ECE 255: L32

Low Frequency Response

(Sedra and Smith, 7th Ed., Sec. 10.1)

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Announcements

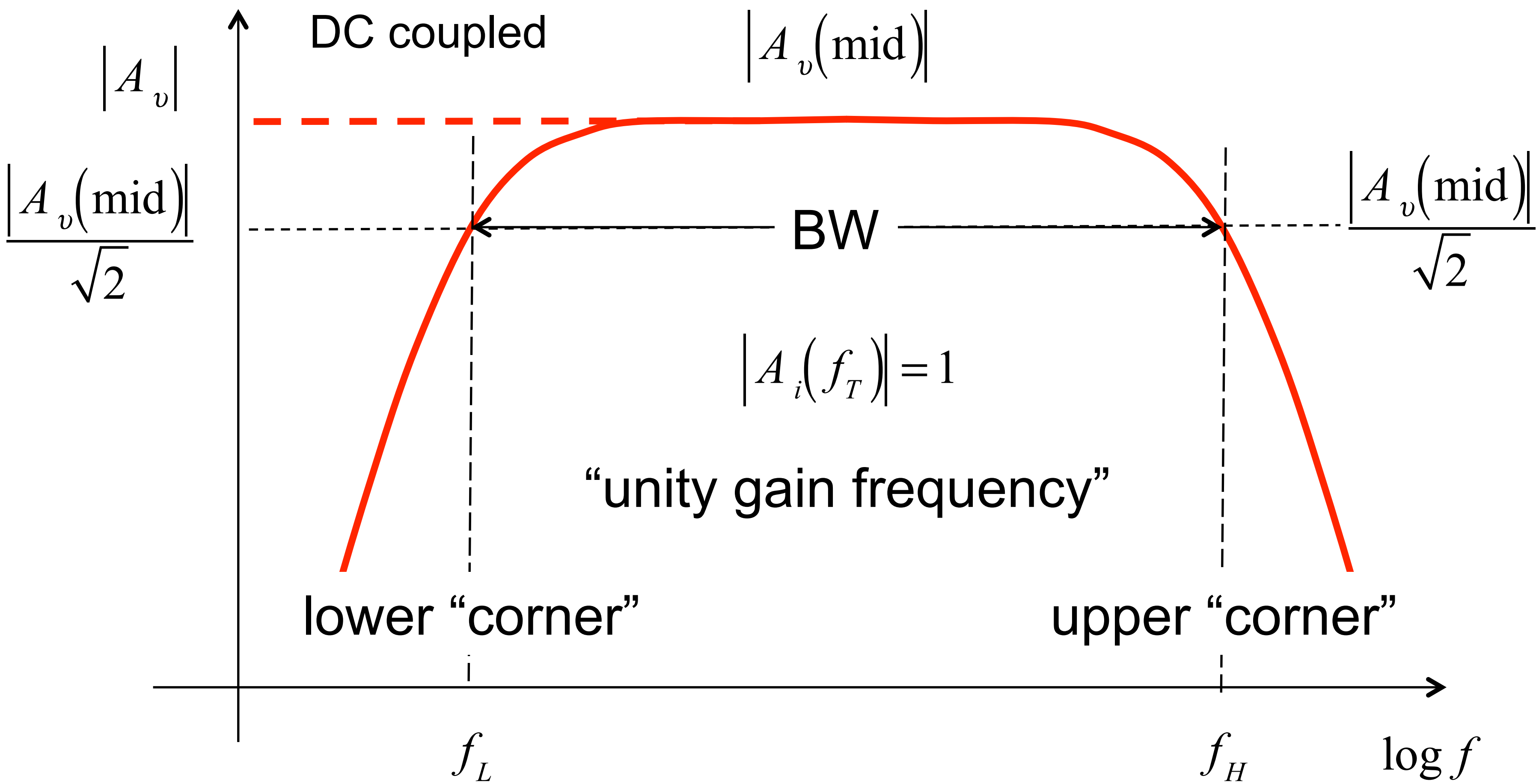
HW10 Due 5:00 PM Friday, April 19 in EE-209 dropbox

LTSpice Project III Due 5:00 PM Wed, April 17

Outline

1) Decibels
2) High pass filter / STC circuits
3) LF response of CS
4) Design Example
5) LF Response of CE
6) Exact CS analysis
7) Summary
8) Appendix

Bode plot


DC coupled
$|A_v|$
$|A_v(\text{mid})|$
$\frac{|A_v(\text{mid})|}{\sqrt{2}}$
$\frac{|A_v(\text{mid})|}{\sqrt{2}}$
BW
$|A_i(f_T)| = 1$
"unity gain frequency"
lower "corner"
upper "corner"
f_L
f_H
$\log f$

Decibels

Power

$$A_p = \frac{P_{out}}{P_{in}} = \frac{\upsilon_o i_o}{\upsilon_i i_i} = \frac{\upsilon_o \upsilon_o / R_L}{\upsilon_i \upsilon_i / R_{in}} = \left(\frac{\upsilon_o}{\upsilon_i}\right)^2 \frac{R_{in}}{R_L}$$

$$A_p\Big|_{dB} = 10\log_{10}\left(\frac{P_{out}}{P_{in}}\right)$$

Ap	Ap (dB)
0.01	-20 dB
0.1	-10 dB
0.5	-3 dB
1	0 dB
10	10 dB
100	20 dB

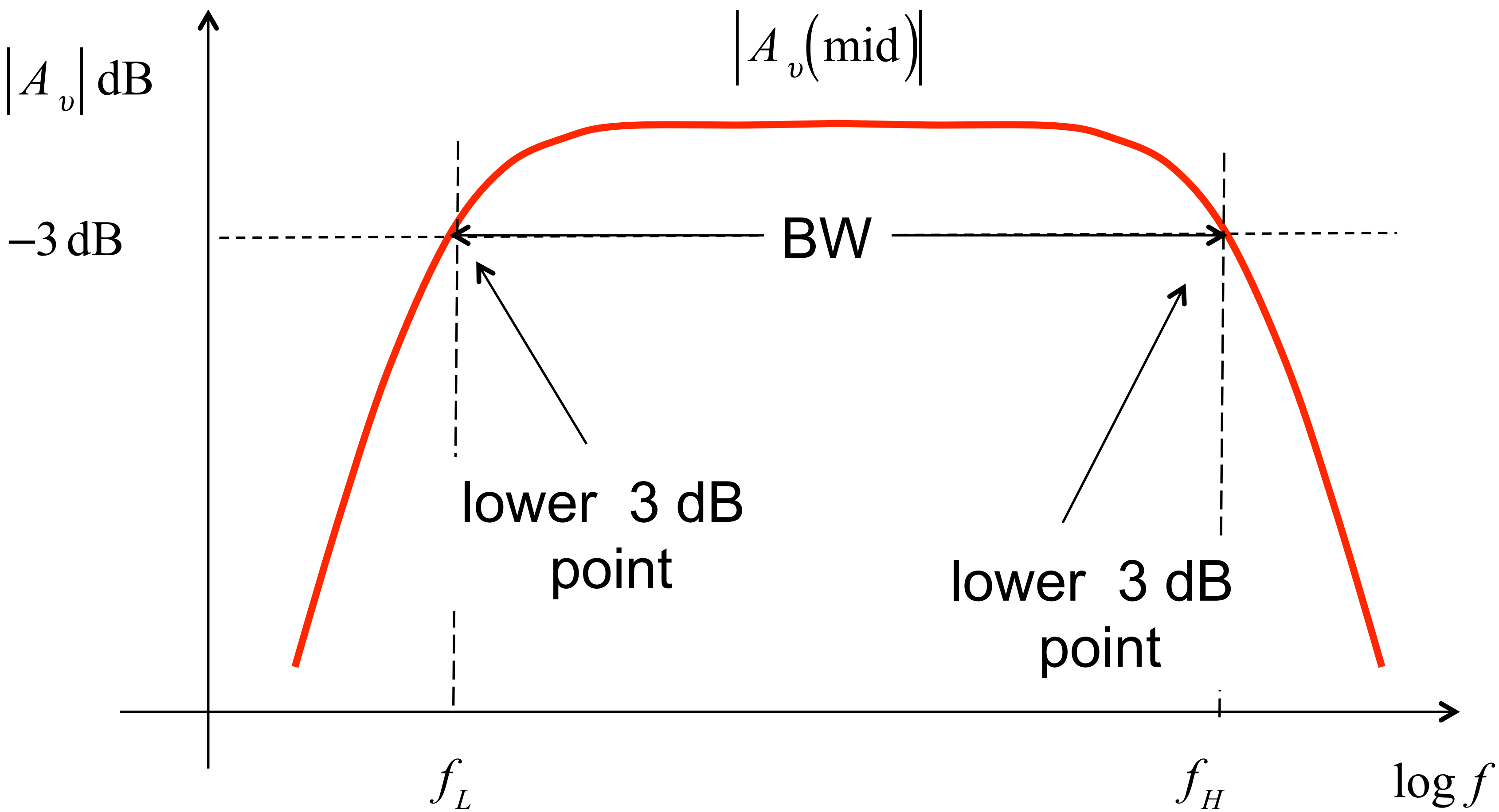
Voltage

$$A_\upsilon = \frac{\upsilon_o}{\upsilon_i}$$

$$A_p\Big|_{dB} = 20\log_{10}\left(\frac{\upsilon_o}{\upsilon_i}\right)$$

Av	Av (dB)
0.01	-40 dB
0.1	-20 dB
1/sqrt(2)	-3 dB
1	0 dB
10	20 dB
100	40 dB

Bode plot



$|A_v|$ dB
$|A_v(\text{mid})|$
–3 dB
BW
lower 3 dB point
lower 3 dB point
f_L
f_H
log f

