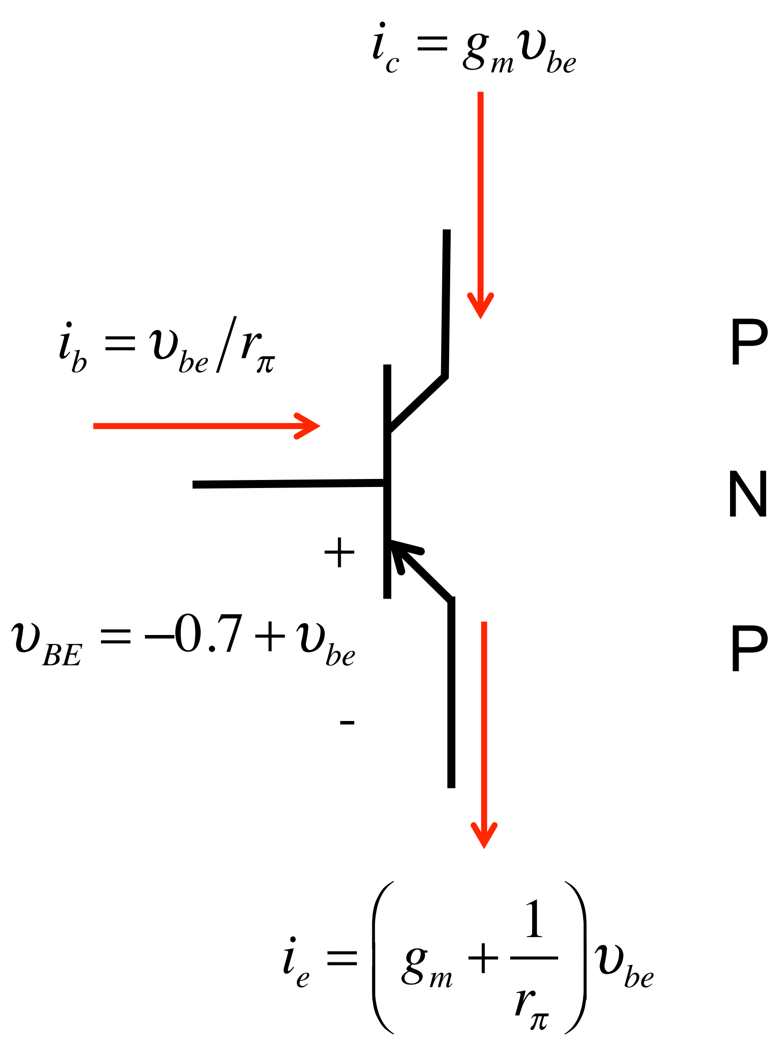
NPN DC to AC


$I_C = \beta I_B$
$I_B = I_C/\beta$
+
$V_{CE} > 0$
-
+
$V_{BE} \approx 0.7 \text{ V}$
-
N
P
N
$I_E = \dfrac{\beta+1}{\beta} I_C$


$i_c = g_m \upsilon_{be}$
$i_b = \upsilon_{be}/r_\pi$
N
P
+
υ_{be}
-
N
$i_e = \left(g_m + \dfrac{1}{r_\pi} \right) \upsilon_{be}$

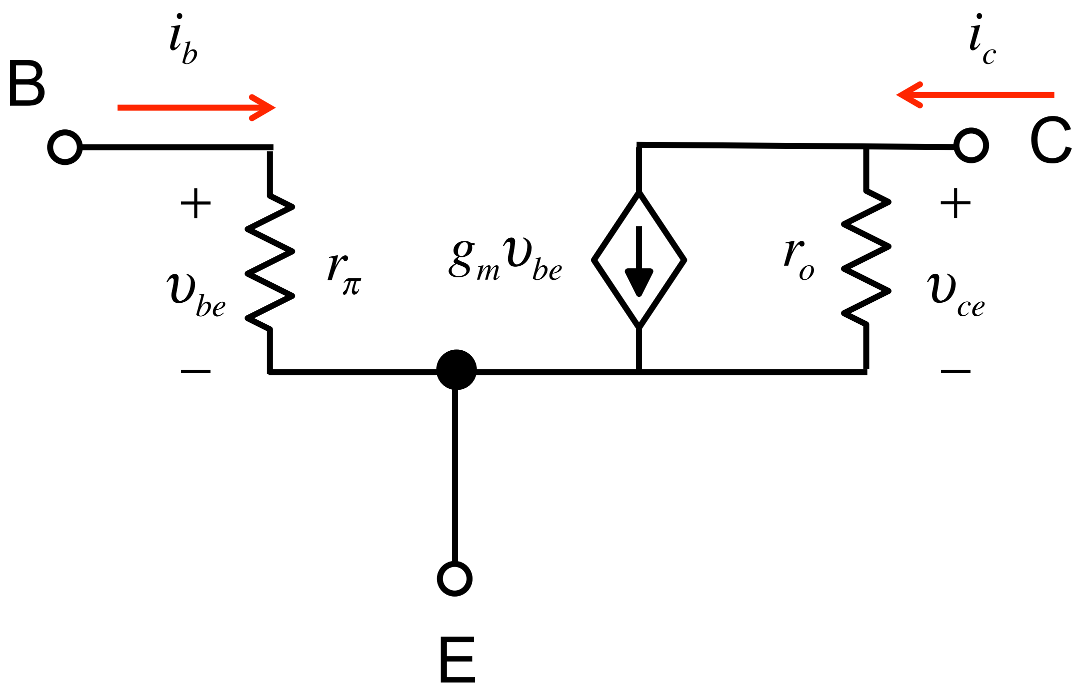
PNP DC to AC





Lundstrom: 2019

Small signal hybrid pi model for NPN **or** PNP

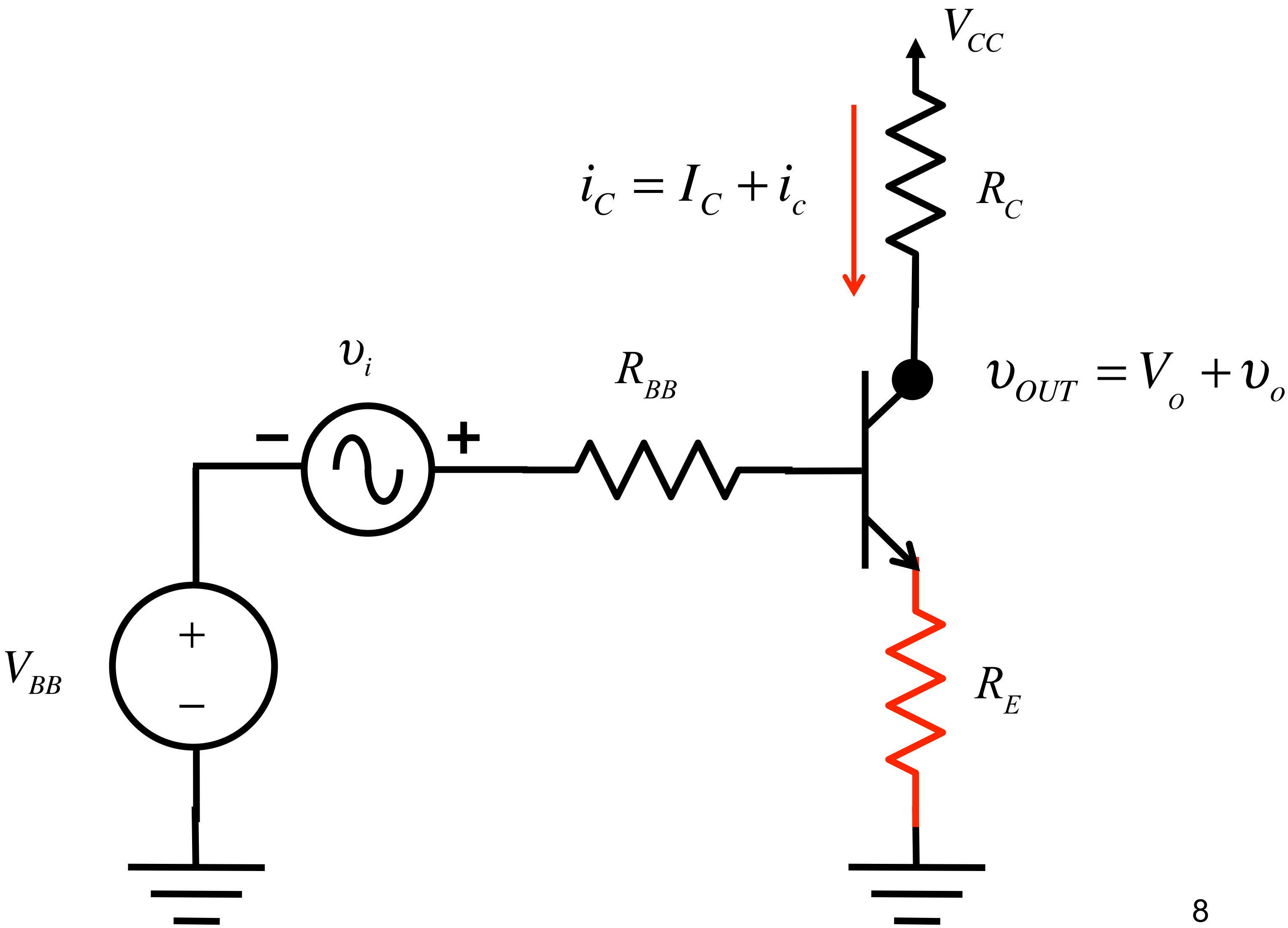


$$g_m = I_C / V_T$$

$$g_m r_\pi = \beta$$

$$r_o = V_A / I_C$$

Quiz: Draw the s.s. equivalent circuit



Outline

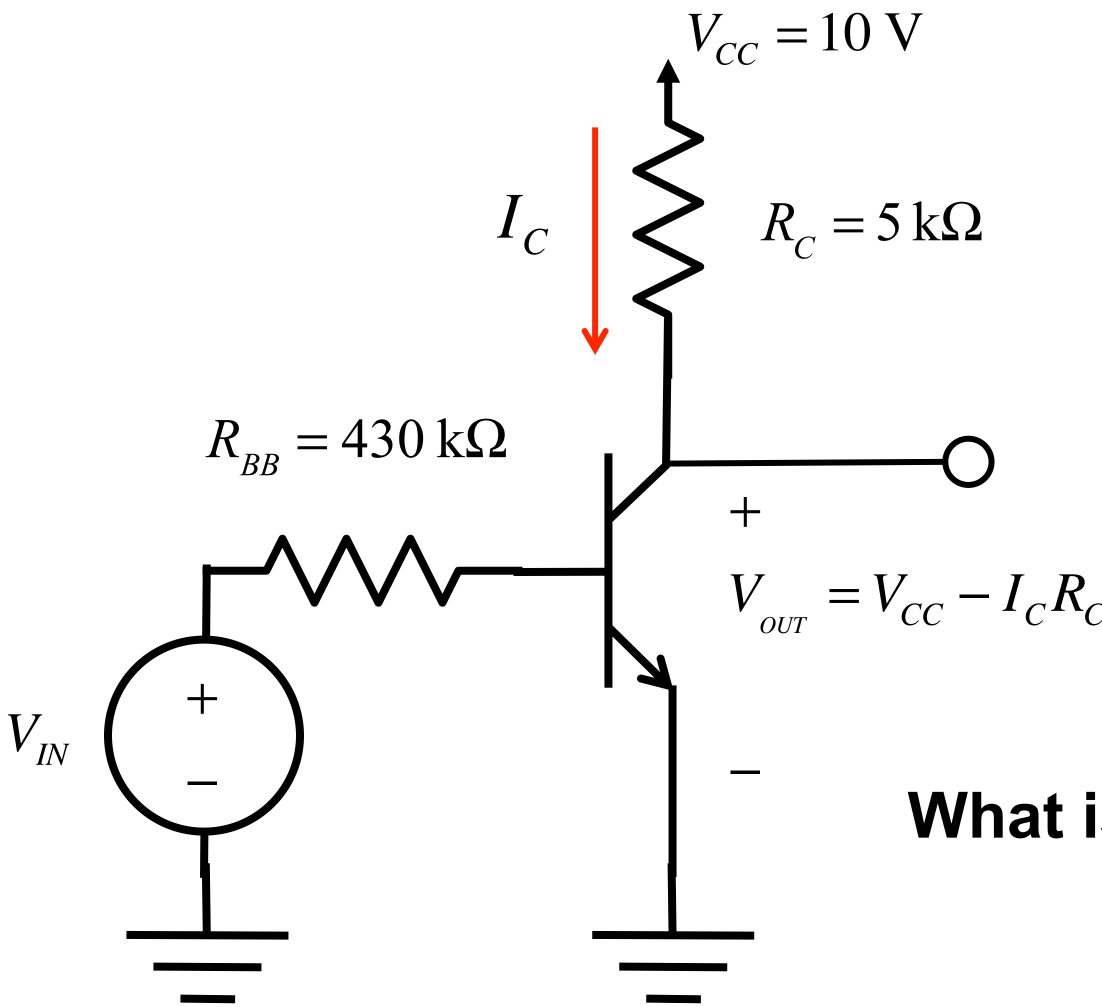
1) Voltage transfer characteristic (MOSFET)

2) Small signal model (hybrid pi)

3) Small signal analysis

4) Small signal model (T-model)

Voltage Transfer Characteristic (BJT)



What is V_{OUT} vs. V_{IN}?