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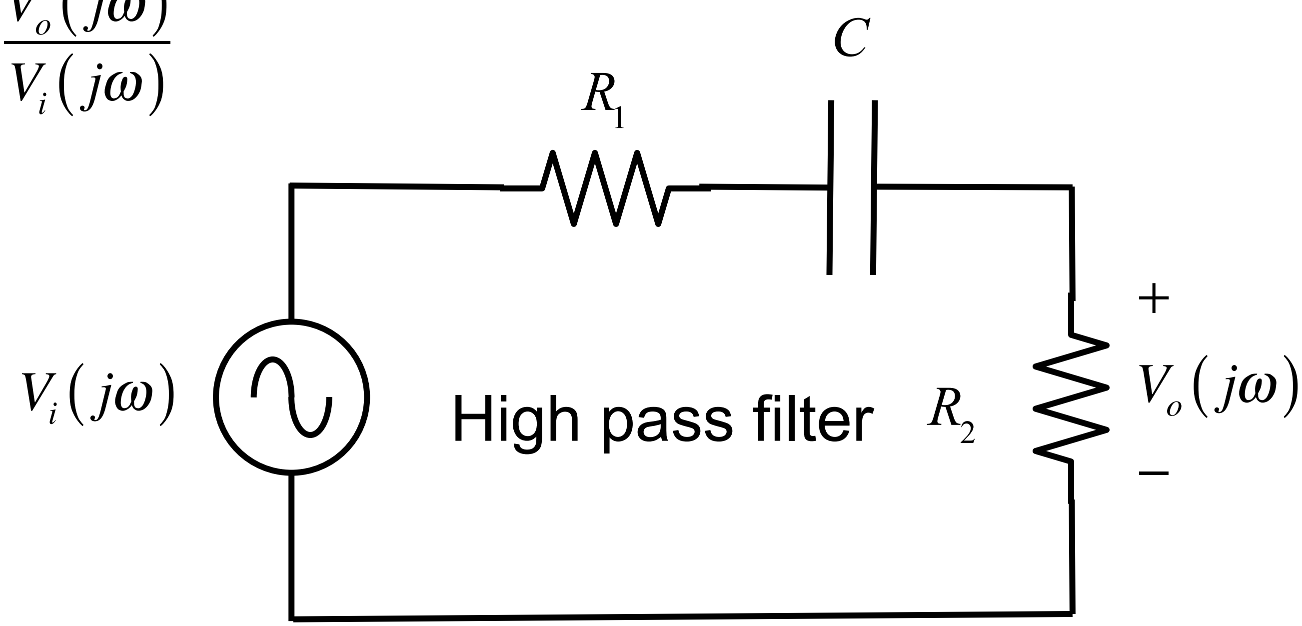
Single time constant (STC) circuit

$$T(j\omega) \equiv \frac{V_o(j\omega)}{V_i(j\omega)}$$



$$T(j\omega) = \frac{R_2}{R_1 + R_2 + 1/j\omega C}$$

$$T(j\omega) = \frac{R_2/(R_1+R_2)}{1 - j\frac{1}{\omega C(R_1+R_2)}}$$

$$\omega_L = \frac{1}{(R_1+R_2)C}$$

$$T(j\omega) = \frac{T_{mid}}{1 - j\omega_L/\omega}$$

(STC) circuit: Bode plot

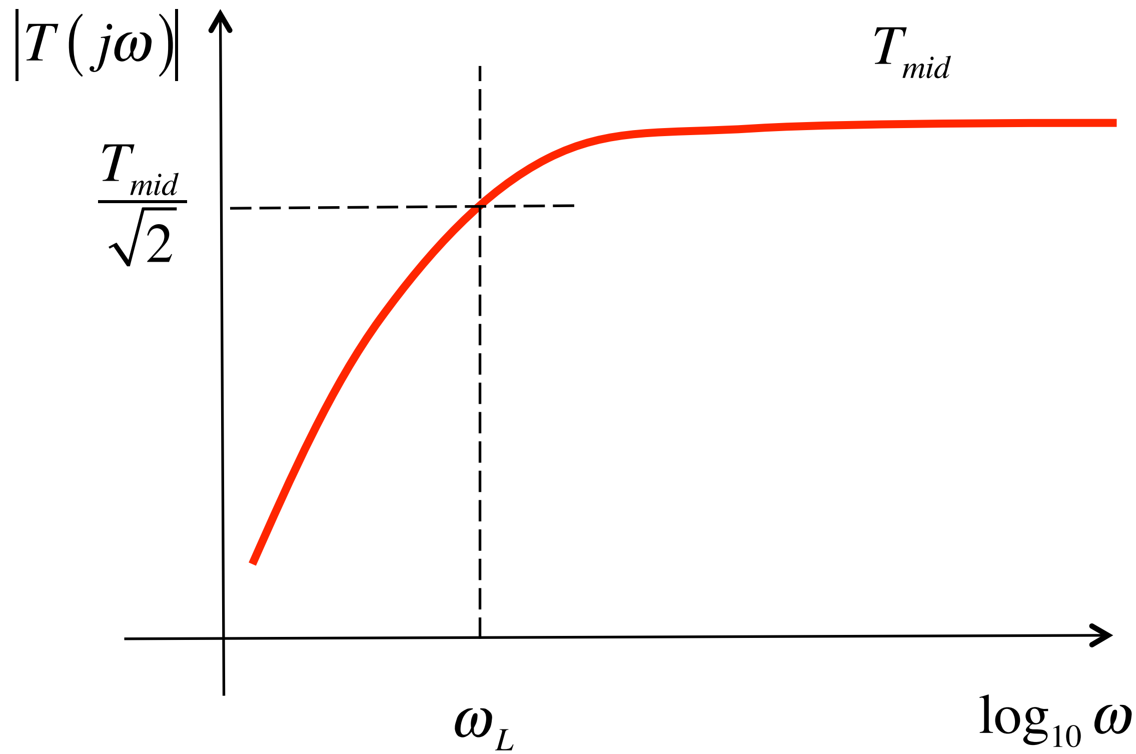
$$T(j\omega) = \frac{T_{mid}}{1 - j\omega_L/\omega}$$

$$|T(j\omega)| = \frac{T_{mid}}{\sqrt{1 + \omega_L^2/\omega^2}}$$

$$\omega_L = \frac{1}{(R_1 + R_2)C}$$

$$\omega \gg \omega_L \quad |T(j\omega)| = T_{mid}$$

$$\omega \ll \omega_L \quad |T(j\omega)| = T_{mid}\frac{\omega}{\omega_L}$$



20 dB /decade or 6 dB per octave

(STC) circuit: phase

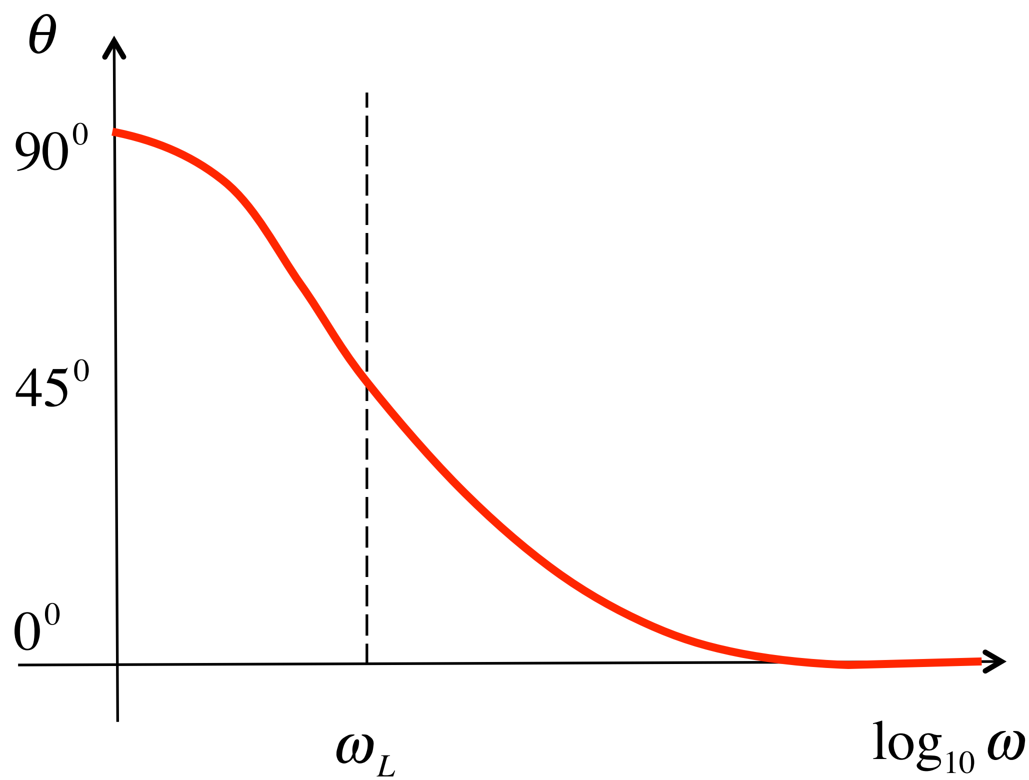
$$T(j\omega) = \frac{T_{mid}}{1 - j\omega_L/\omega}$$

$$T(j\omega) = \frac{T_{mid}}{1 + \omega_L^2/\omega^2}\left(1 + j\omega_L/\omega\right)$$