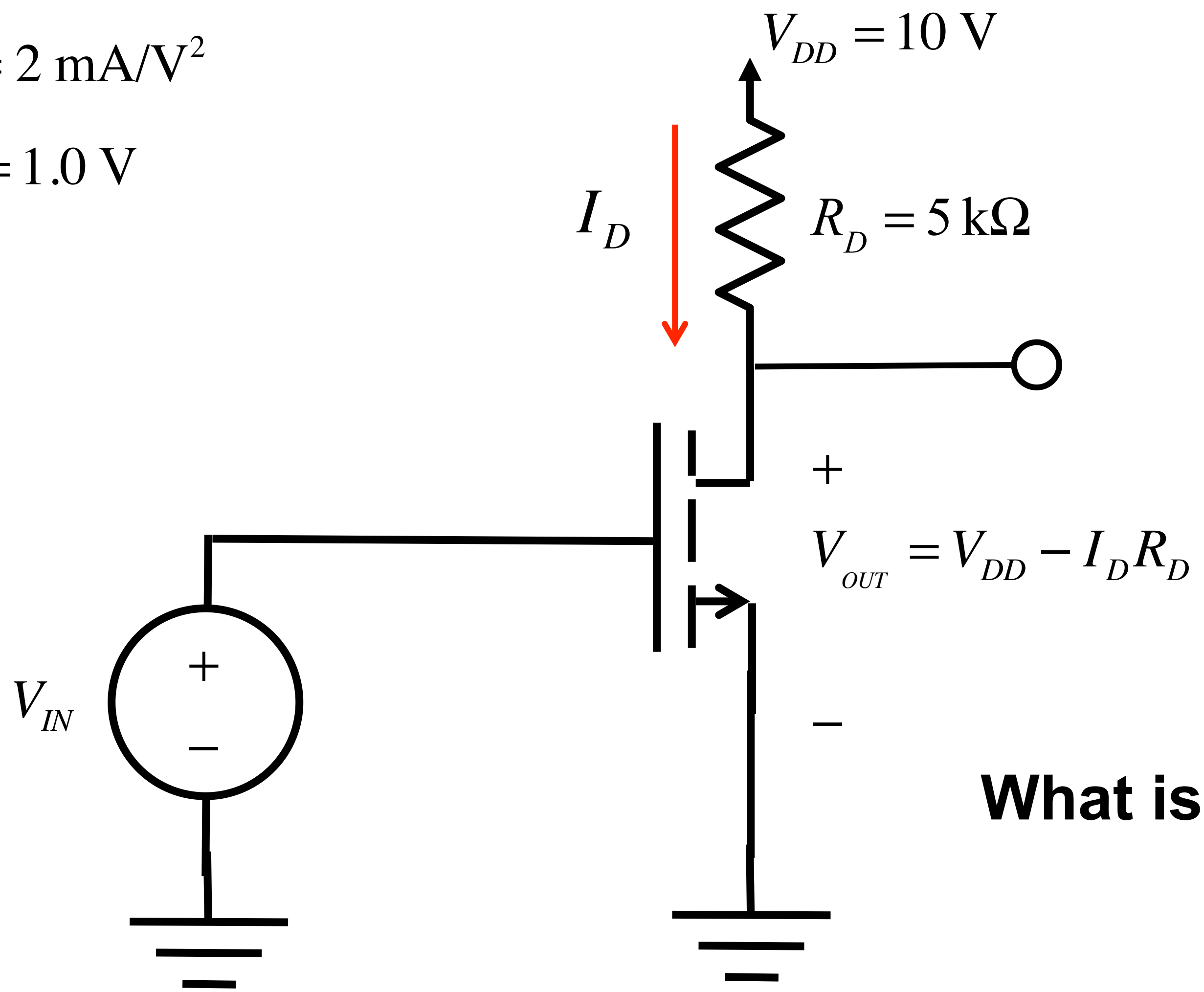
$\beta = 100$

$V_{BE}(\text{ON}) = 0.7\text{ V}$

Voltage Transfer Characteristic (**MOSFET**)

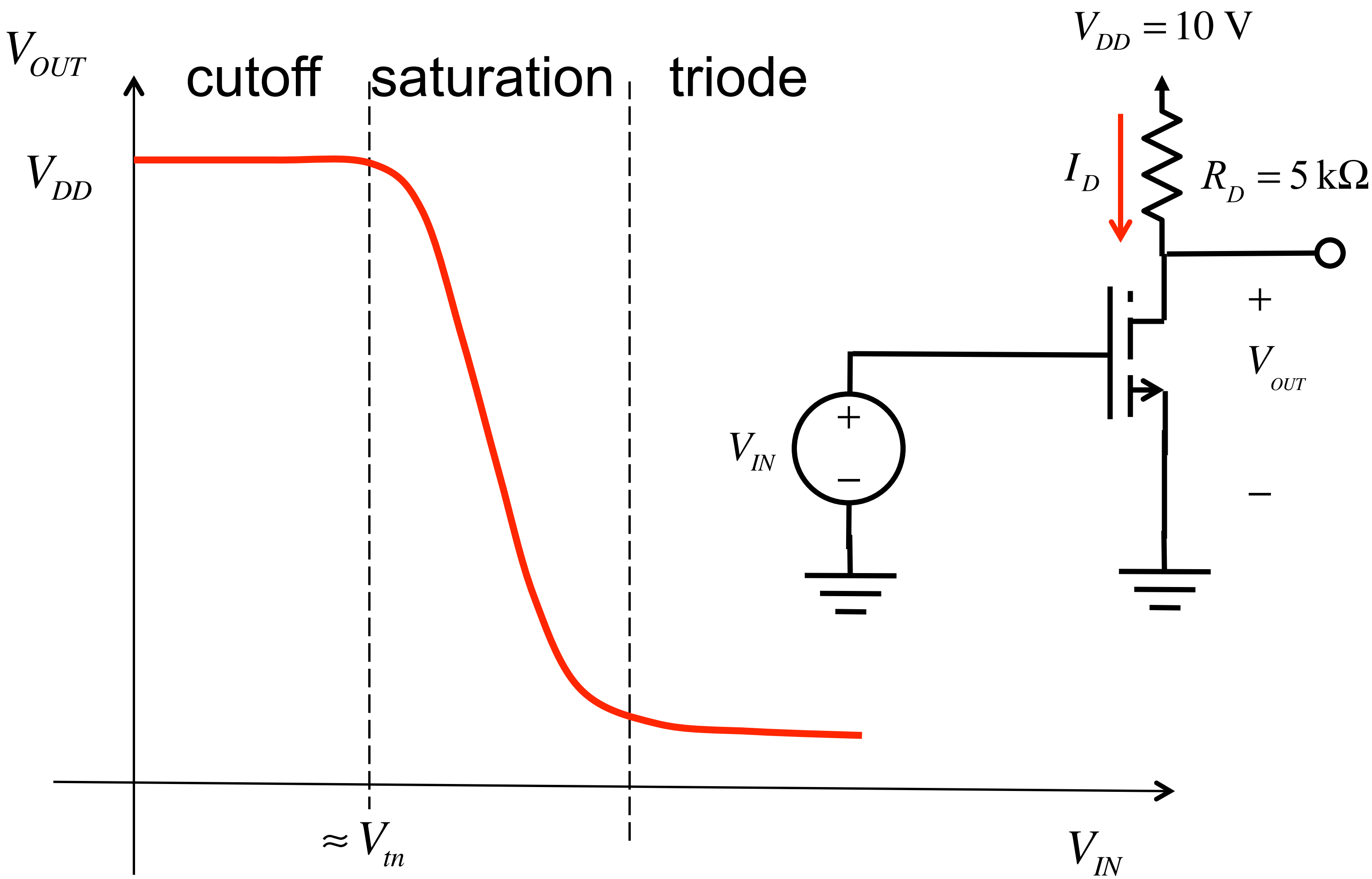
$k_n = 2\ \text{mA/V}^2$

$V_{tn} = 1.0\ \text{V}$



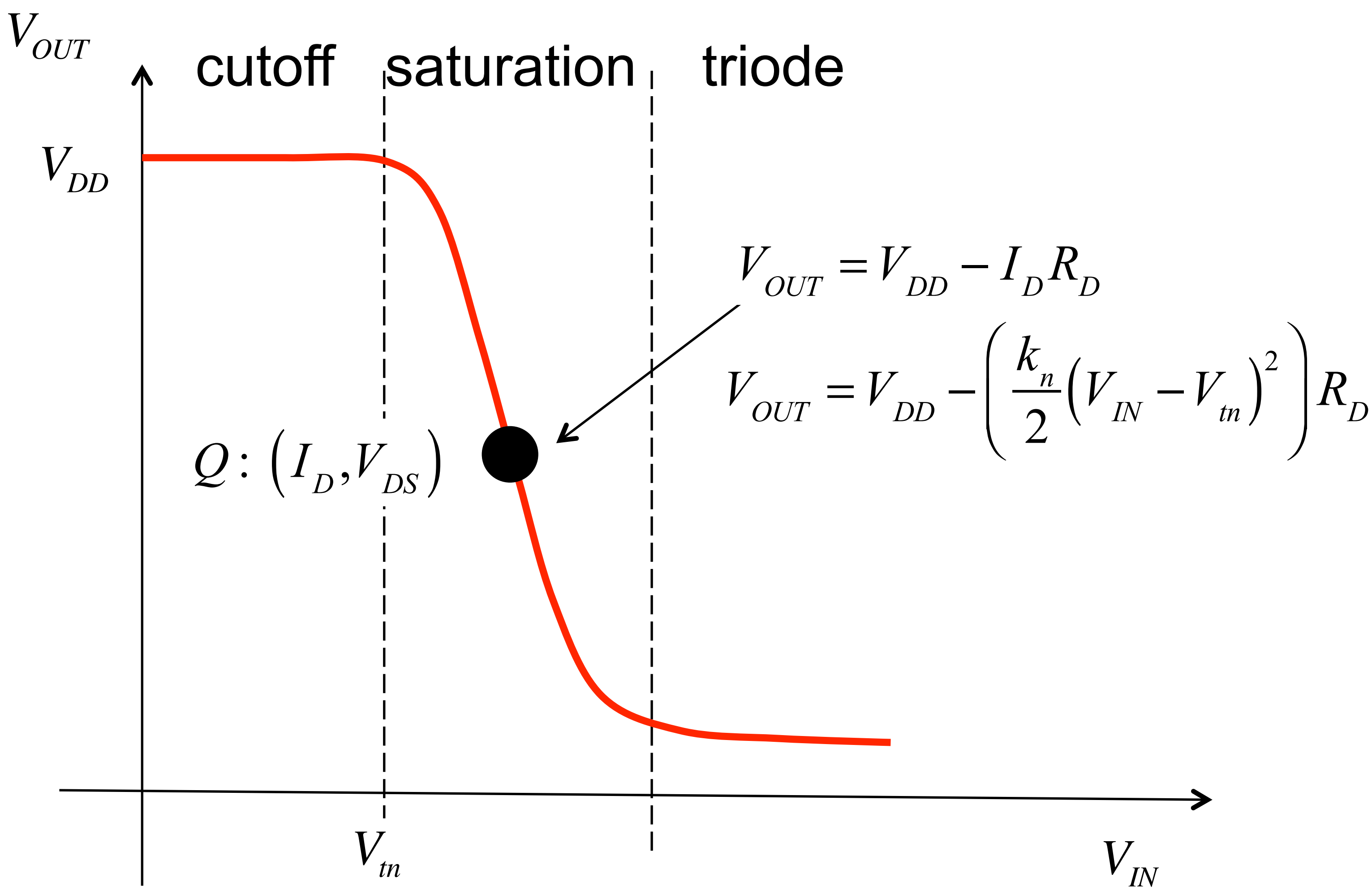
What is V_{OUT} vs. V_{IN}?

VTC



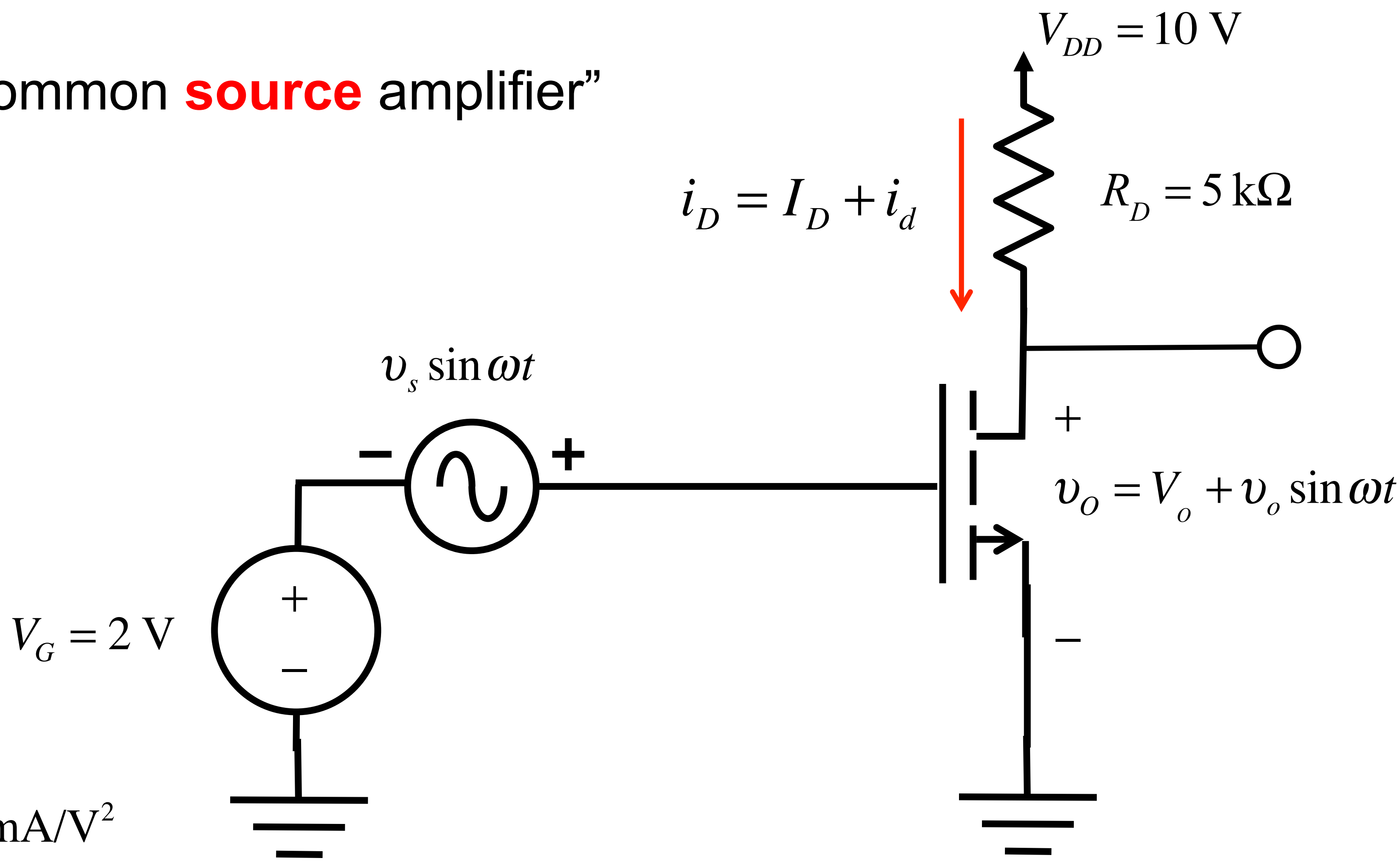
V_{OUT}
cutoff
saturation
triode
V_{DD}
$\approx V_{tn}$
V_{IN}
$V_{DD} = 10\ \text{V}$
I_D
$R_D = 5\ \text{k}\Omega$
+
V_{OUT}
–
V_{IN}
+
–

Biasing in the active region



Add ac small signal

"Common **source** amplifier"