$\omega << \omega_L$: phase $= 90^0$

$\omega = \omega_L$: phase $= 45^0$

$\omega >> \omega_L$: phase $= 0^0$



"Poles and zeros"

$$T(j\omega) = \frac{T_{mid}}{1 + \omega_L / j\omega}$$

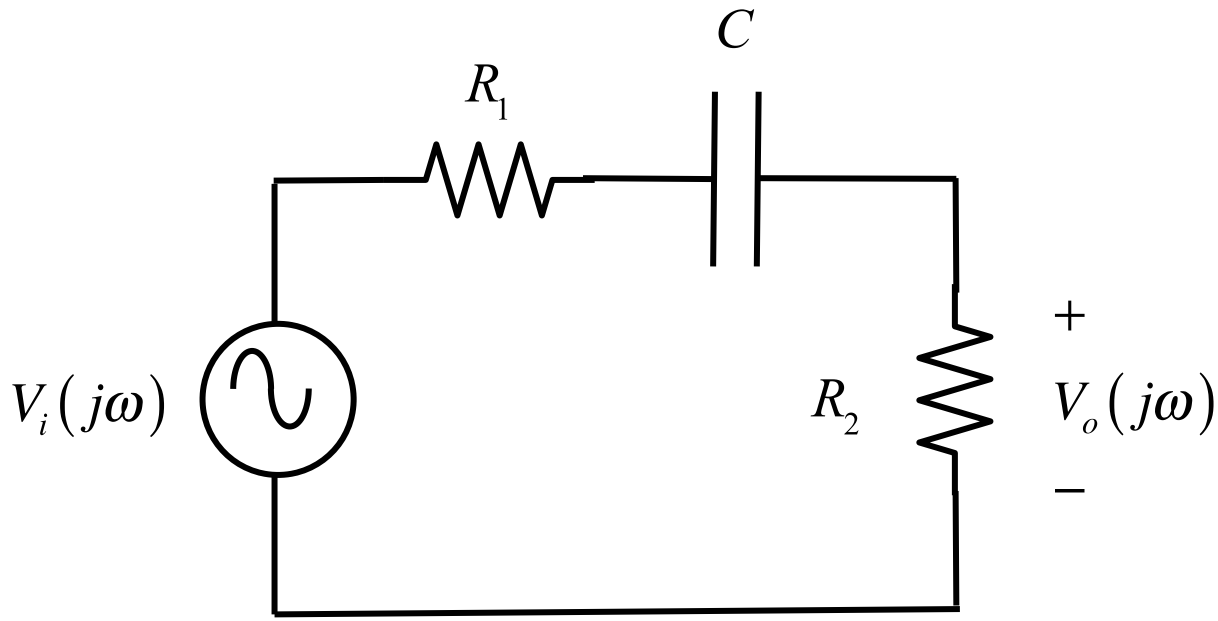
$$s = j\omega$$

$$T(s) = \frac{T_{mid}}{1 + \omega_L / s}$$

$$T(s) = T_{mid} \times \frac{s}{s + \omega_L}$$

STC has one pole

Short cut



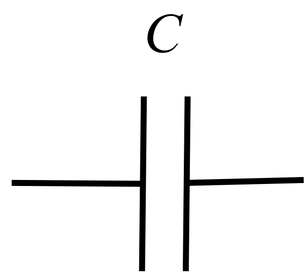
$$T(j\omega) = \frac{T_{mid}}{1 + \omega_L / j\omega}$$

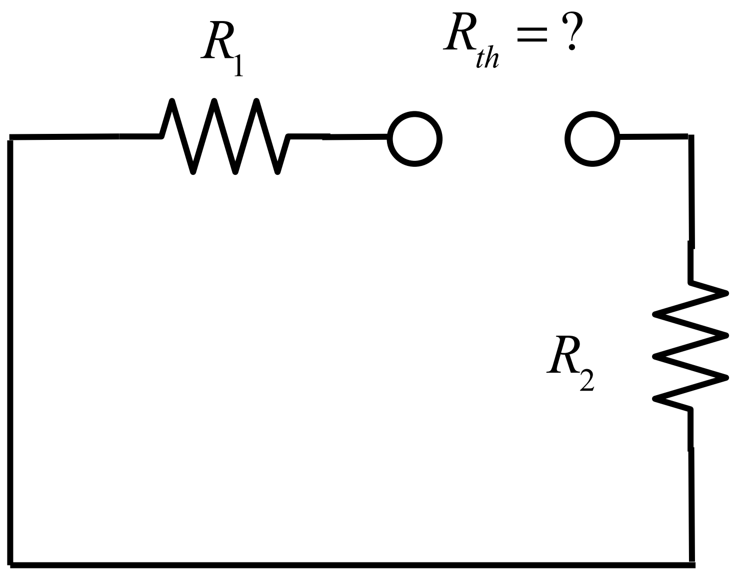
$$\omega_L = \frac{1}{(R_1 + R_2)C}$$

The corner frequency (the pole) is one over a time constant − just find the RC time constant.

Kill the voltage source and find the Thevinen equivalent resistance that the capacitor sees.

Short cut





$$R_{th} = R_1 + R_2$$

$$\omega_L = \frac{1}{R_{th}C} = \frac{1}{(R_1 + R_2)C}$$

Outline

1) Decibels
2) High pass filter / STC circuits
3) LF response of a **discrete** CS
4) Design Example
5) LF Response of CE
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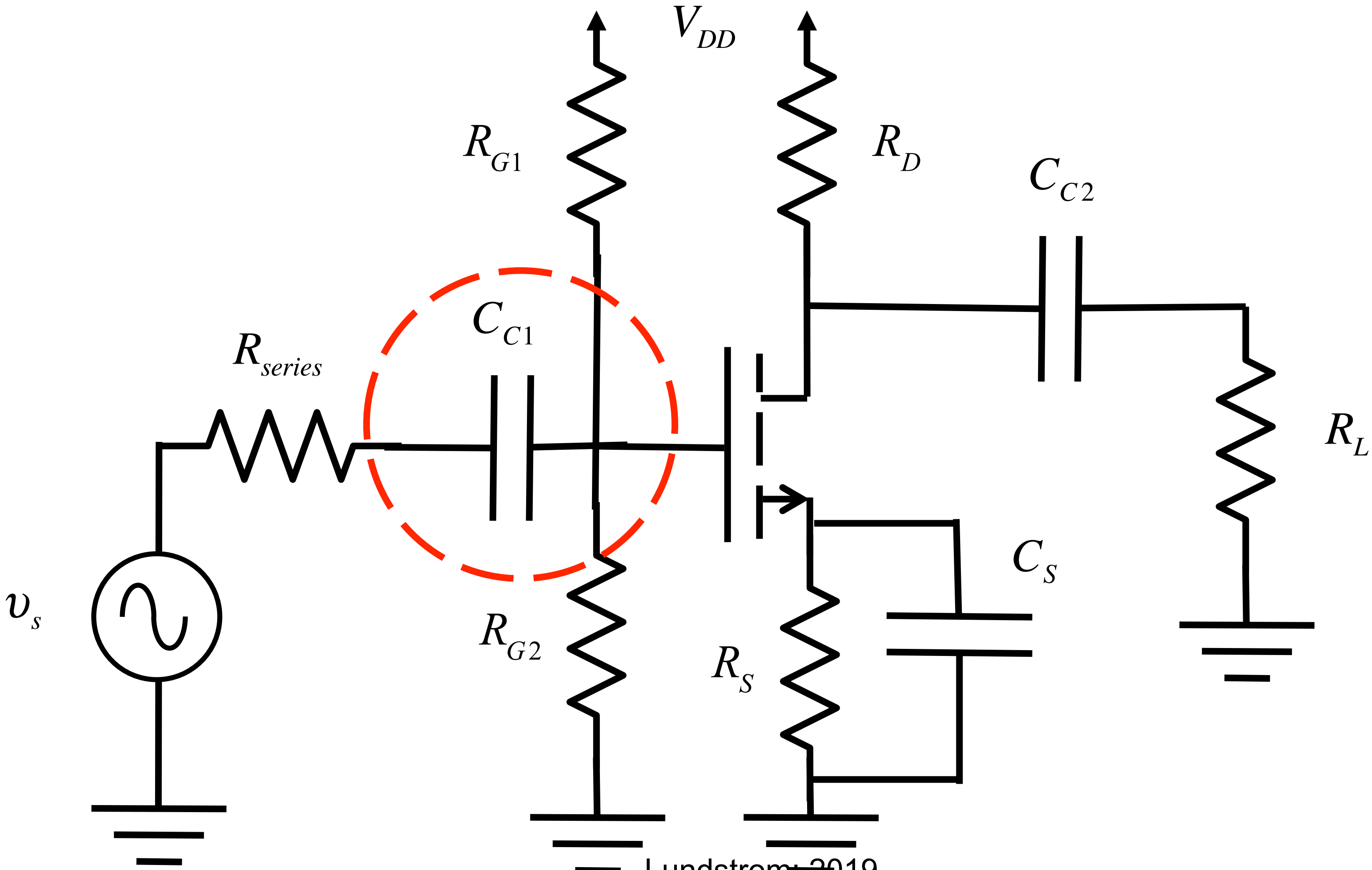
Now analyze this circuit