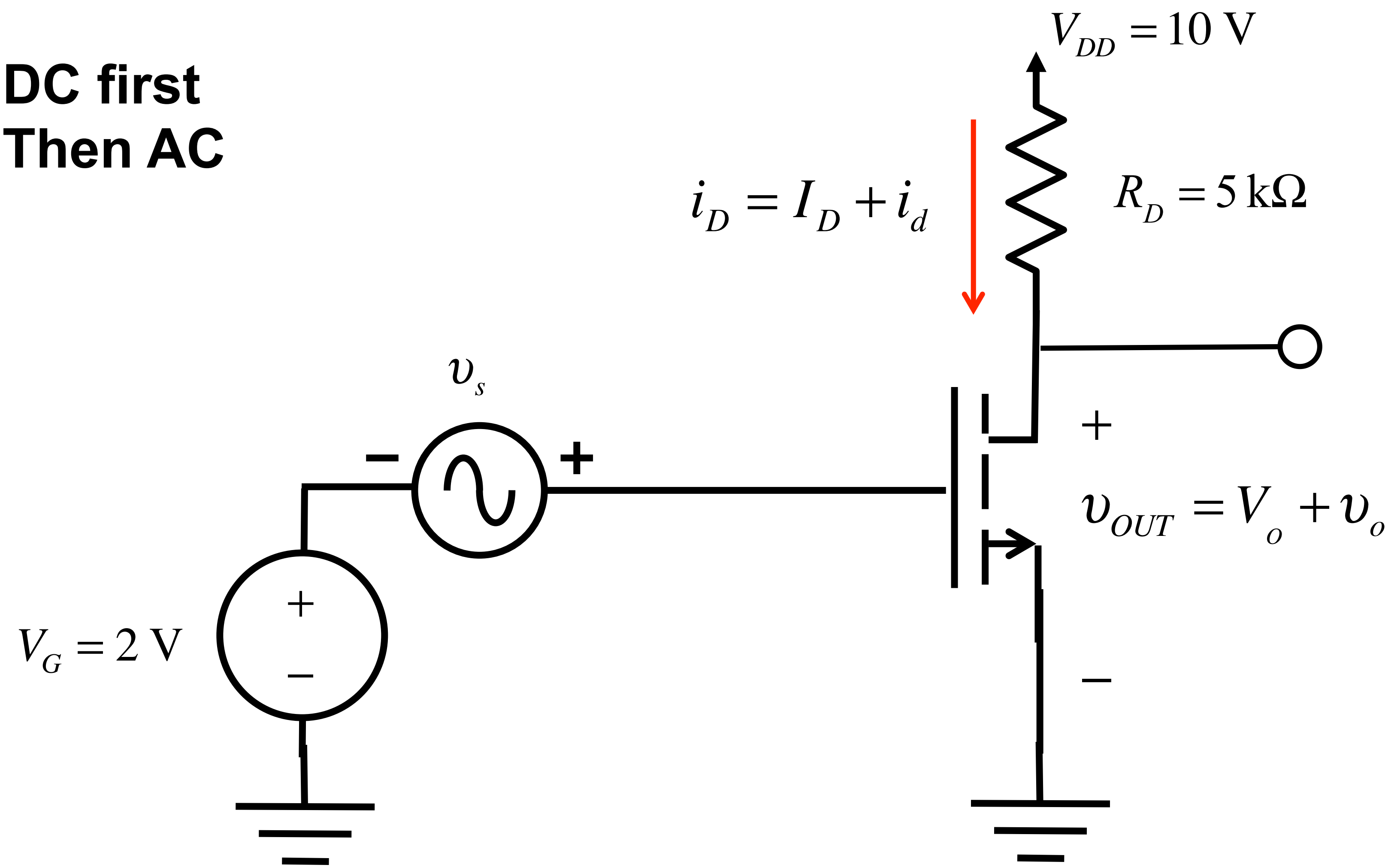


$k_n = 2\text{ mA/V}^2$

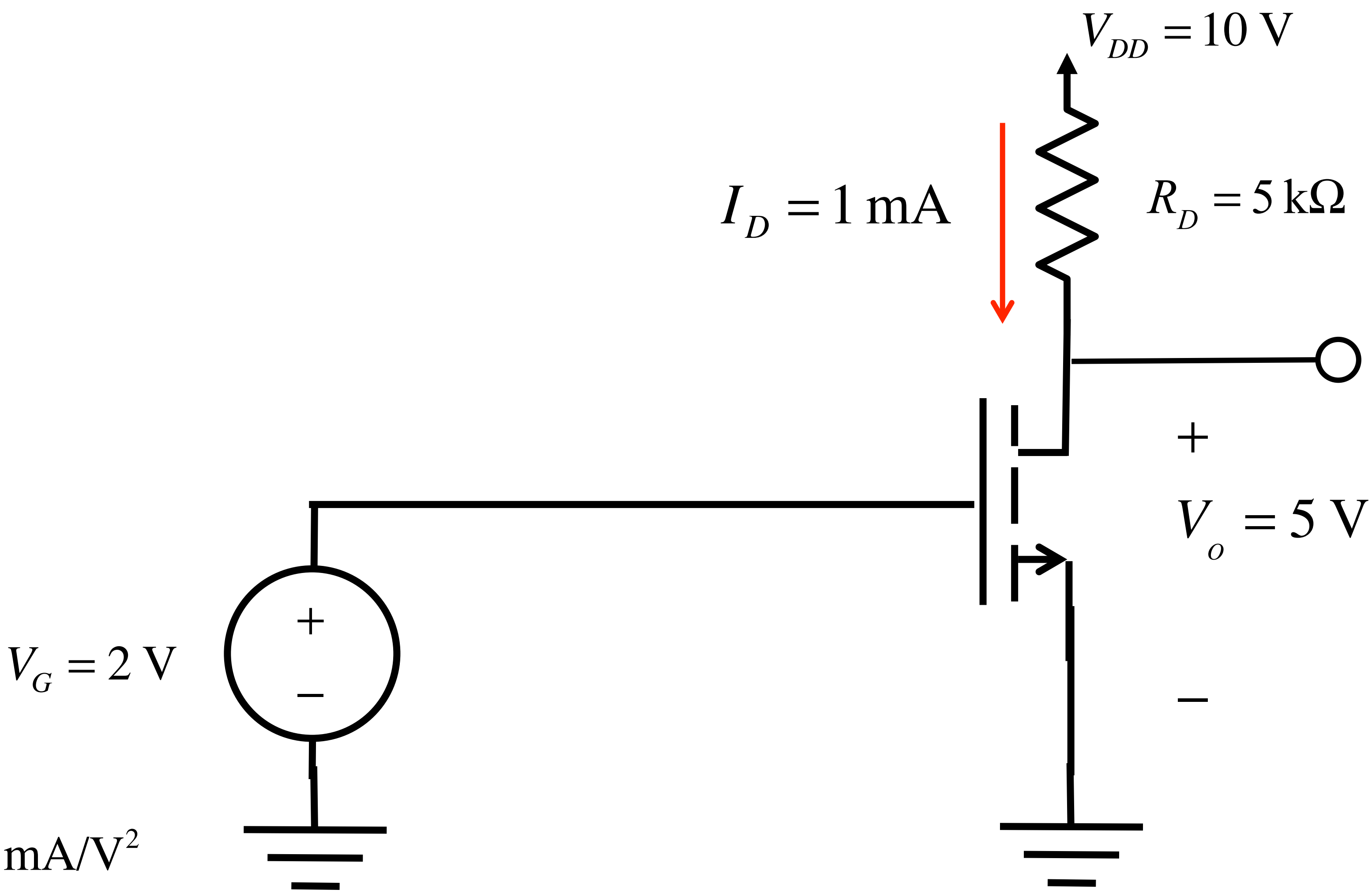
$V_{tn} = 1.0\text{ V}$

Analysis by superposition

1) **DC first**
2) **Then AC**



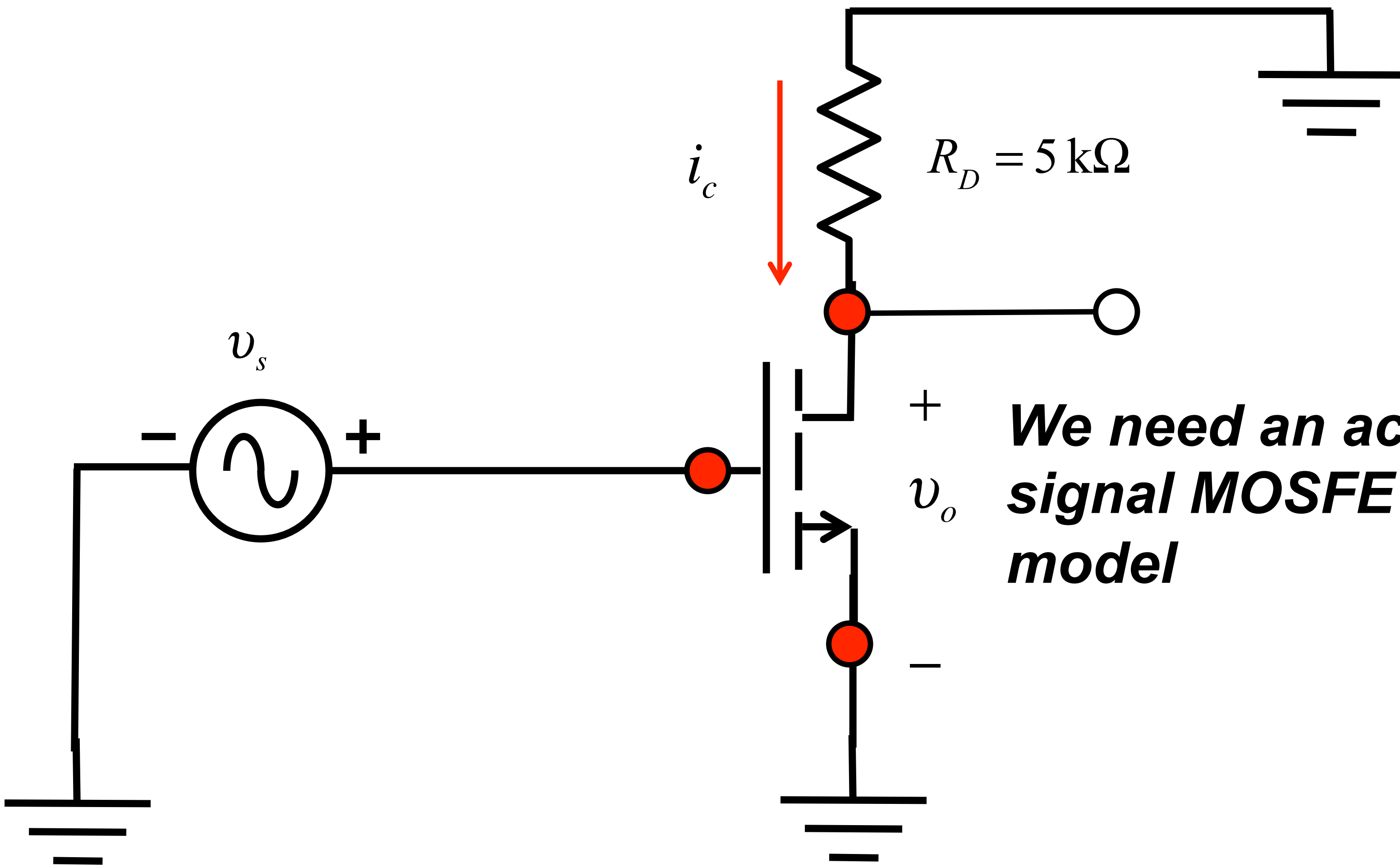
1) DC analysis



$k_n = 2\text{ mA/V}^2$

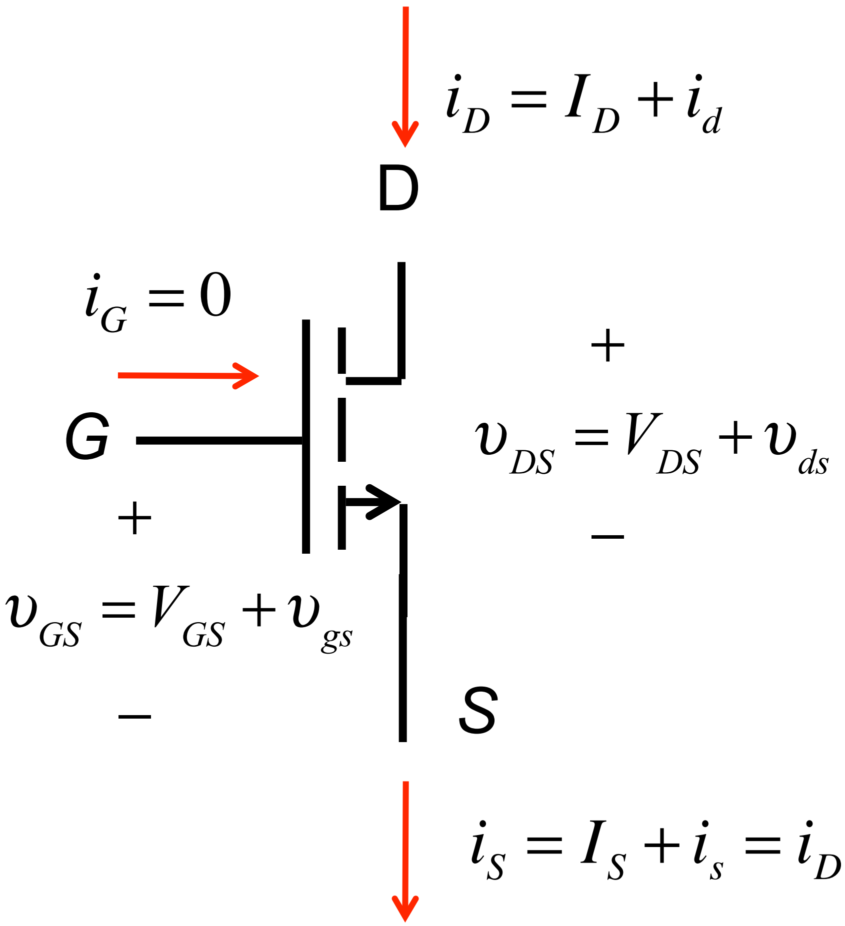
$V_{tn} = 1.0\text{ V}$

2) AC analysis



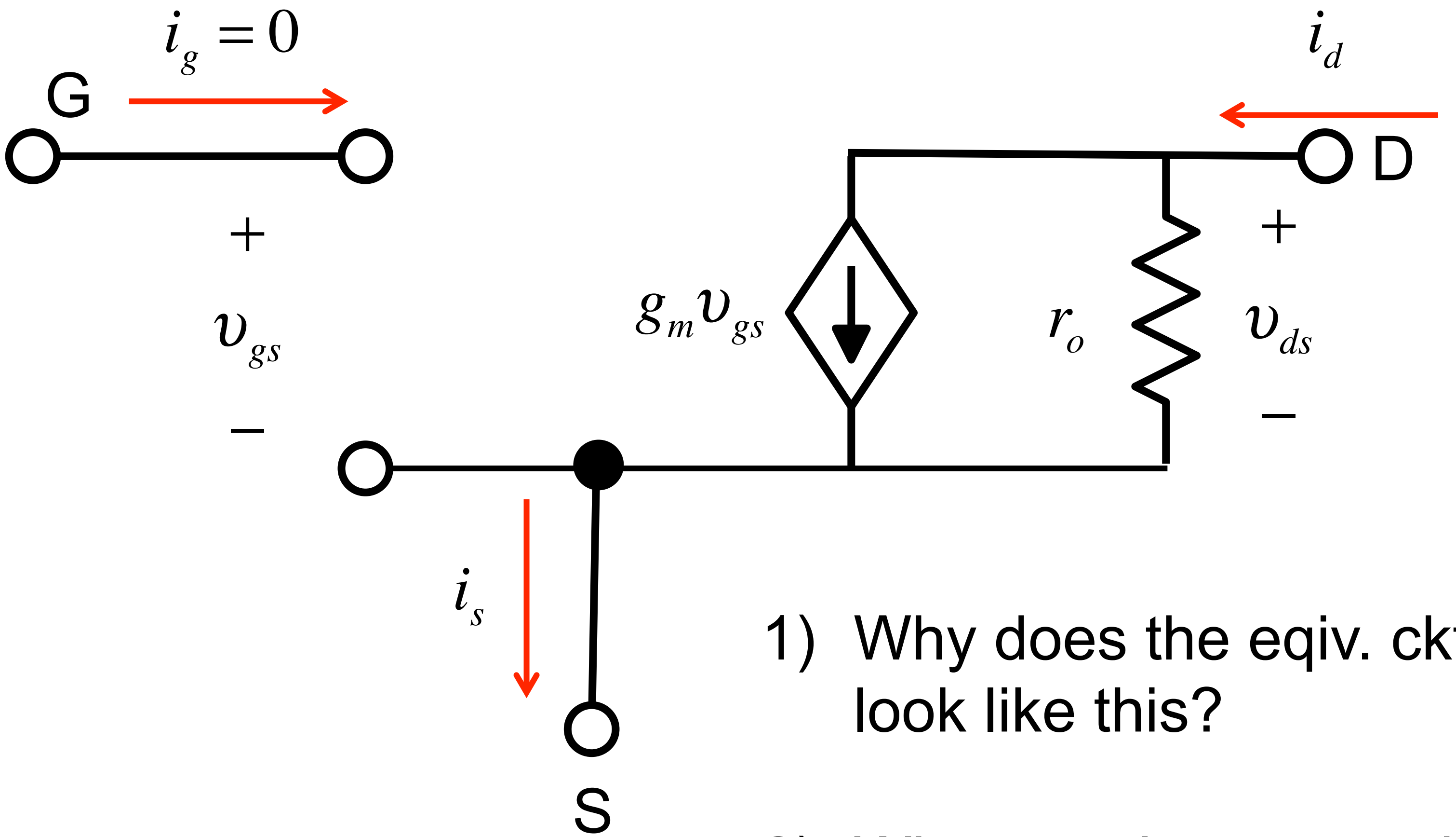
We need an ac small signal MOSFET model

MOSFET s.s. model



$$\Rightarrow \begin{pmatrix} i_d, i_g, i_s \\ \upsilon_{gs}, \upsilon_{ds} \end{pmatrix}$$

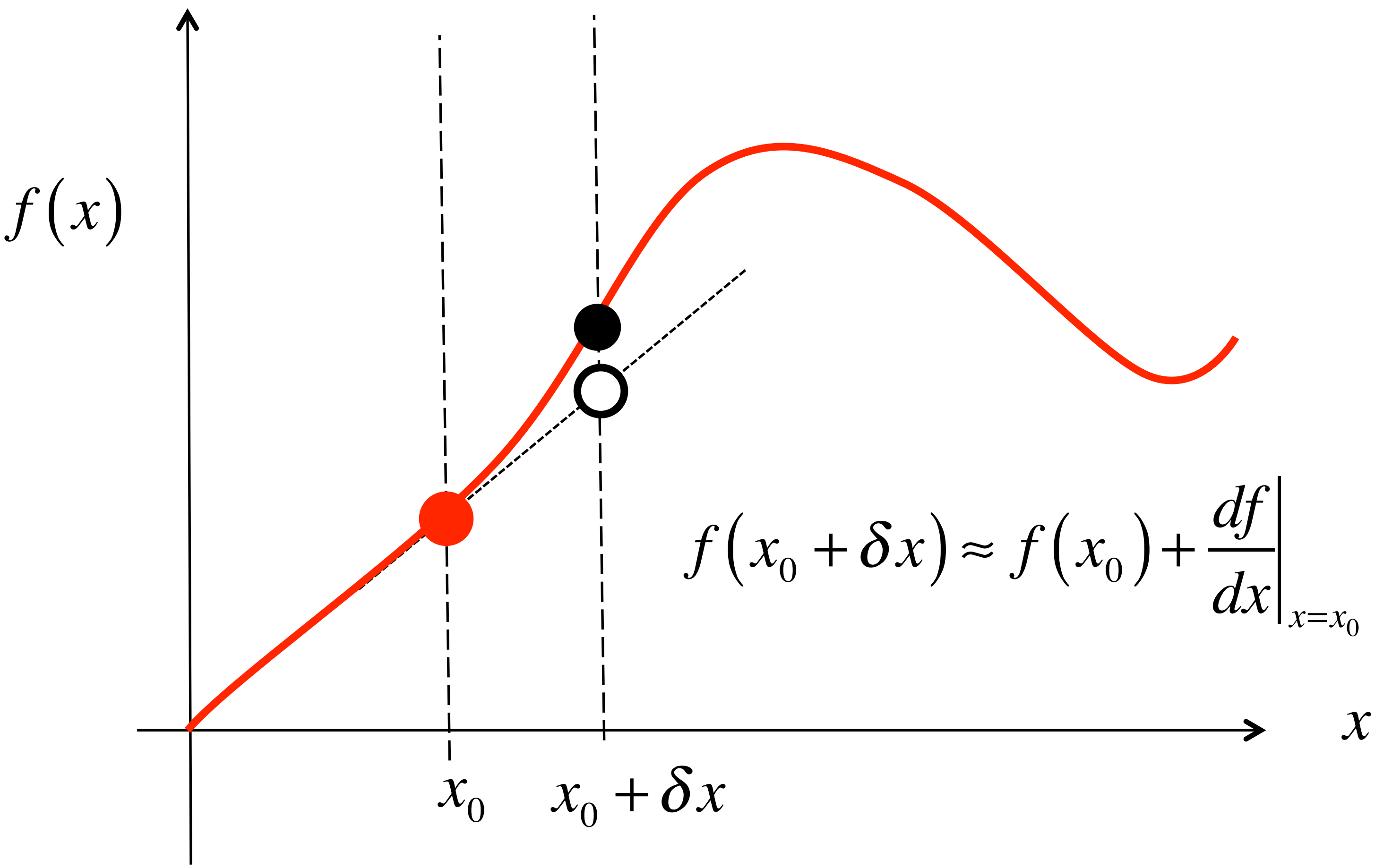
Result: MOSFET hybrid pi model



1) Why does the eqiv. ckt. look like this?

2) What are the expressions for g_m and r_o?

Taylor series



$$f(x_0 + \delta x) \approx f(x_0) + \left.\frac{df}{dx}\right|_{x=x_0} \delta x + \ldots$$

Taylor series in two variables

$f(x,y)$ $\qquad$ $f(x_0,y_0)$ $\qquad$ $f(x_0+\delta x, y_0+\delta y)=?$

$$f(x_0+\delta x, y_0+\delta y) \approx f(x_0,y_0) + \left.\frac{\partial f}{\partial x}\right|_{x_0,y_0} \delta x + \left.\frac{\partial f}{\partial y}\right|_{x_0,y_0} \delta y$$

$$\delta f = f(x_0+\delta x, y_0+\delta y) - f(x_0,y_0)$$

$$\delta f = \left.\frac{\partial f}{\partial x}\right|_{x_0,y_0} \delta x + \left.\frac{\partial f}{\partial y}\right|_{x_0,y_0} \delta y$$

Taylor series expansion of I_D

$$I_D\left(V_{GS},V_{DS}\right)=\frac{k_n}{2}\left(V_{GS}-V_{tn}\right)^2\left(1+V_{DS}/V_A\right) \qquad \left[f\left(x_0,y_0\right)\right]$$

$$I_D\left(V_{GS}+\delta V_{GS},V_{DS}+\delta V_{DS}\right)=? \qquad \left[f\left(x_0+\delta x,y_0+\delta y\right)\right]$$

$$I_D\left(V_{GS}+\delta V_{GS},V_{DS}+\delta V_{DS}\right)=I_D\left(V_{GS},V_{DS}\right) + \left.\frac{\partial I_D}{\partial V_{GS}}\right|_{V_{GS},V_{DS}}\delta V_{GS} + \left.\frac{\partial I_D}{\partial V_{DS}}\right|_{V_{GS},V_{DS}}\delta V_{DS}+\ldots$$