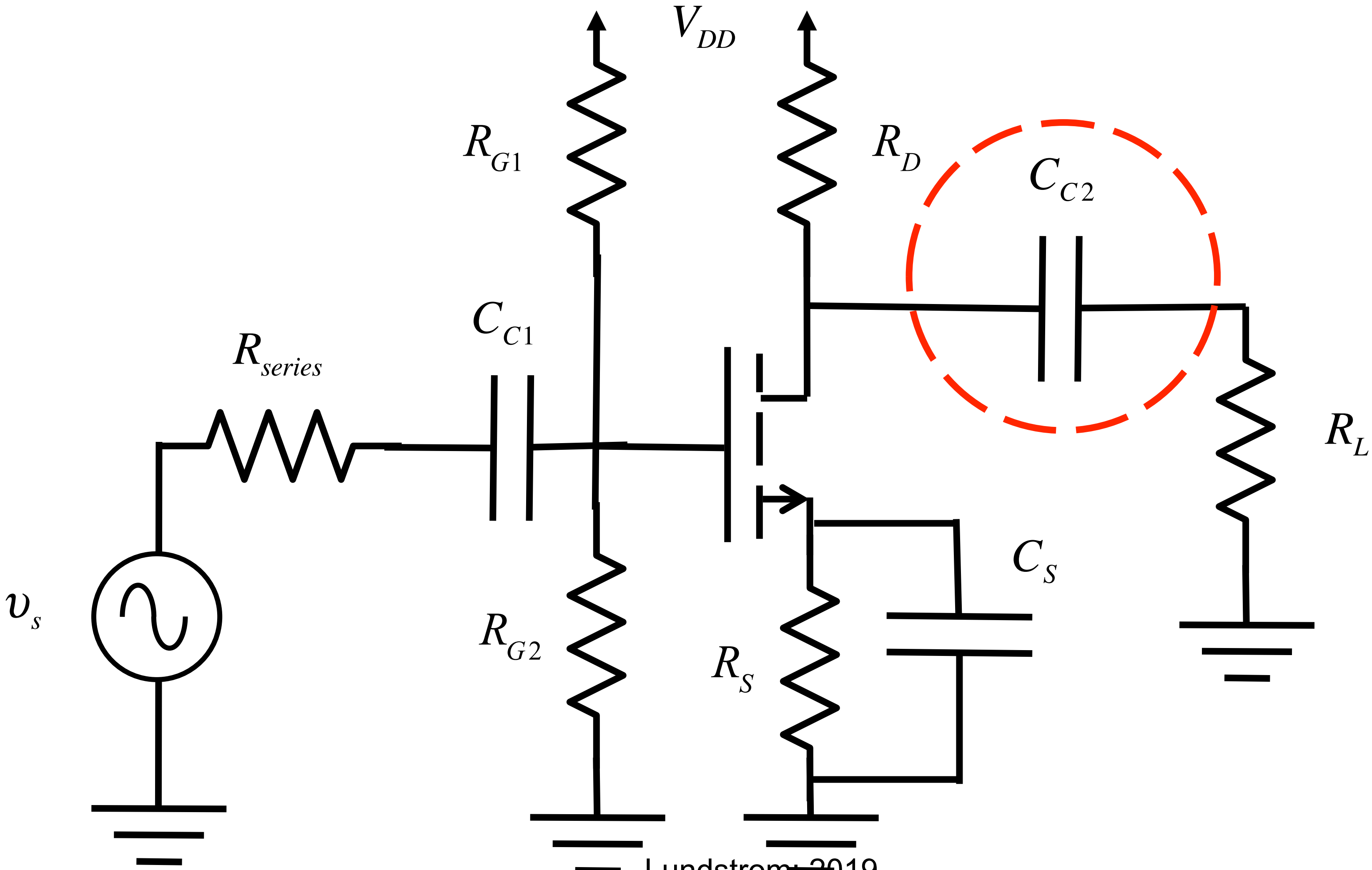
First corner frequency

$$\omega_{L1} = \frac{1}{C_{C1}\left(R_{series} + R_{G1} \parallel R_{G2}\right)}$$

Expected be small (which is good)

Important: While we compute this corner frequency, we assume that the other C's are shorted − this produces a STC circuit.

Second coupling capacitor

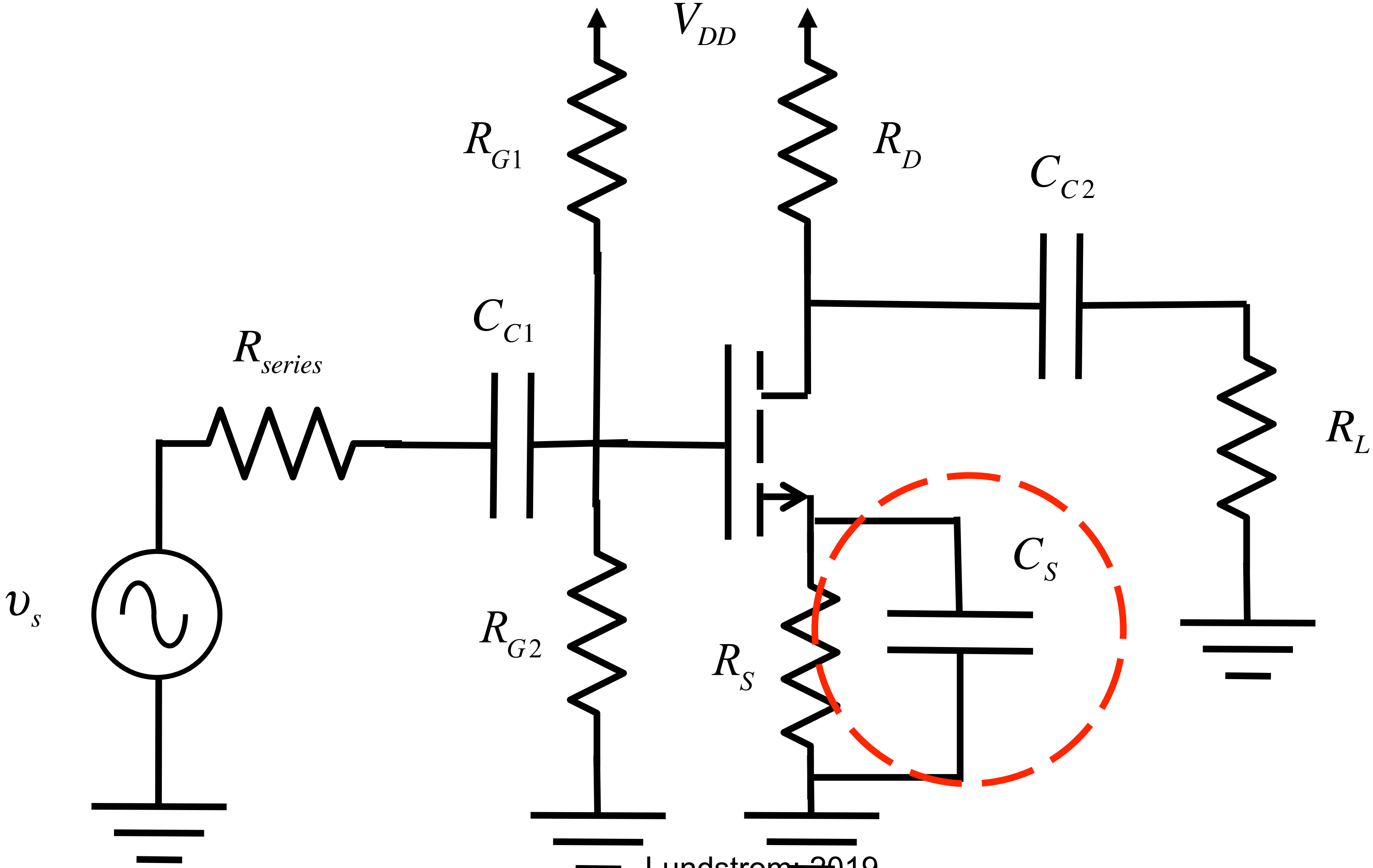


Second corner frequency

$$\omega_{L2} = \frac{1}{C_{C2}\left(R_D + R_L\right)}$$

Expected be reasonably small (which is also good)