$$\delta I_D = I_D\left(V_{GS}+\delta V_{GS},V_{DS}+\delta V_{DS}\right)-I_D\left(V_{GS},V_{DS}\right)$$

$$\delta I_D = \left.\frac{\partial I_D}{\partial V_{GS}}\right|_{V_{GS},V_{DS}}\delta V_{GS} + \left.\frac{\partial I_D}{\partial V_{DS}}\right|_{V_{GS},V_{DS}}\delta V_{DS}$$

Taylor series (ii)

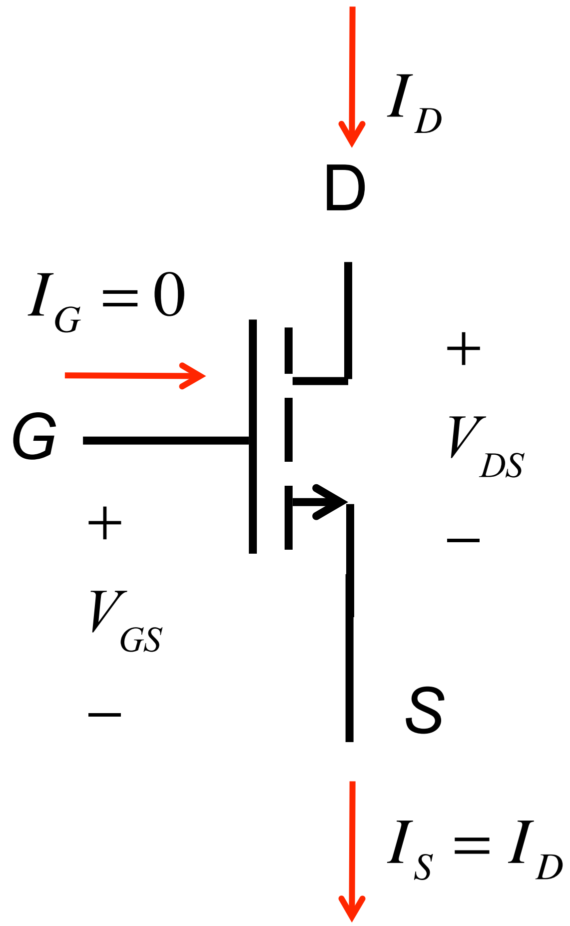
$$\delta I_D = \left.\frac{\partial I_D}{\partial V_{GS}}\right|_{V_{GS},V_{DS}} \delta V_{GS} + \left.\frac{\partial I_D}{\partial V_{DS}}\right|_{V_{GS},V_{DS}} \delta V_{DS}$$

$$\delta I_D = g_m \delta V_{GS} + g_d \delta V_{DS}$$

$$\delta I_D = g_m \delta V_{GS} + \frac{\delta V_{DS}}{r_o}$$

$$g_m \equiv \left.\frac{\partial I_D}{\partial V_{GS}}\right|_{V_{GS},V_{DS}} \qquad i_d = g_m \upsilon_{gs} + \frac{1}{r_o}\upsilon_{ds} \qquad \frac{1}{r_o} \equiv \left.\frac{\partial I_D}{\partial V_{DS}}\right|_{V_{GS},V_{DS}}$$

MOSFET in saturation (DC)



$$I_D = \frac{k_n}{2}\left(V_{GS} - V_{tn}\right)^2\left(1 + \lambda V_{DS}\right)$$

$$k_n = \frac{W}{L}\mu_n C_{ox} \qquad \lambda = \frac{1}{V_A}$$

$$V_{DS} > \left(V_{GS} - V_{tn}\right) \qquad V_{GS} > V_{tn}$$

Transconductance

$$g_m \equiv \left.\frac{\partial I_D}{\partial V_{GS}}\right|_{V_{GS},V_{DS}}$$

$$I_D = \frac{k_n}{2}\left(V_{GS} - V_{tn}\right)^2\left(1 + \lambda V_{DS}\right)$$

$$g_m = k_n\left(V_{GS} - V_{tn}\right)\left(1 + V_{DS}/V_A\right)$$

$$I_D \approx \frac{k_n}{2}\left(V_{GS} - V_{tn}\right)^2$$

$$\left(V_{GS} - V_{tn}\right) = \sqrt{\frac{2I_D}{k_n}}$$

$$g_m = \frac{2I_D}{\left(V_{GS} - V_{tn}\right)}$$

$$g_m \approx k_n\left(V_{GS} - V_{tn}\right)$$

$$g_m \approx \sqrt{2k_n I_D}$$

Output resistance

$$\frac{1}{r_o} \equiv \left.\frac{\partial I_D}{\partial V_{DS}}\right|_{V_{GS},V_{DS}}$$

$$I_D\left(V_{GS},V_{DS}\right) = \frac{k_n}{2}\left(V_{GS} - V_{tn}\right)^2\left(1 + \frac{V_{DS}}{V_A}\right)$$

$$\frac{1}{r_0} = \frac{k_n}{2}\left(V_{GS} - V_{tn}\right)^2\left(1/V_A\right) \approx \frac{I_D}{V_A}$$