Third coupling capacitor

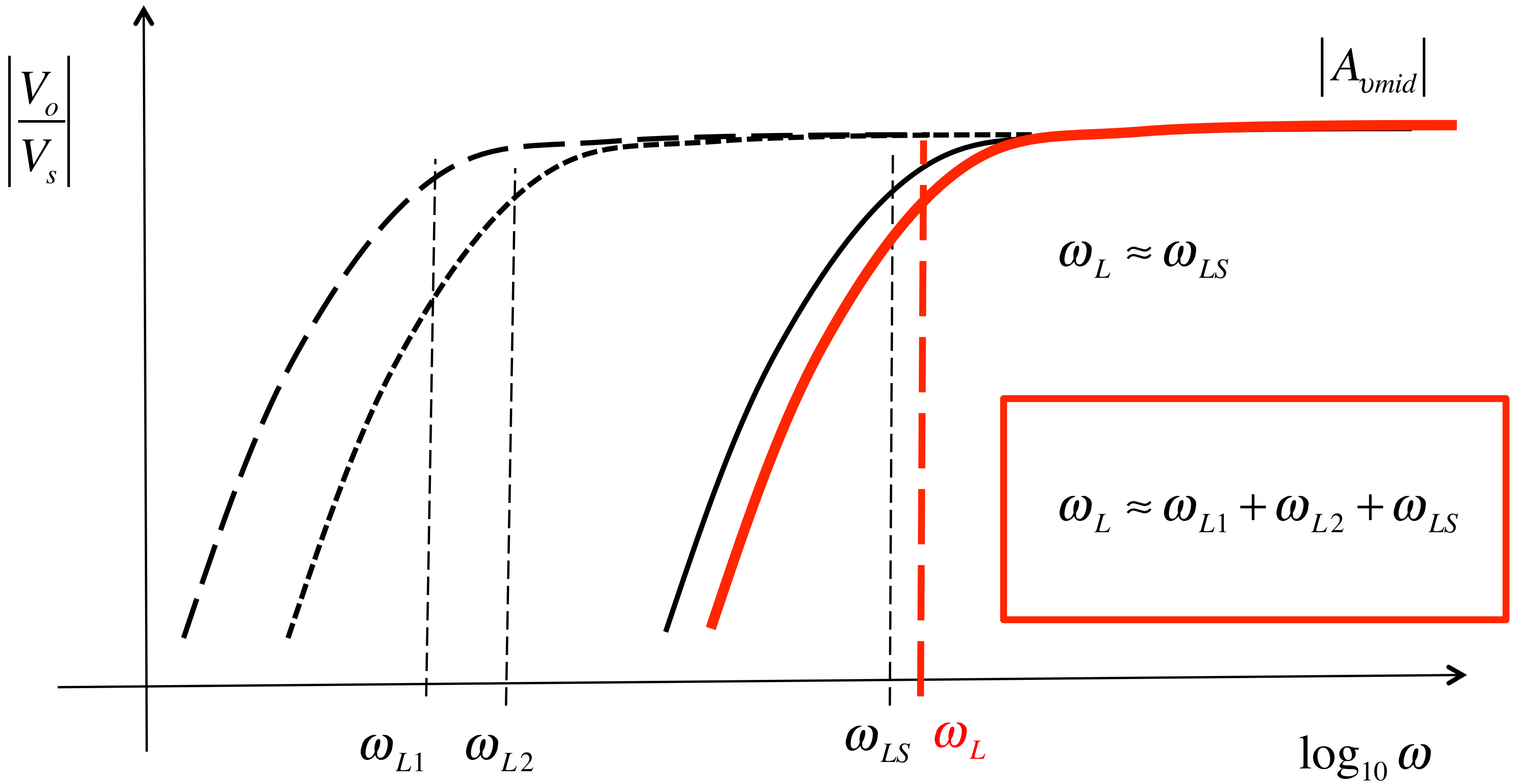


Third corner frequency

$$\omega_{LS} = \frac{1}{C_S\left(R_S \parallel \left(1/g_m\right)\right)}$$

Expected be the **largest** of the three.

Result



$\left|\frac{V_o}{V_s}\right|$
$\left|A_{vmid}\right|$
$\omega_L \approx \omega_{LS}$
$\omega_L \approx \omega_{L1} + \omega_{L2} + \omega_{LS}$
ω_{L1}
ω_{L2}
ω_{LS}
ω_L
$\log_{10}\omega$

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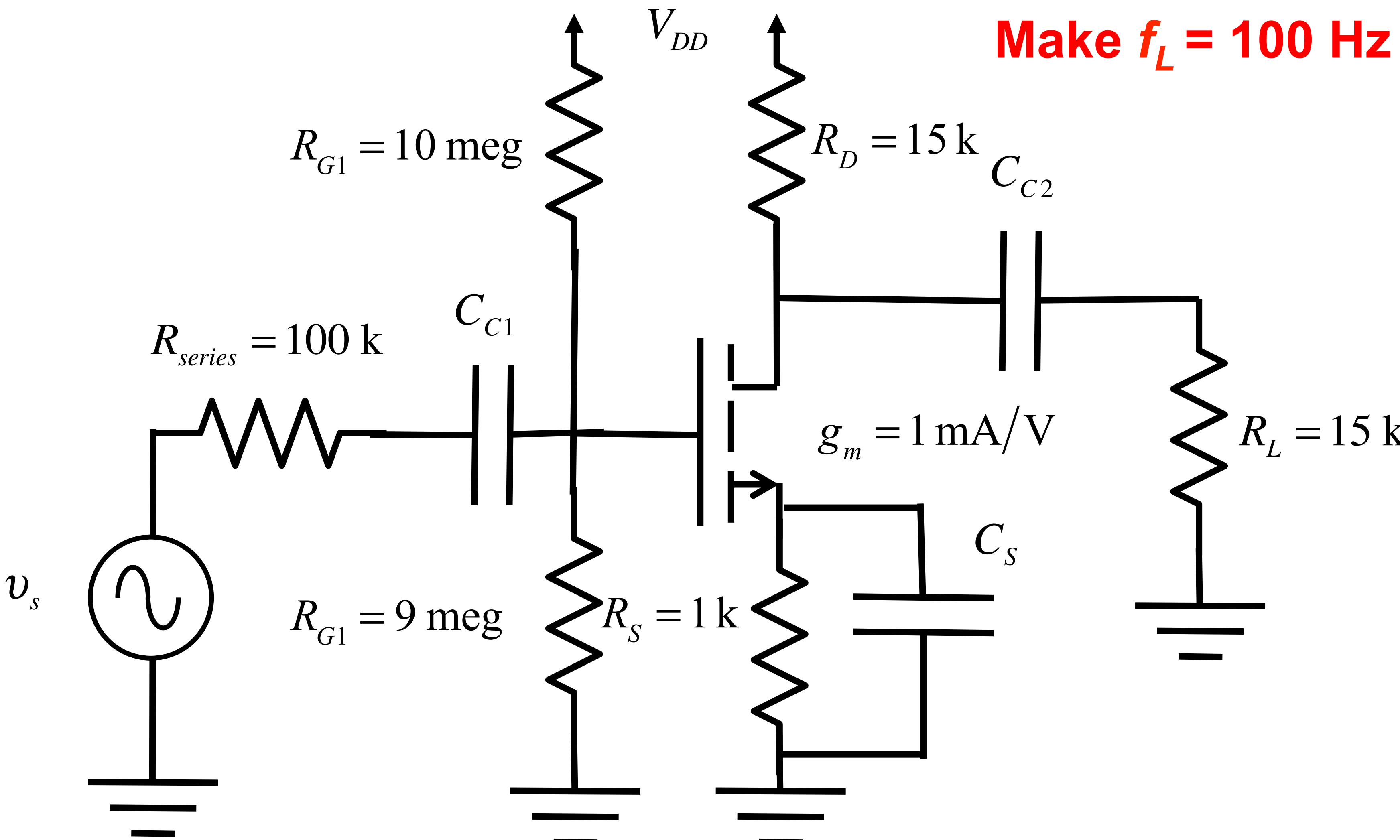
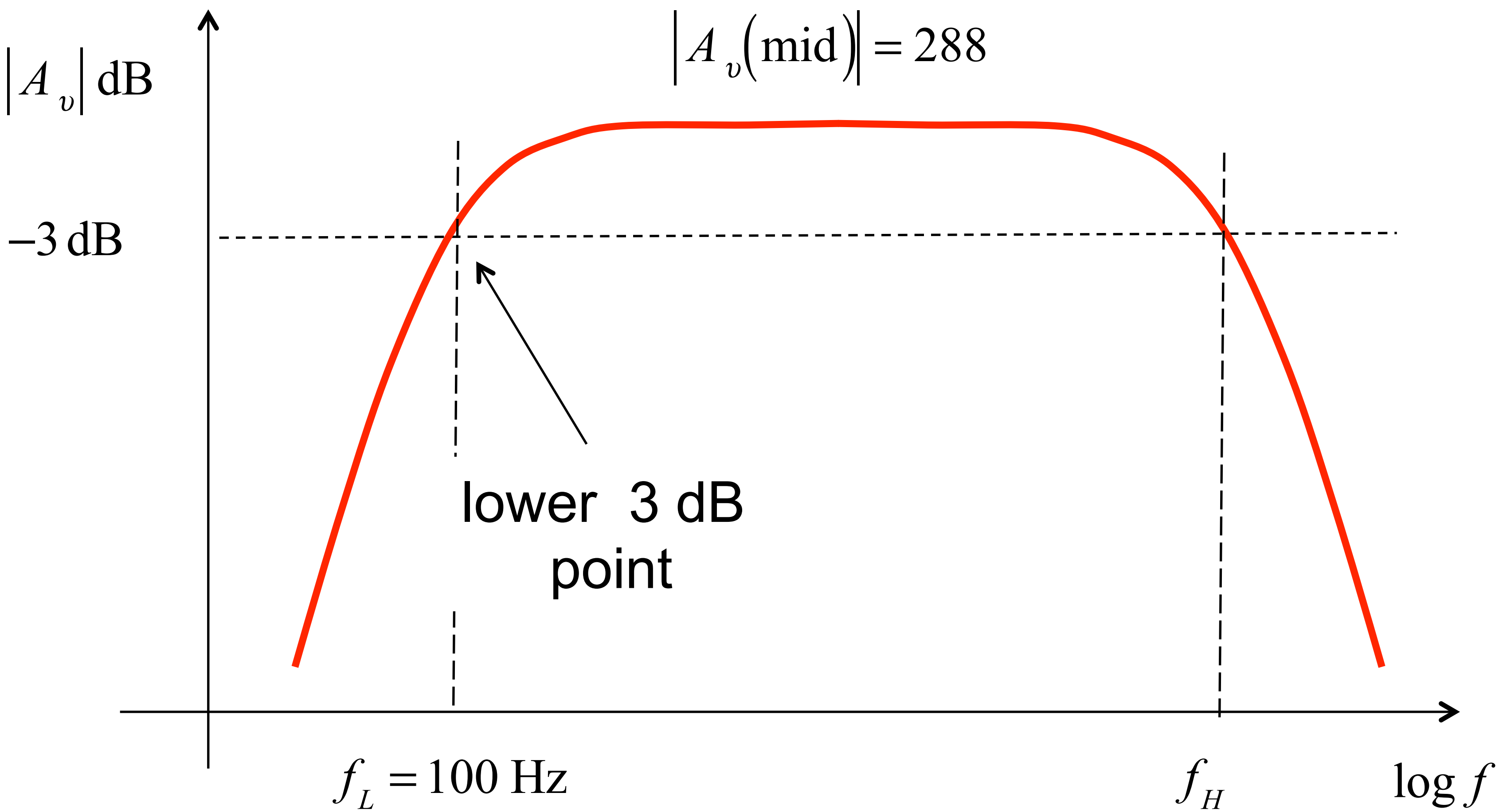
Select the capacitors to:

Make f_L = 100 Hz

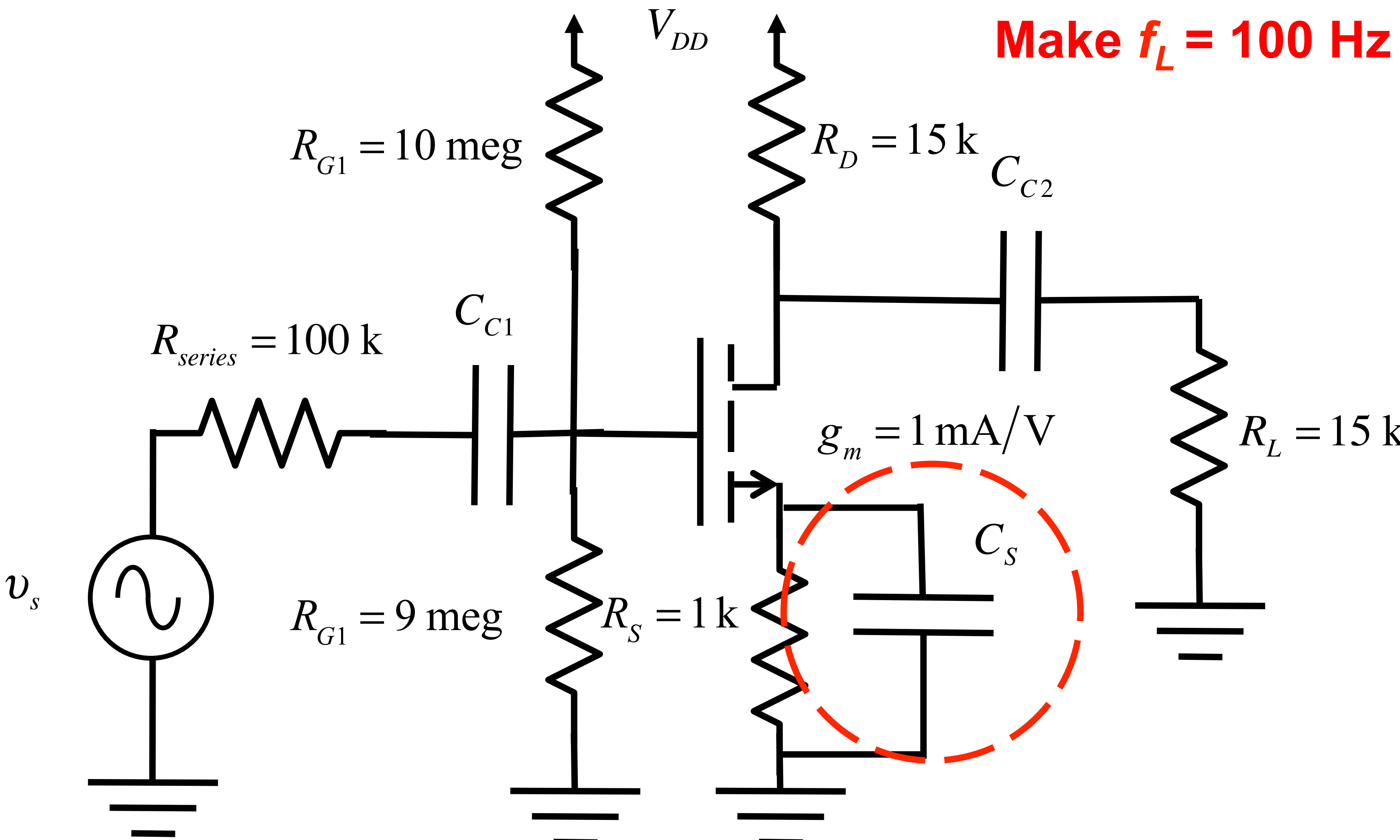


Bode plot



$\left|A_v(\text{mid})\right| = 288$
$\left|A_v\right|$ dB
-3 dB
lower 3 dB
point
$f_L = 100$ Hz
f_H
$\log f$

Select the first capacitor



Source capacitance

C_S is expected be the largest of the three. It require the largest C to make the Corner Frequency spec.

$$\omega_{LS} = \frac{1}{C_S\left(R_S \parallel \left(1/g_m\right)\right)} = 2\pi \times 100$$