Final result in equations

$$i_d = g_m \upsilon_{gs} + \frac{1}{r_0} \upsilon_{ds}$$

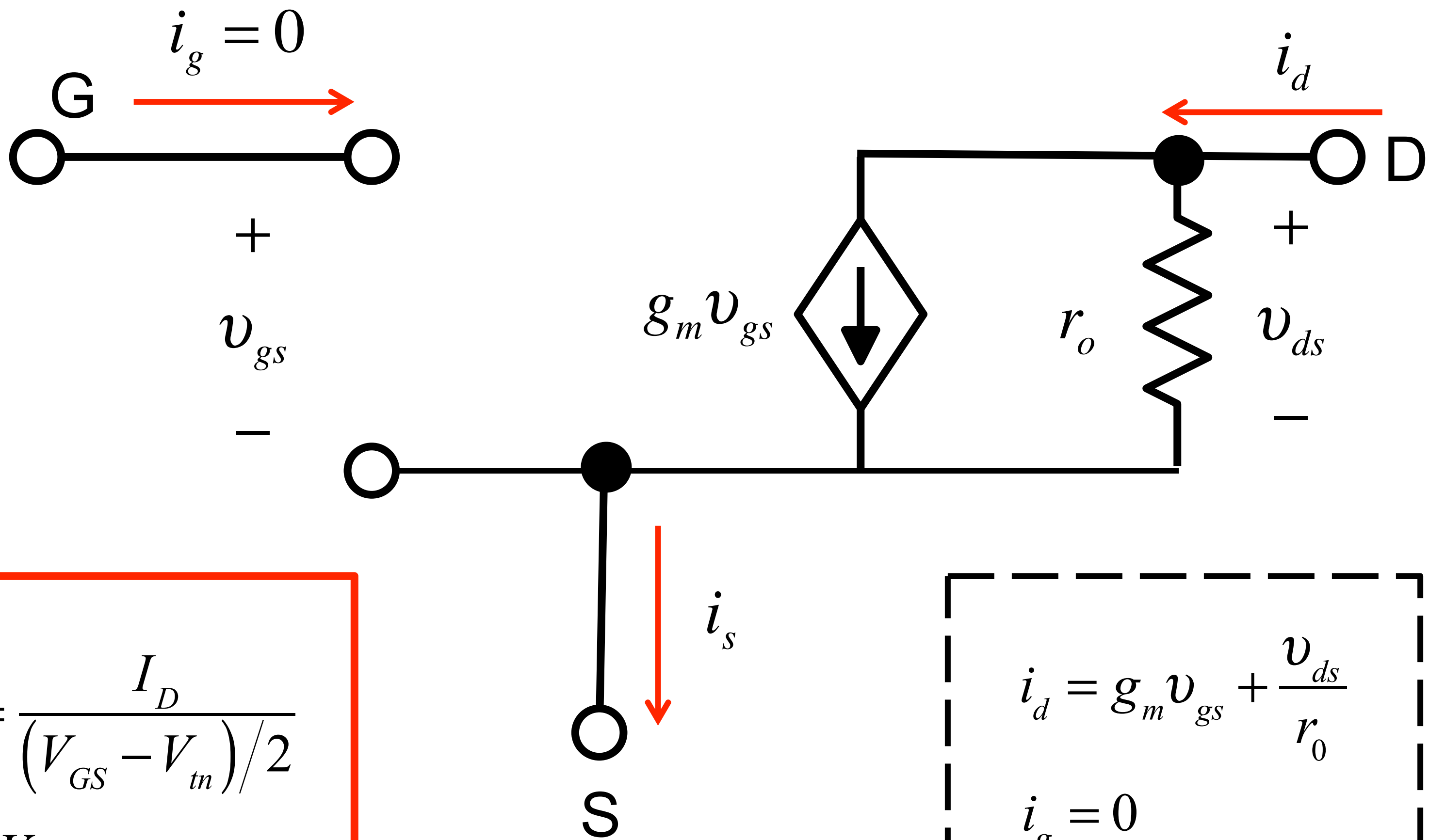
$$i_g = 0$$

$$i_s = i_d$$

$$g_m = \frac{I_D}{\left(V_{GS} - V_{tn}\right)/2}$$

$$r_o \approx \frac{V_A}{I_D}$$

Final result in circuit form (hybrid pi model)



$$g_m = \frac{I_D}{\left(V_{GS} - V_{tn}\right)/2}$$

$$r_o = \frac{V_A}{I_D}$$

$$i_d = g_m \upsilon_{gs} + \frac{\upsilon_{ds}}{r_0}$$

$$i_g = 0$$

$$i_s = i_d$$

BJT vs. MOSFET

BJT

$$g_m = \frac{I_C}{V_T}$$

$$I_C = 1\,\text{mA}$$

$$V_T = 0.025\text{ V}$$

$$g_m = 40\,\frac{\text{mA}}{\text{V}} = 40\text{ mS}$$

MOSFET

$$g_m = \frac{I_D}{\left(V_{GS} - V_{tn}\right)/2}$$

$$I_D = 1\,\text{mA} \quad V_{GS} = 2.0\text{ V} \qquad V_{tn} = 1\text{ V}$$

$$g_m = 2\ \text{mS}$$

BJT vs. MOSFET

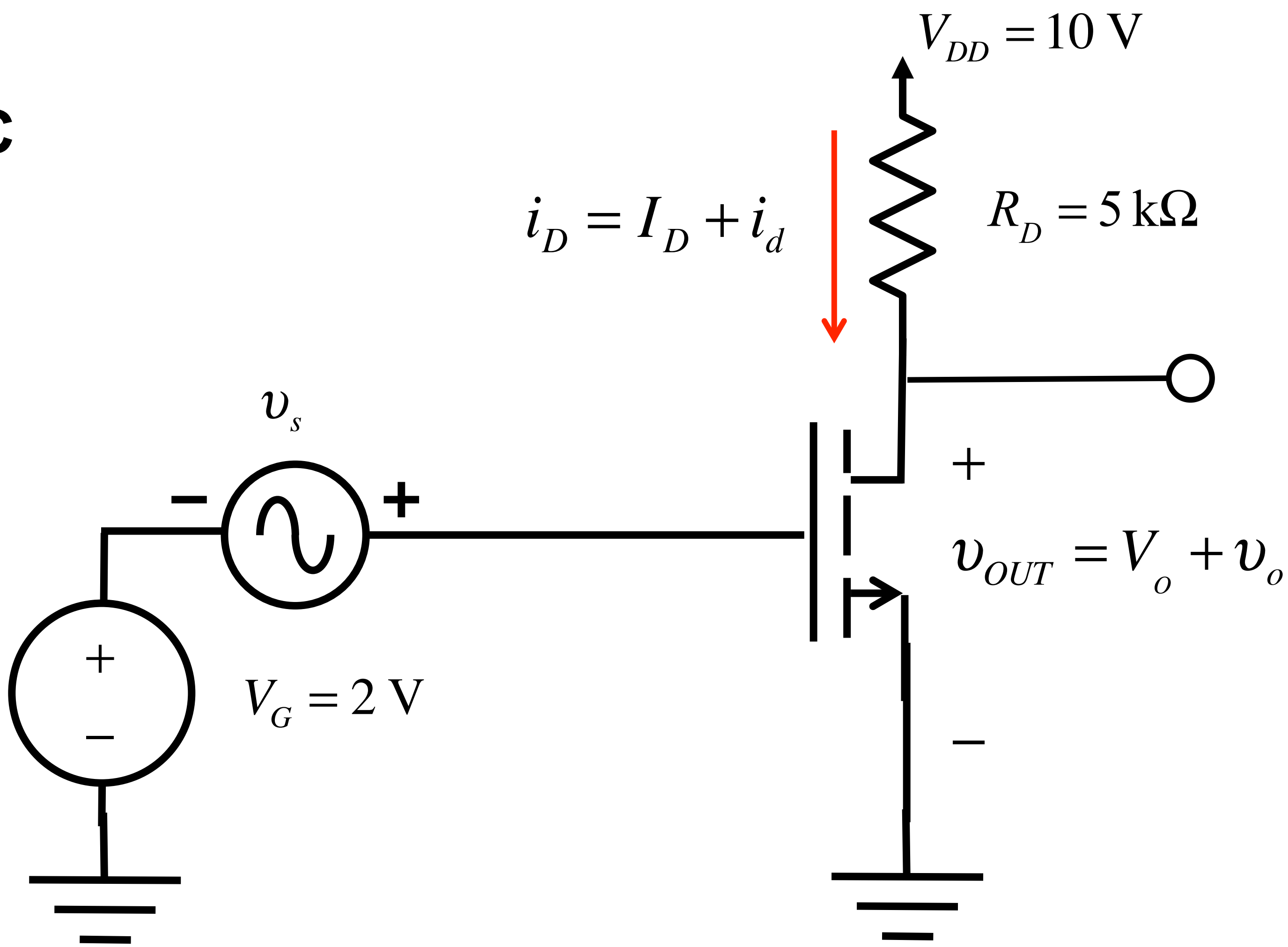
Bipolar transistors are much better amplifying devices than MOSFETs.

A typical BJT provides roughly 10 X the transconductance of a typical MOSFET.

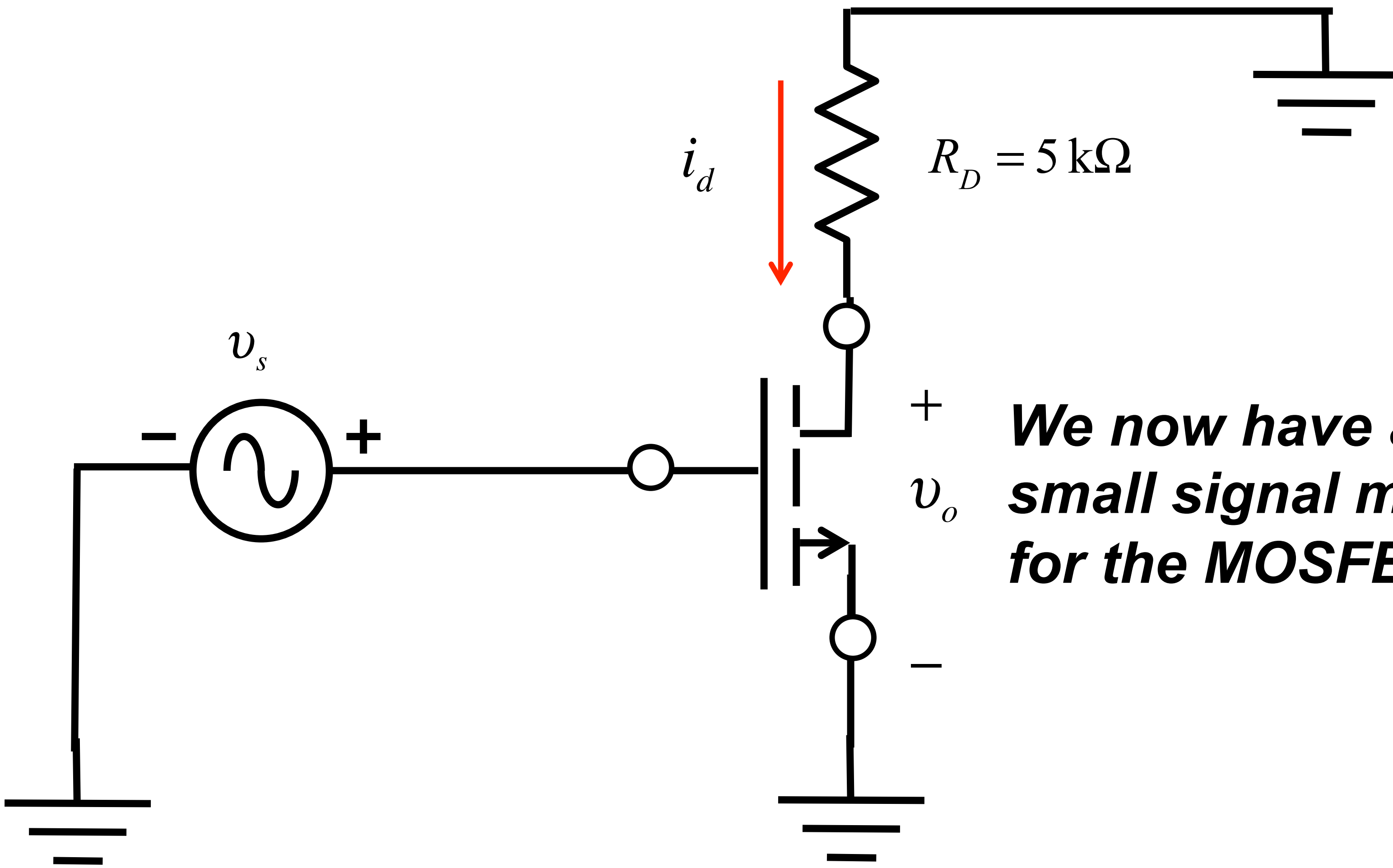
But the hybrid pi models are very similar.
(For the MOSFET, $r_\pi \rightarrow$ infinity.)