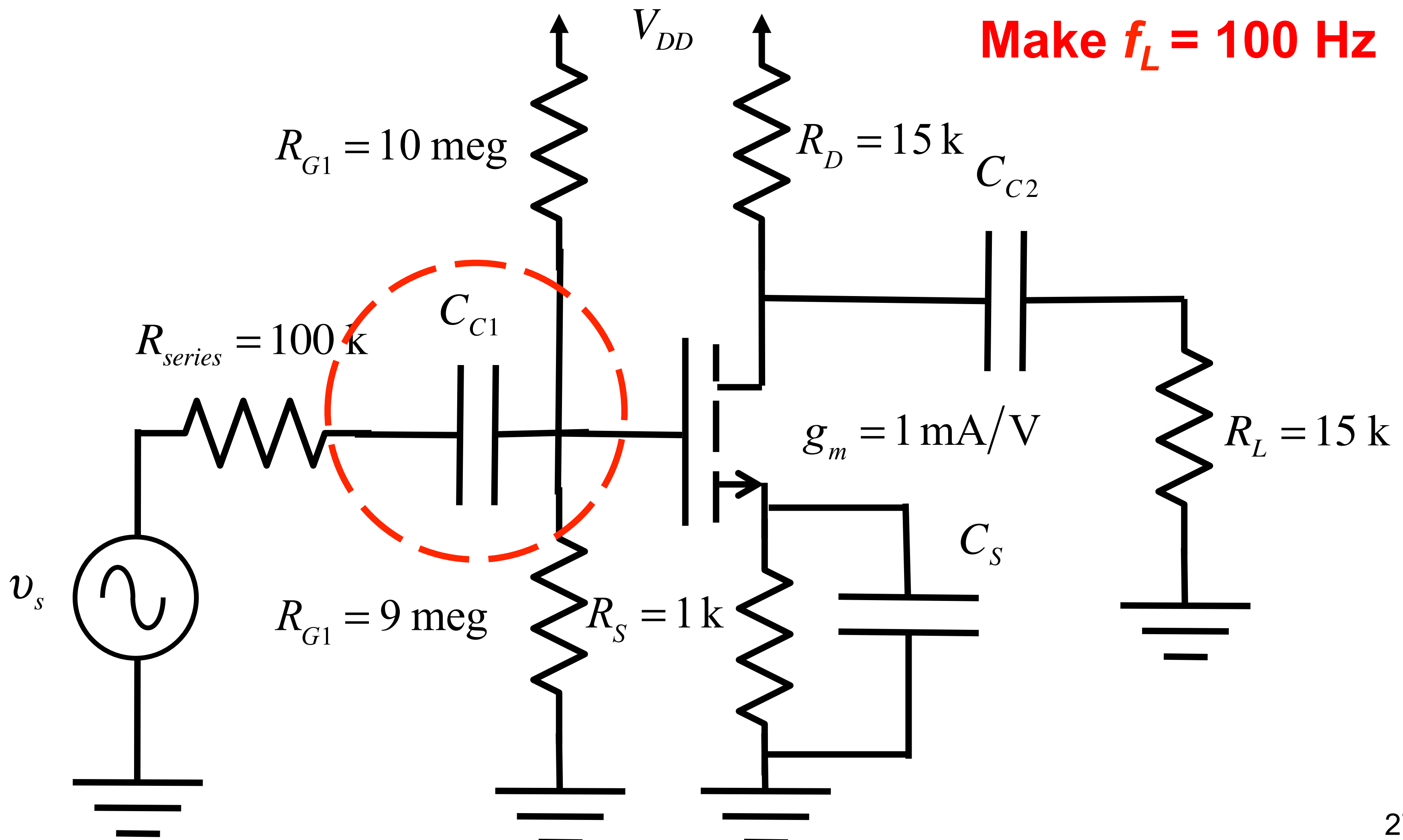
$$1/g_m = 26\ \Omega$$

$$\frac{1}{C_S} = 628 \times 26$$

$$C_S = 62 \times 10^{-6}\ \mathrm{F}$$

Now make the other two corner frequencies 5-10 X smaller.

Select the second capacitors



The second capacitor

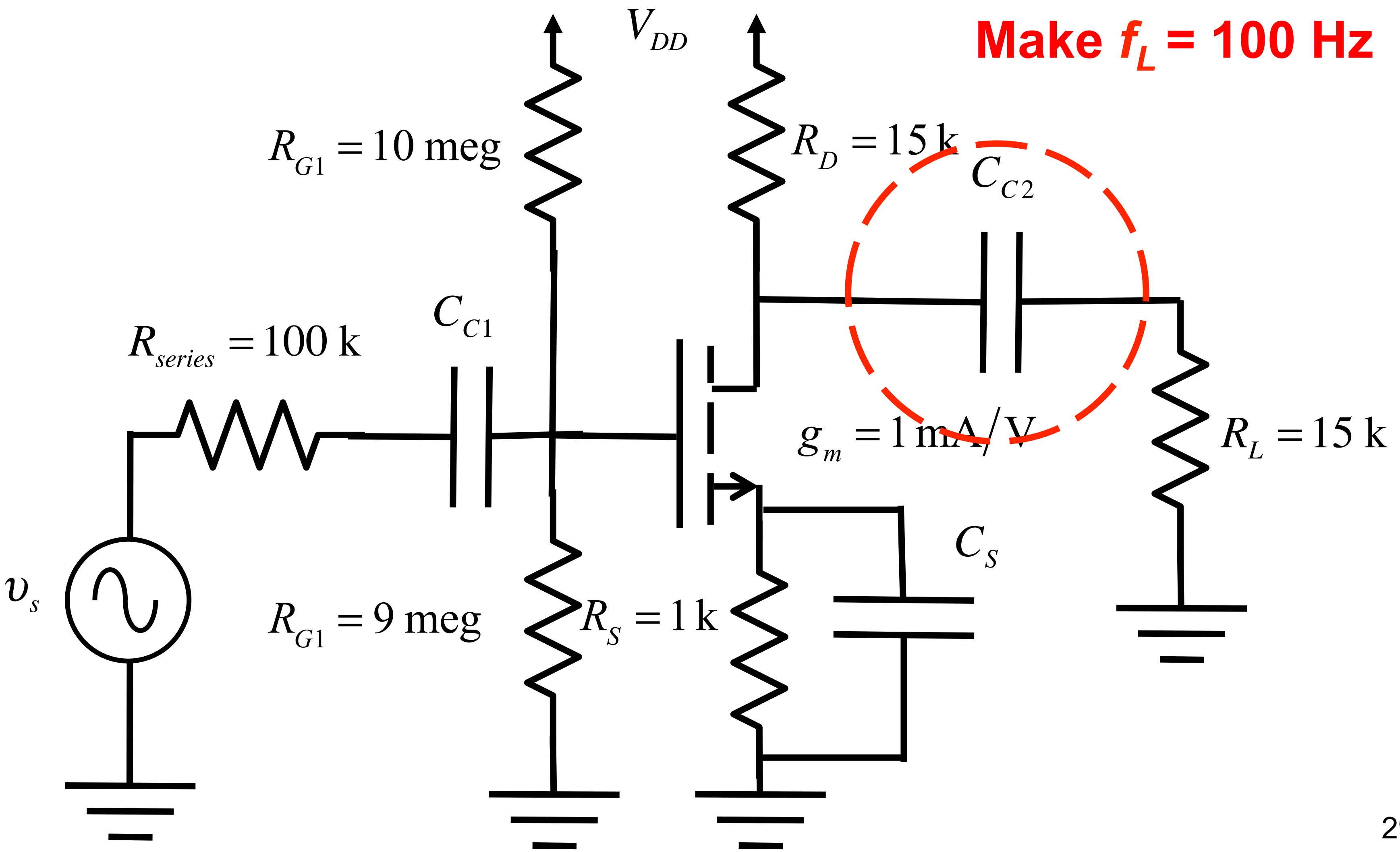
$$\omega_{L1} = \frac{1}{C_{C1}\left(R_{series} + R_{G1} \parallel R_{G2}\right)} = 2\pi \times 10$$

10 X smaller

$$C_{C1} = \left[62.8 \times \left(100 \times 10^{3} + 4.7 \times 10^{6}\right)\right]^{-1}$$

$$C_{C1} = 3.3 \times 10^{-9} \text{ F}$$

Select the third capacitor



The third capacitor

$$\omega_{L2} = \frac{1}{C_{C2}\left(R_D + R_D\right)} = 2\pi \times 10$$

10 X smaller

$$C_{C2} = \left(62.8 \times 30 \times 10^3\right)^{-1}$$

$$C_{C2} = 0.5 \times 10^{-6}\ \text{F}$$

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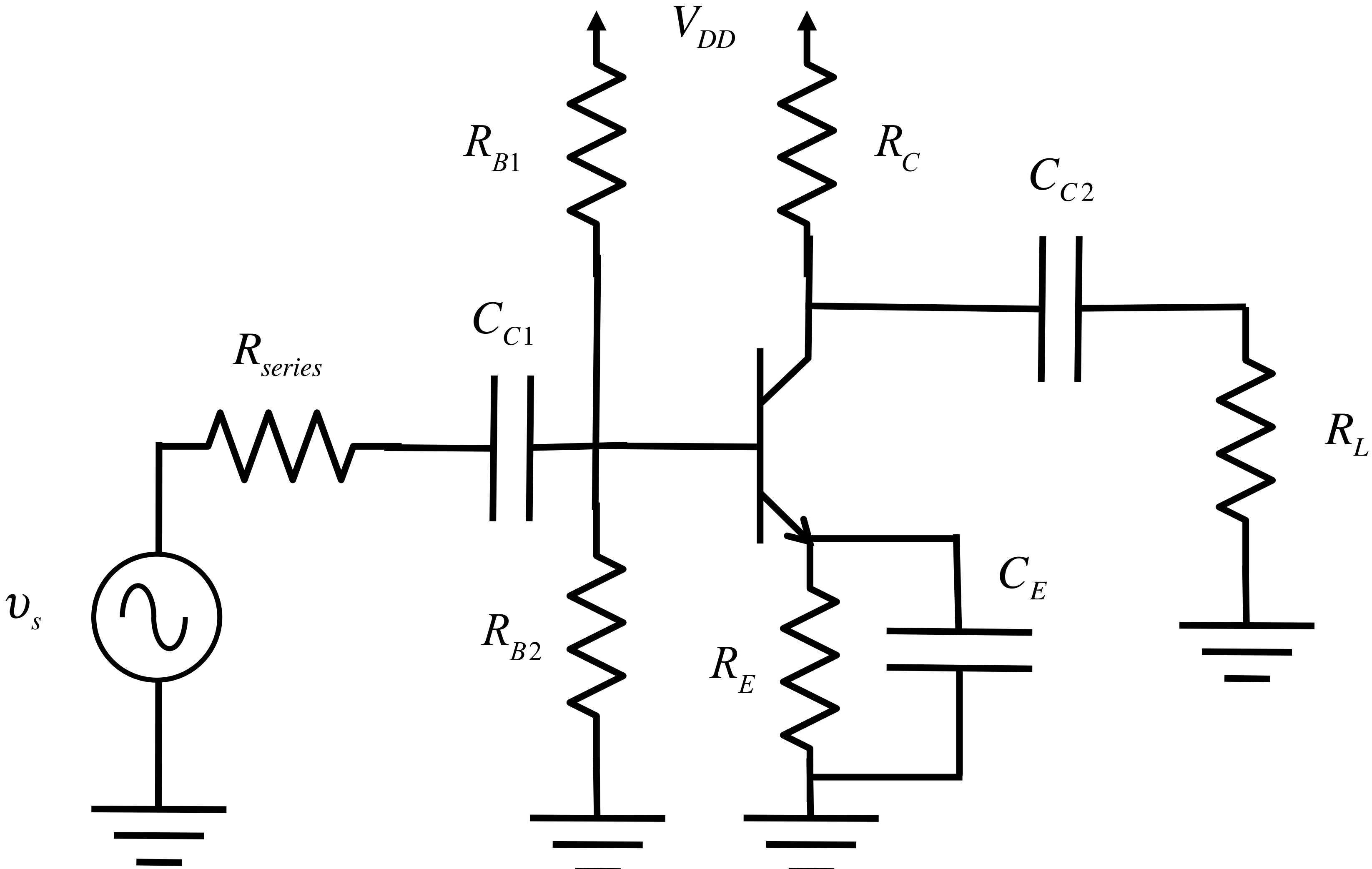
Now analyze this circuit