Analysis by superposition

1) **DC first**
2) **Then AC**

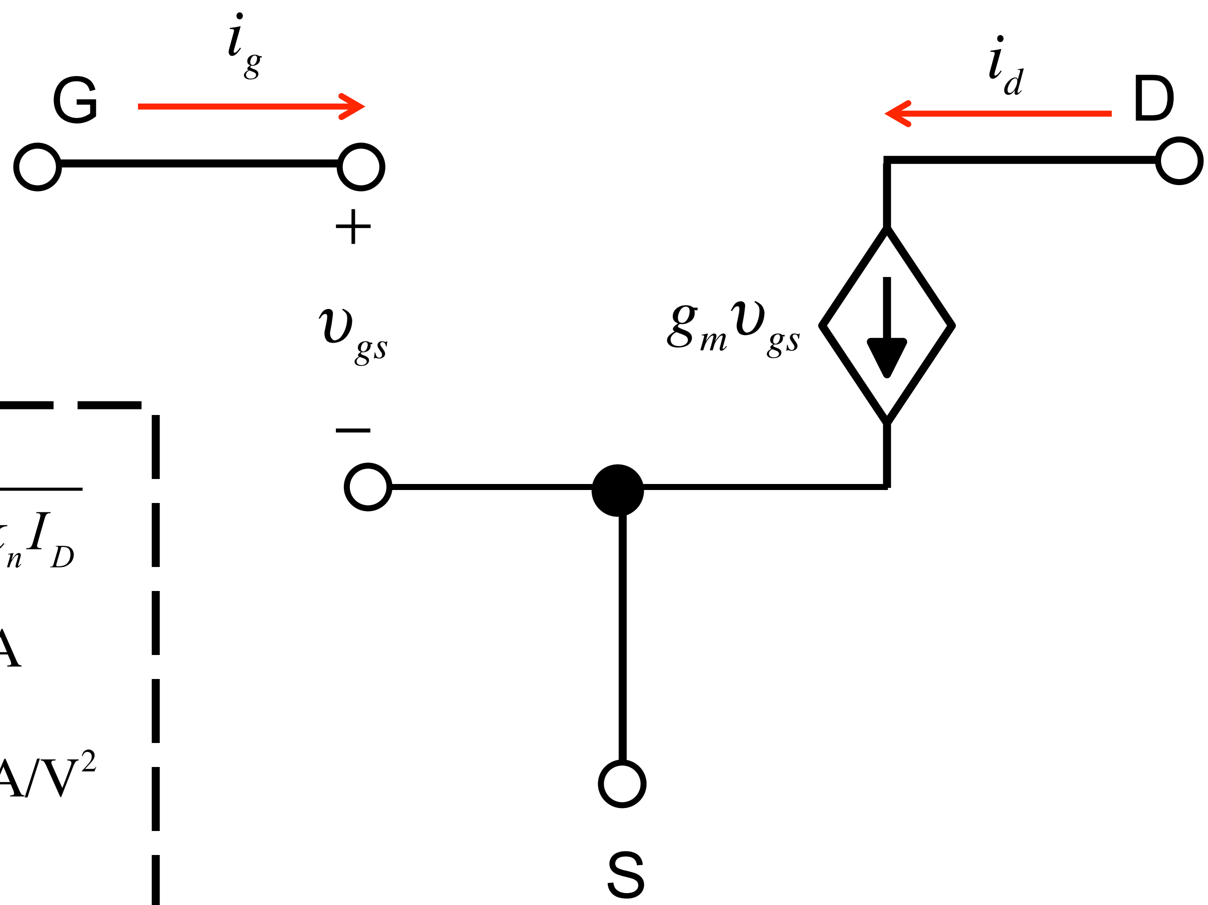


Small signal circuit



We now have an ac small signal model for the MOSFET

Simple s.s. model



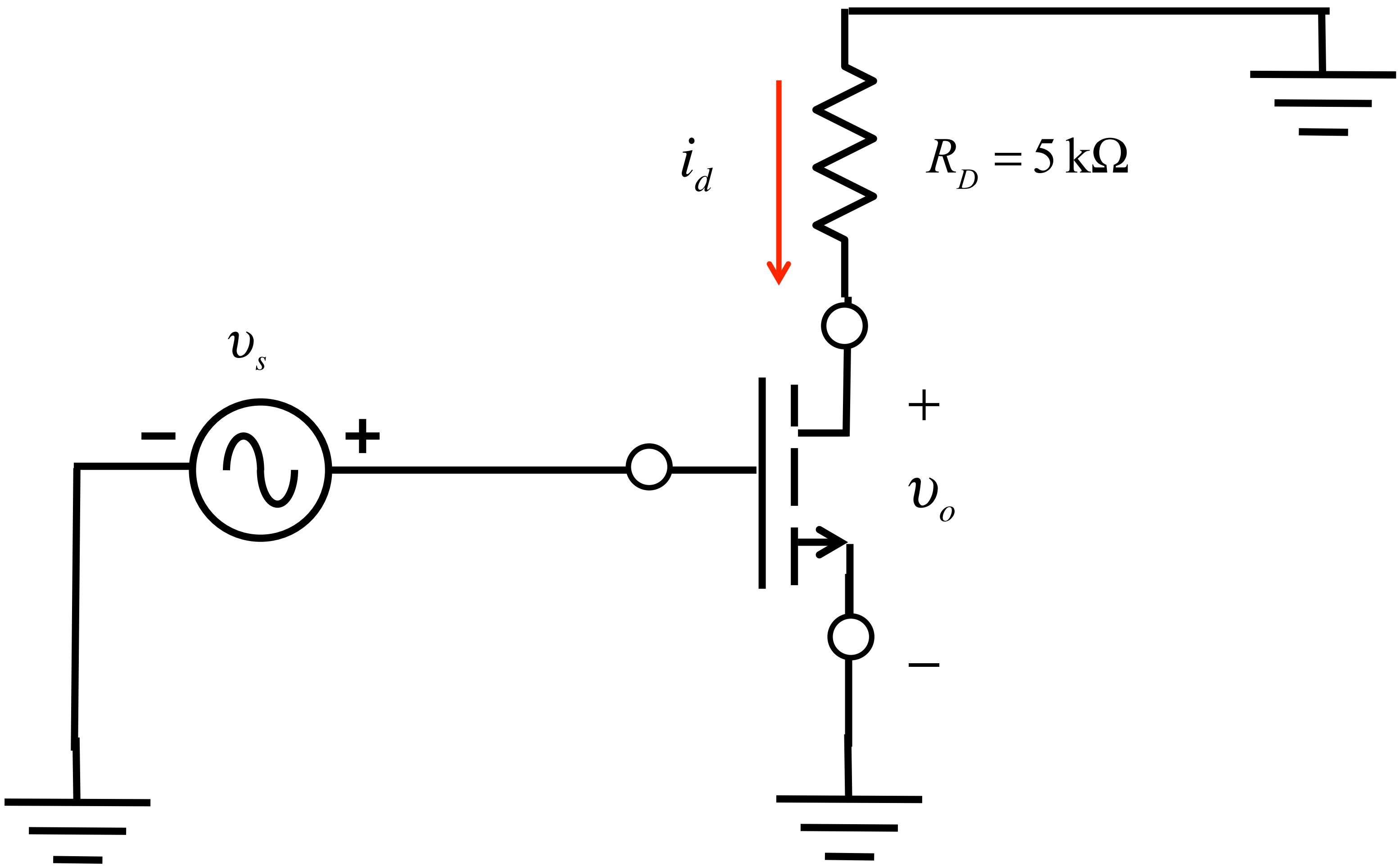
$$g_m \approx \sqrt{2k_n I_D}$$

$$I_D = 1 \text{ mA}$$

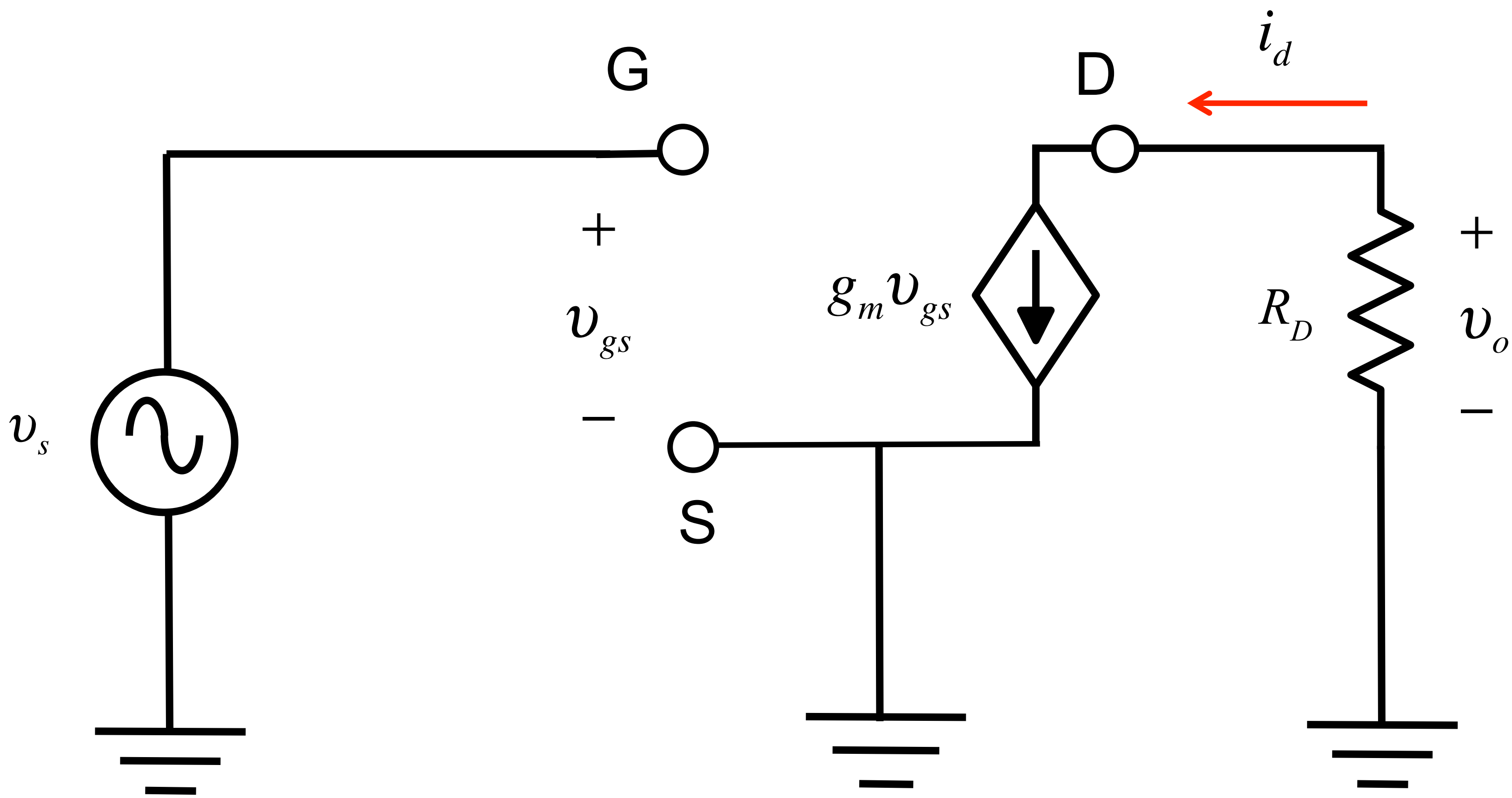
$$k_n = 2 \text{ mA/V}^2$$

$$g_m \approx 4 \text{ mS}$$

Draw the small signal equivalent circuit



Result



ac analysis

$$\upsilon_o = -g_m \upsilon_{gs} R_D$$

$$\upsilon_{gs} = \upsilon_s$$

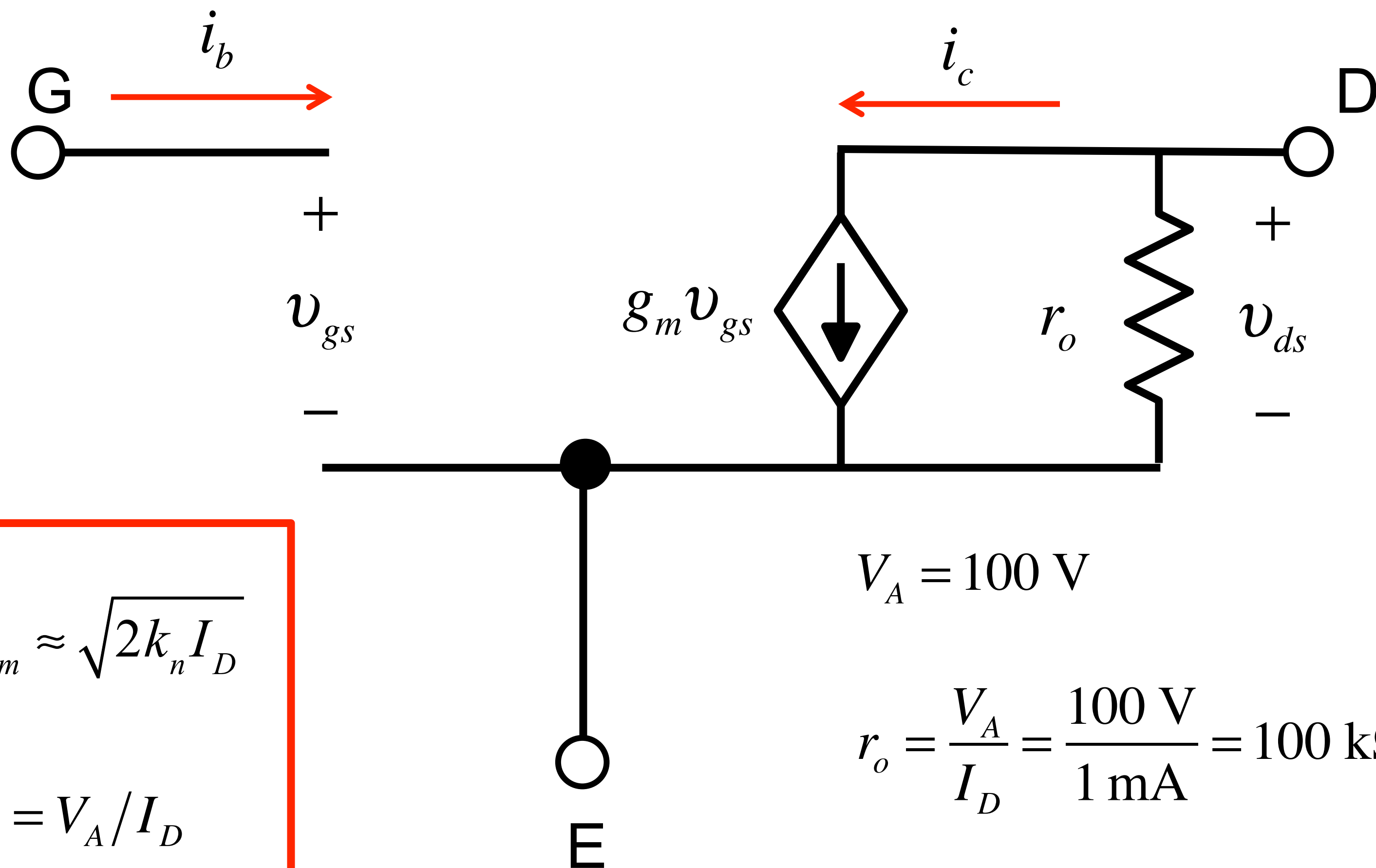
$$\upsilon_o = -g_m R_D \upsilon_s$$

$$\frac{\upsilon_o}{\upsilon_s} = A_{\upsilon_s} = -g_m R_D$$

$$g_m \approx 4 \text{ mS}$$

$$A_{\upsilon_s} = -20$$

Hybrid pi model with output resistance



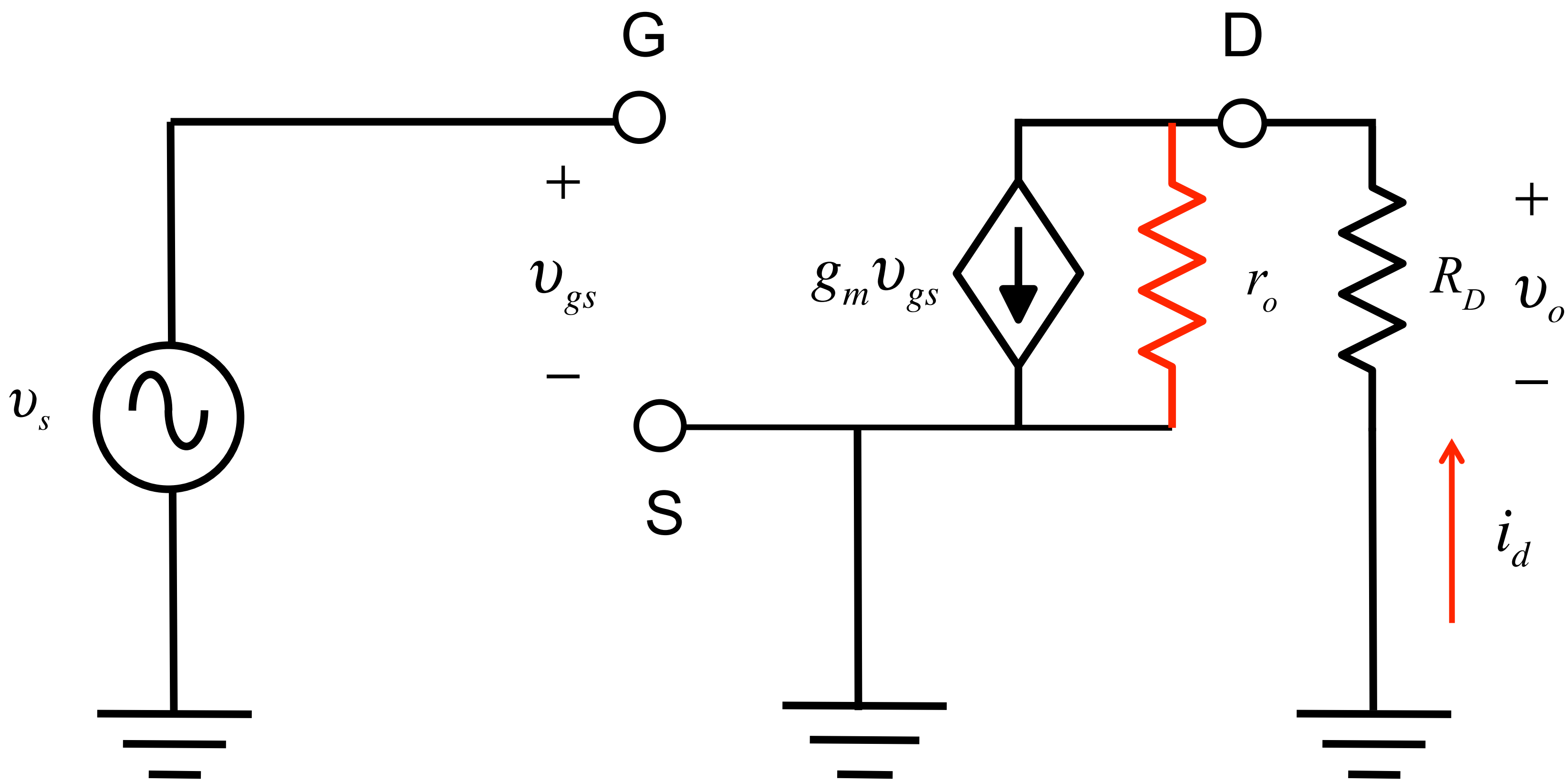
$$g_m \approx \sqrt{2k_n I_D}$$

$$r_o = V_A / I_D$$

$$V_A = 100 \text{ V}$$

$$r_o = \frac{V_A}{I_D} = \frac{100 \text{ V}}{1 \text{ mA}} = 100 \text{ k}\Omega$$

Circuit with output resistance



2) AC analysis with output resistance

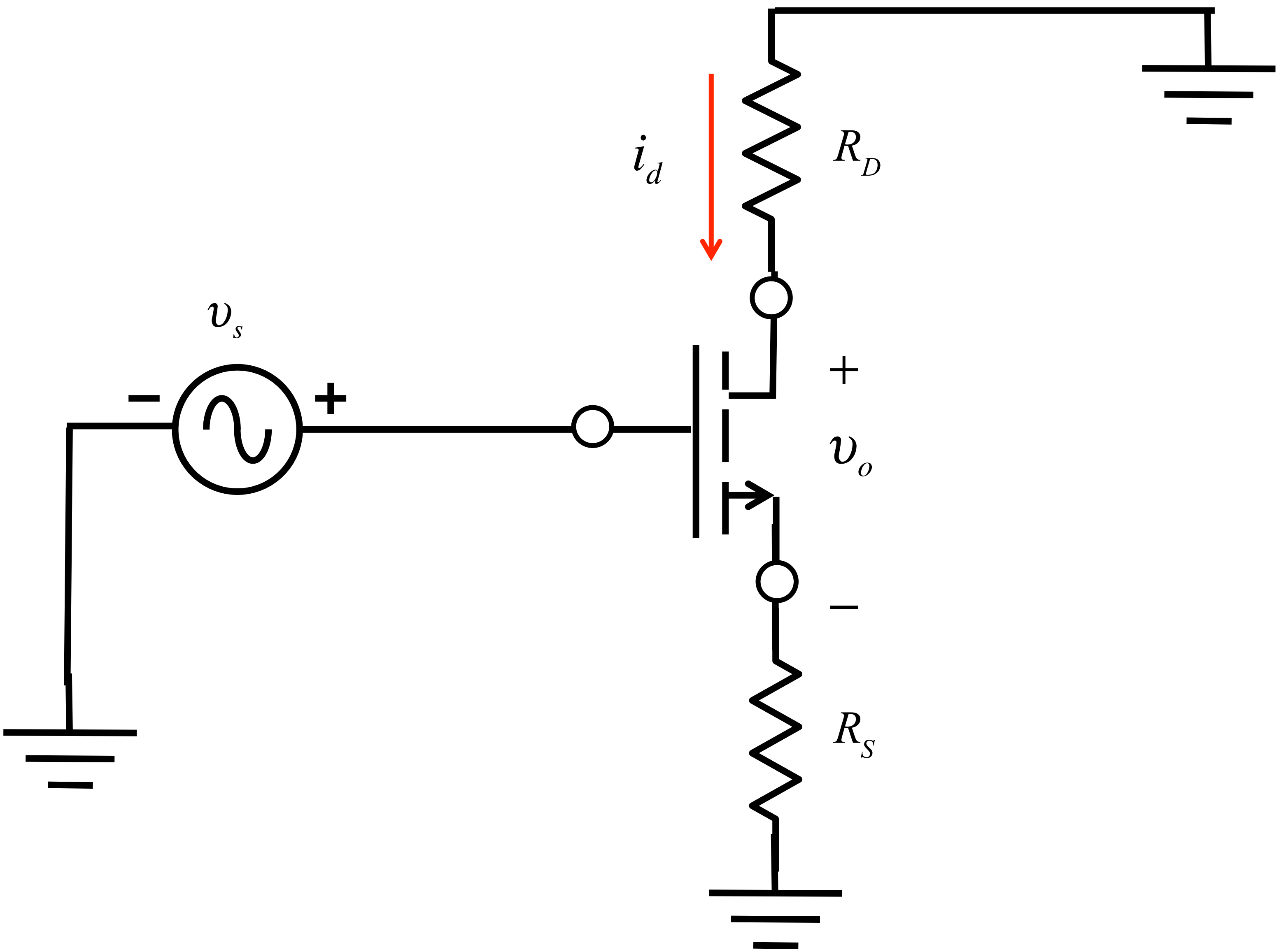
$$\frac{\upsilon_o}{\upsilon_s} = A_{\upsilon_s} = -g_m R_D$$ (without r_o)

$$\frac{\upsilon_o}{\upsilon_s} = A_{\upsilon_s} = -g_m R_D \parallel r_0$$ (with r_o)

$$5\ \text{k}\Omega \parallel 100\ \text{k}\Omega = 4.76\ \text{k}\Omega$$

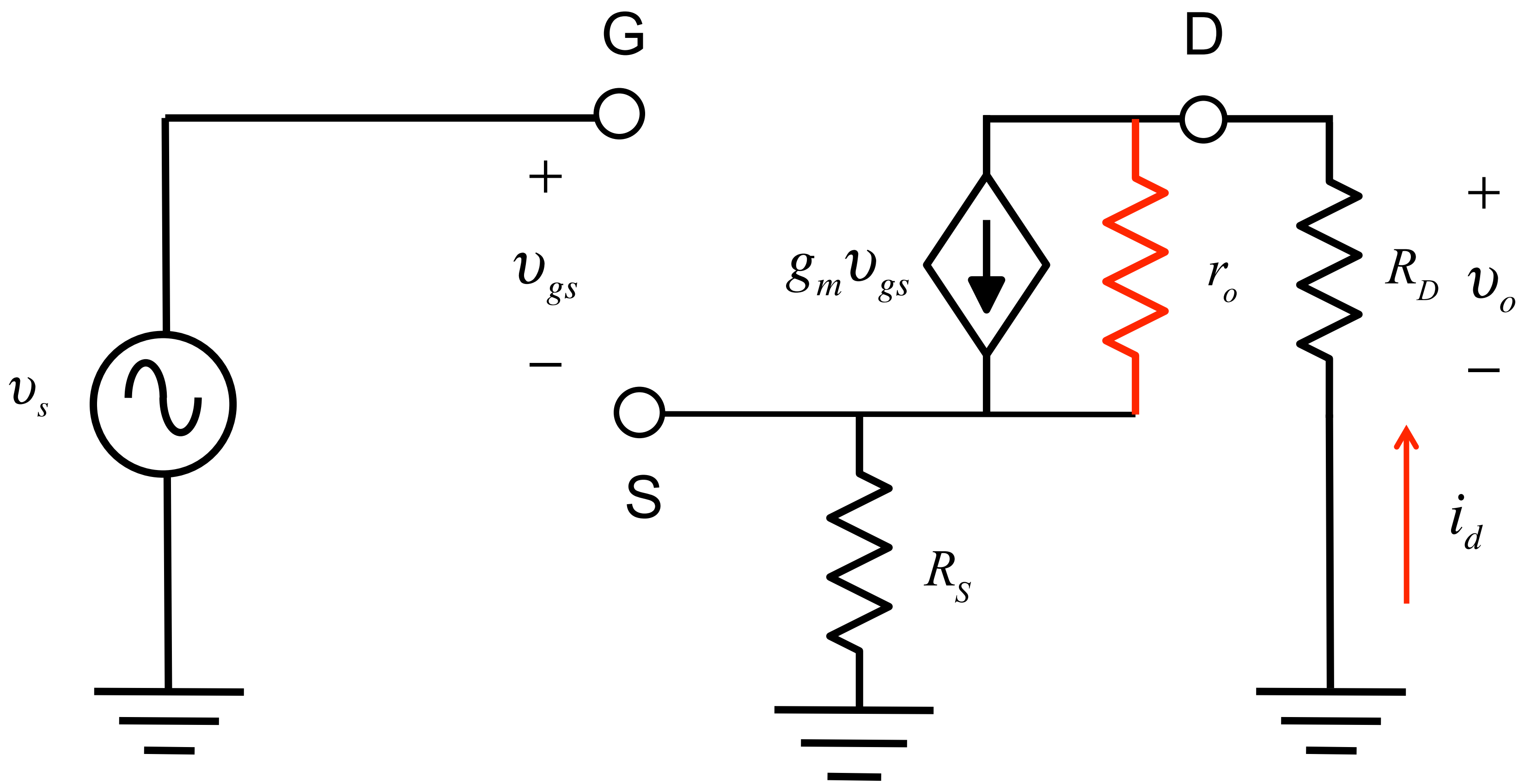
$$A_v = -20 \rightarrow A_v = -19$$

Small signal circuit with source resistor



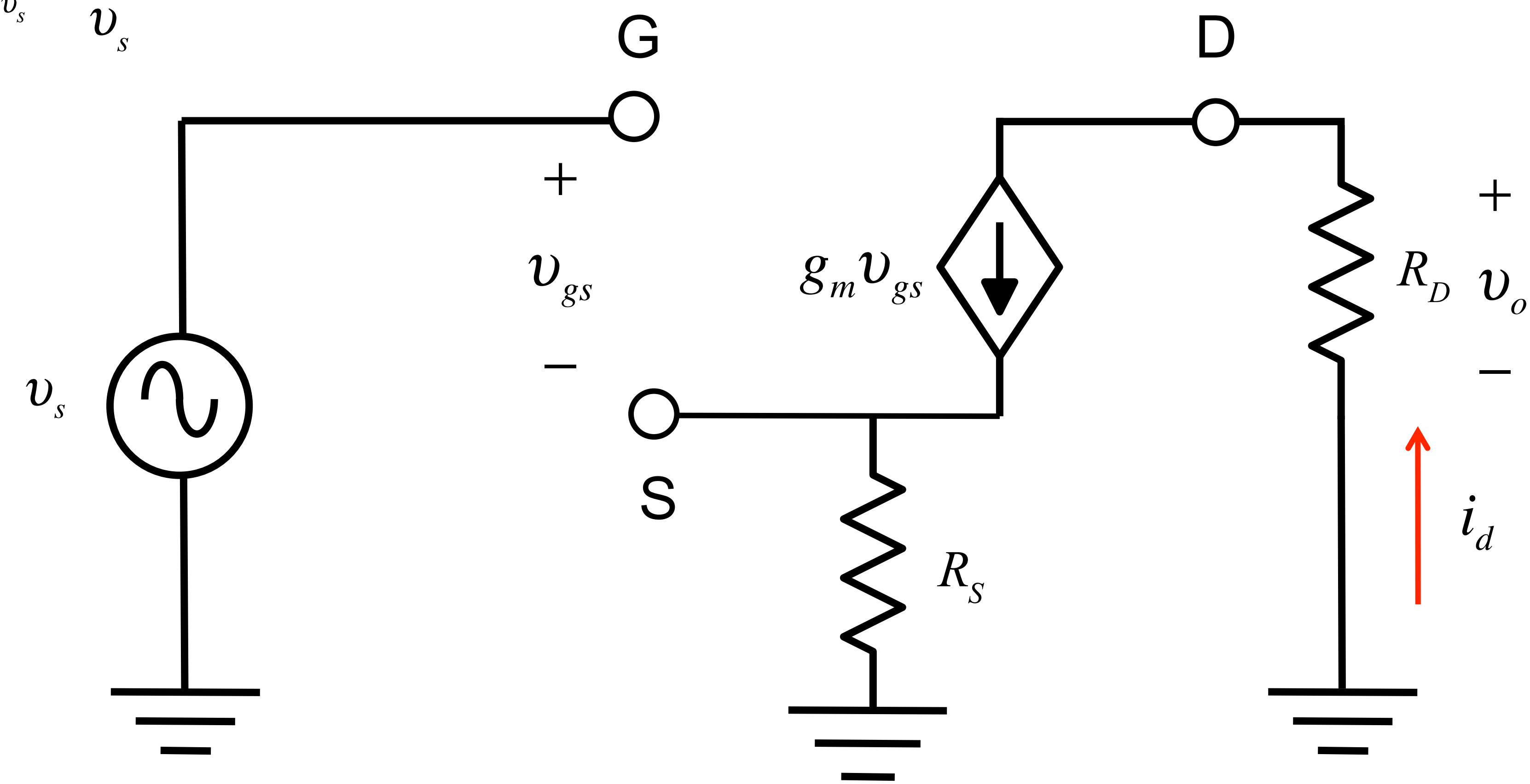
Draw the small signal circuit

The small signal circuit



Compute the voltage gain

$$A_{v_s} = \frac{v_o}{v_s}$$



Result

$$A_{\upsilon_s} = \frac{\upsilon_o}{\upsilon_s}$$

$$A_{\upsilon_s} = -\frac{g_m R_D}{1 + g_m R_S}$$

$$\left|A_{\upsilon_s}\right| < g_m R_D$$

(negative feedback from R_S)

$$g_m R_S >> 1$$

$$A_{\upsilon_s} \to -\frac{R_D}{R_S}$$

(precision amplifier)

Outline

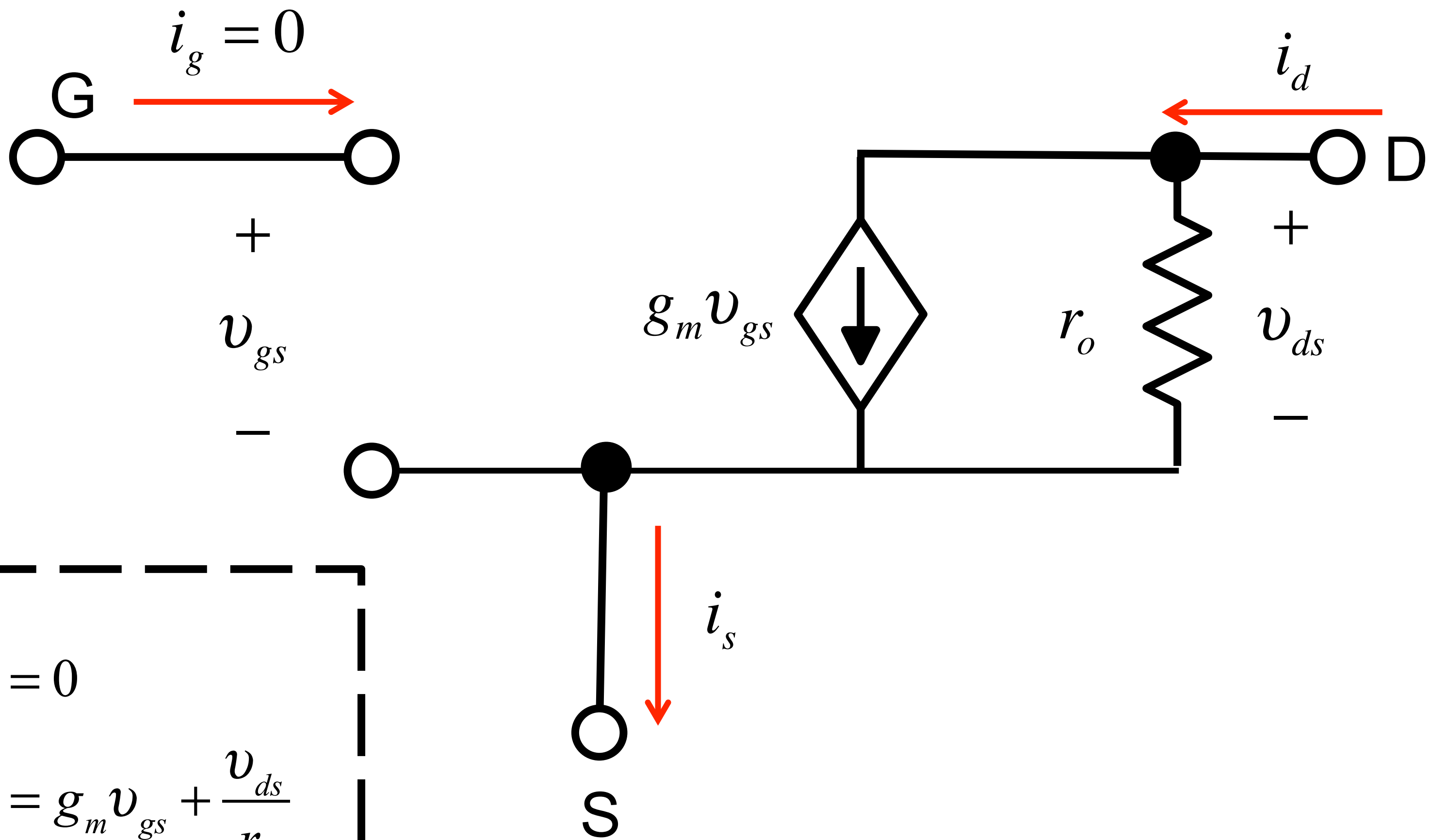
1) Voltage transfer characteristic (MOSFET)

2) Small signal model (hybrid pi)

3) Small signal analysis

4) Small signal model (T-model)

MOSFET hybrid pi model

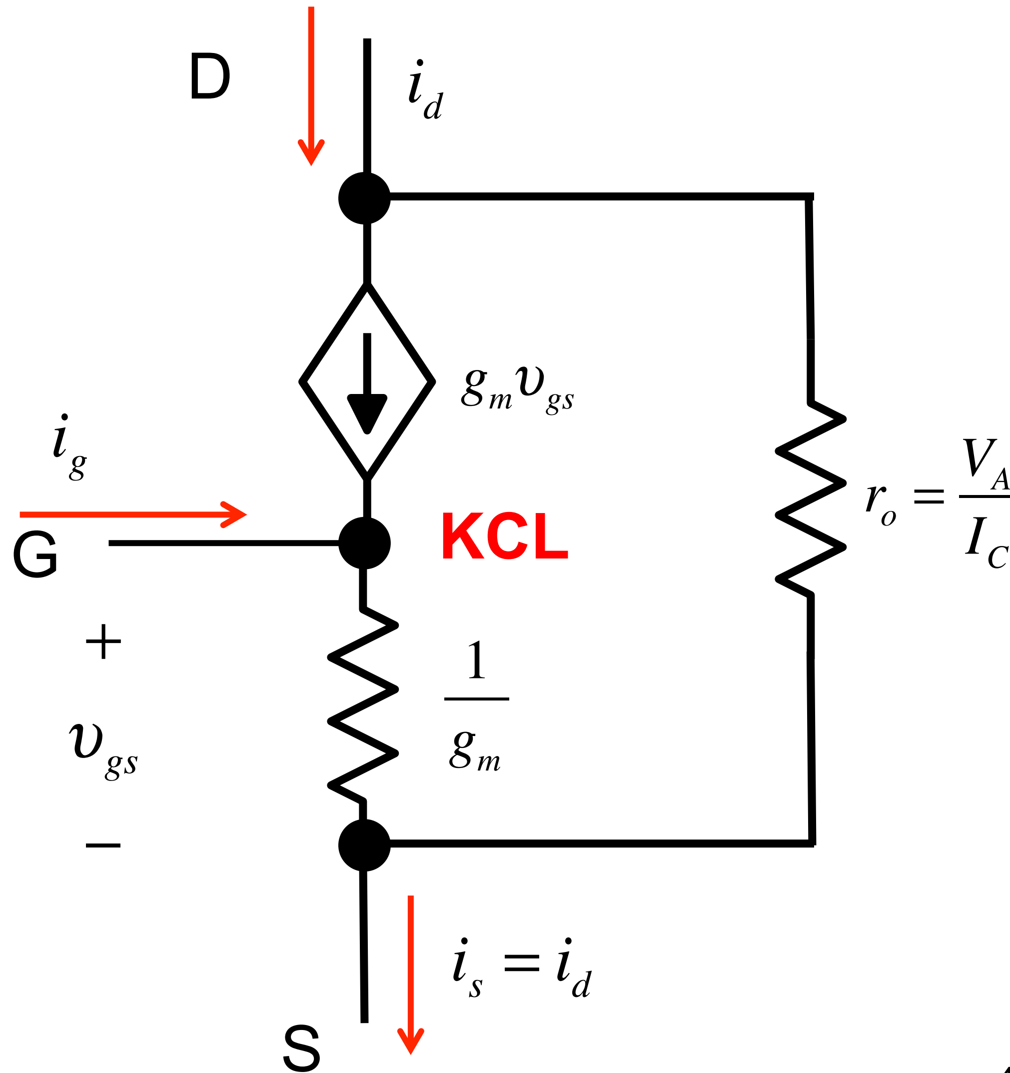


$$i_g = 0$$

$$i_d = g_m v_{gs} + \frac{v_{ds}}{r_0}$$

$$i_s = i_d$$

MOSFET T model

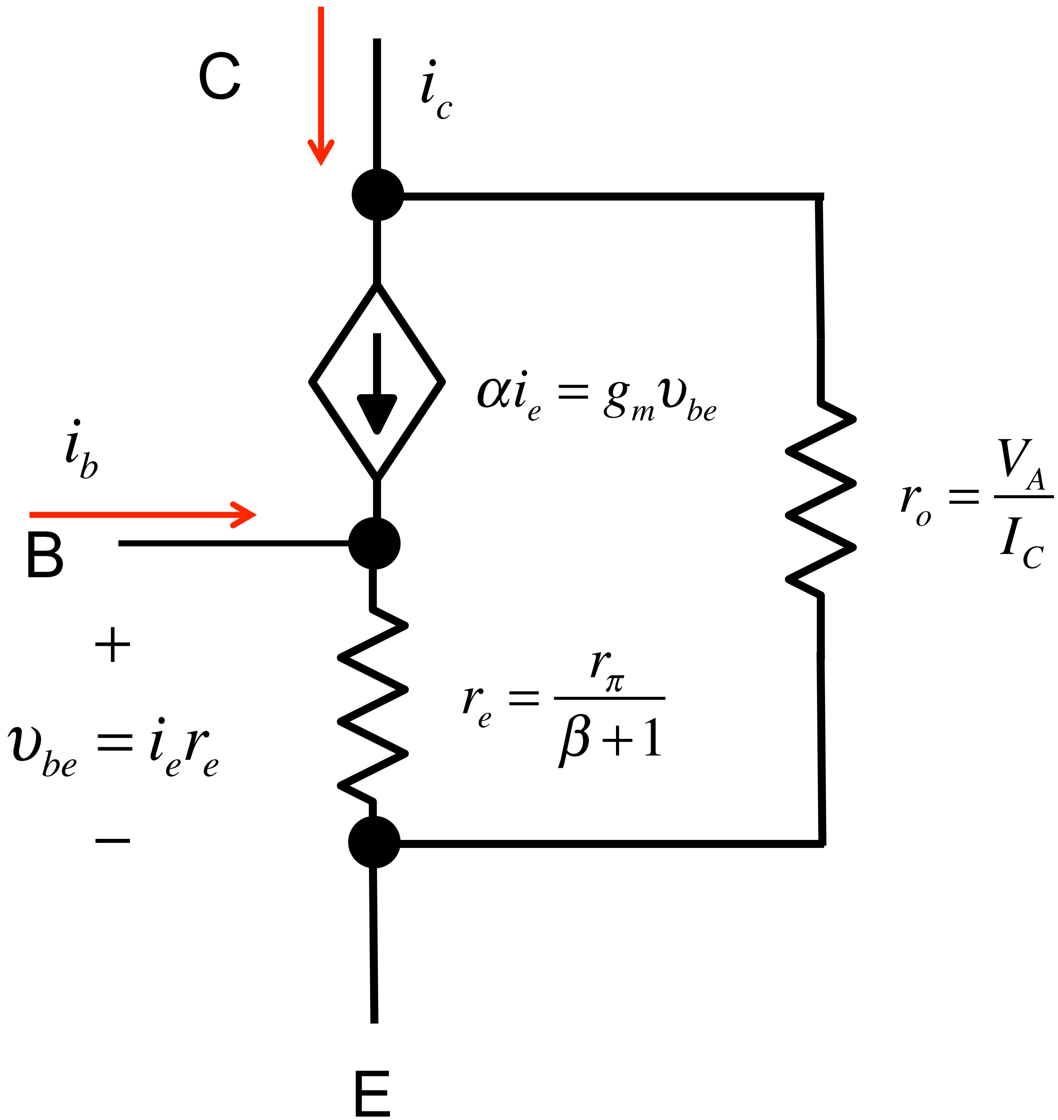


$$i_g = 0$$

$$i_d = g_m v_{gs} + \frac{v_{ds}}{r_0}$$

$$i_s = i_d$$

BJT T-Model



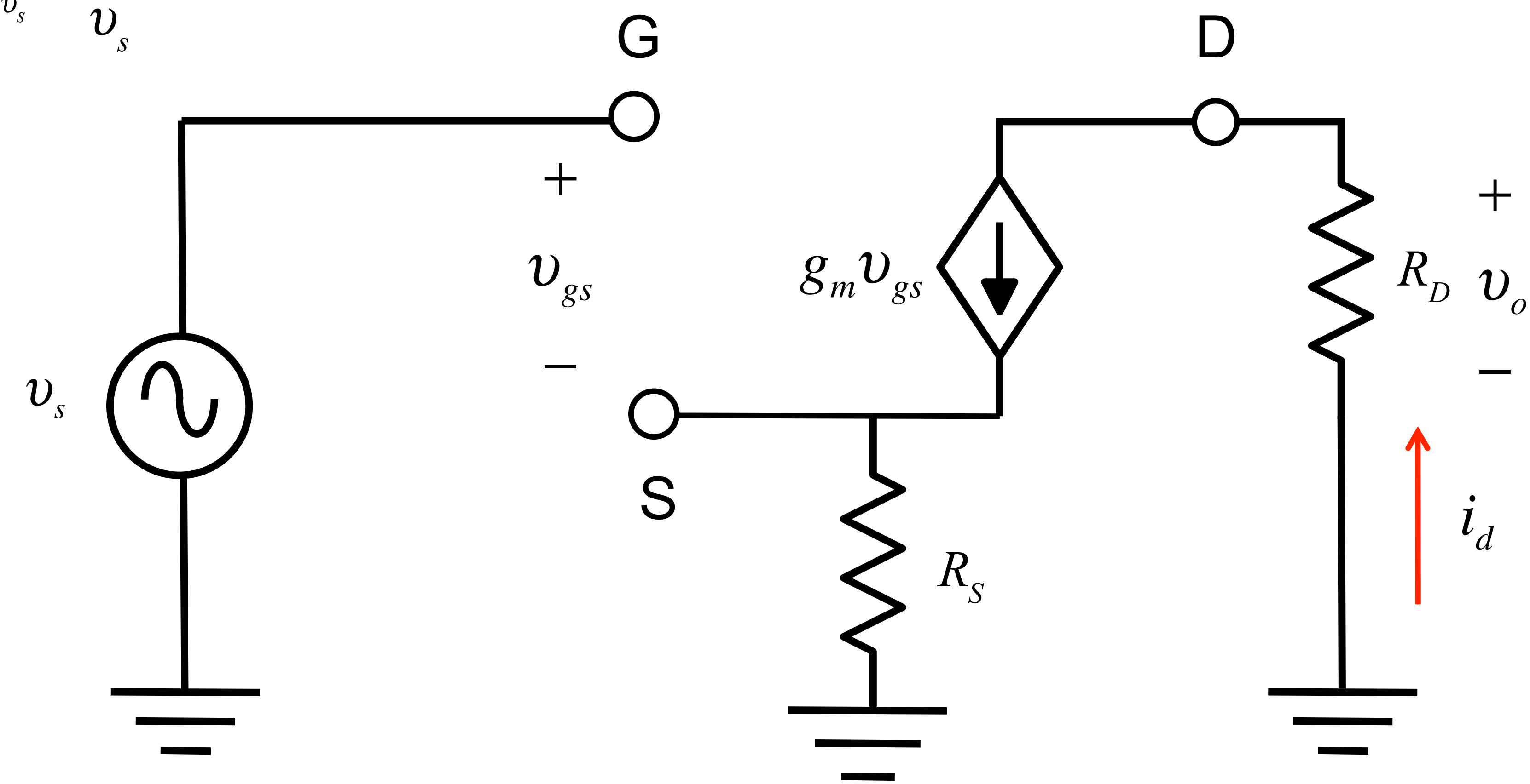
$$i_b = \upsilon_{be} / r_\pi$$

$$i_c = g_m \upsilon_{be} + \frac{\upsilon_{ce}}{r_0}$$

$$i_e = i_b + i_c$$

Exercise: Compute the voltage gain using the T model

$$A_{v_s} = \frac{v_o}{v_s}$$



Summary

The Common Source MOSFET amplifier is much like the CE BJT amplifier.

The transconductance of a MOSFET is typically more than 10 times smaller than that of a BJT.

We will stick with the hybrid pi model for MOSFETs and BJTs, but the book also uses the T model from time to time.

MOS Transistor Amplifiers

1) Voltage transfer characteristic
2) Small signal model (hybrid pi)
3) Small signal analysis
4) Small signal model (T-model)