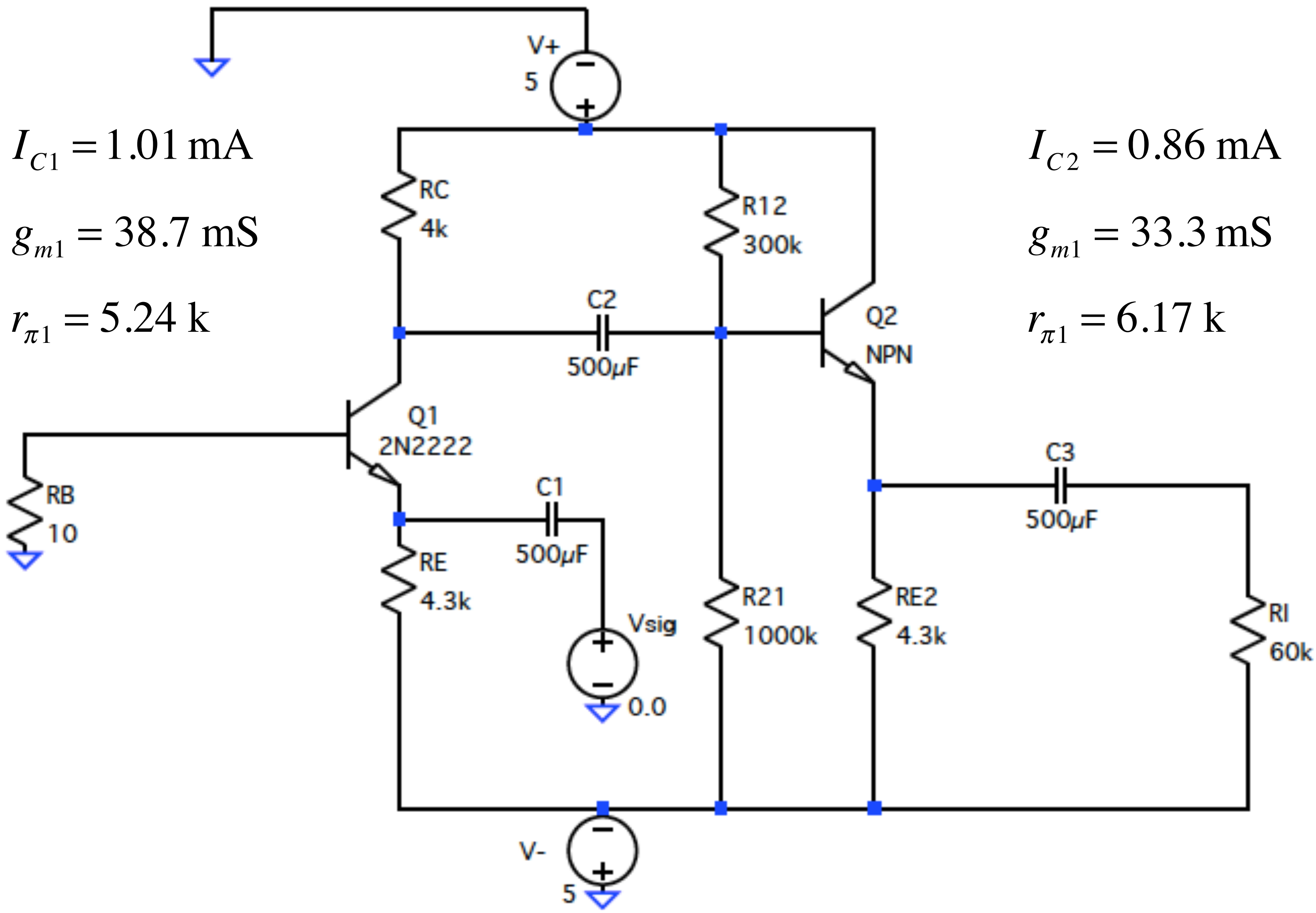
Answers can be written down by inspection

CB: CC Example

$I_{C1} = 1.01\ \text{mA}$

$g_{m1} = 38.7\ \text{mS}$

$r_{\pi 1} = 5.24\ \text{k}$

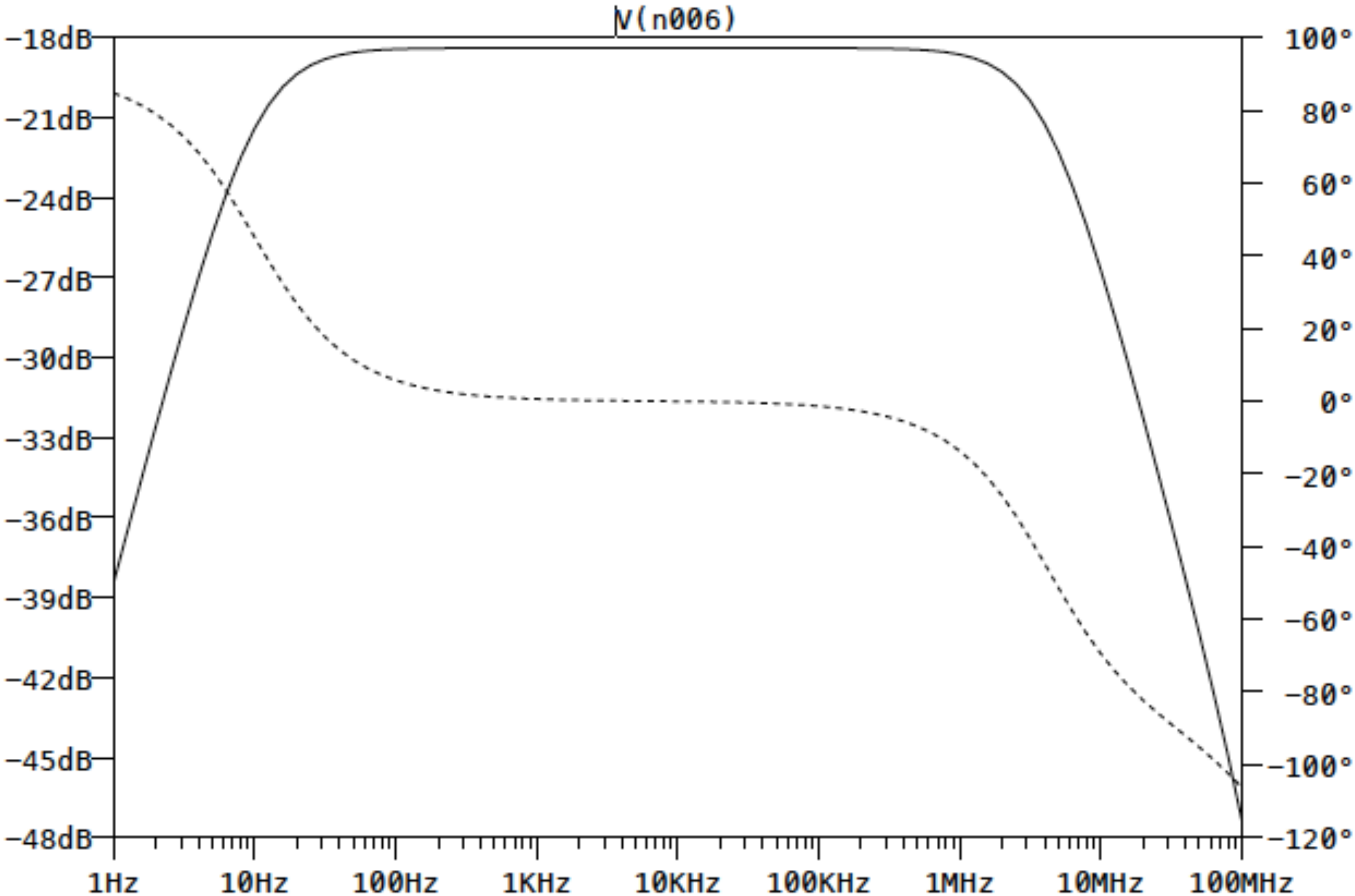


$I_{C2} = 0.86\ \text{mA}$

$g_{m1} = 33.3\ \text{mS}$

$r_{\pi 1} = 6.17\ \text{k}$

Example



V(n006)
-18dB
-21dB
-24dB
-27dB
-30dB
-33dB
-36dB
-39dB
-42dB
-45dB
-48dB
100°
80°
60°
40°
20°
0°
-20°
-40°
-60°
-80°
-100°
-120°
1Hz
10Hz
100Hz
1KHz
10KHz
100KHz
1MHz
10MHz
100MHz

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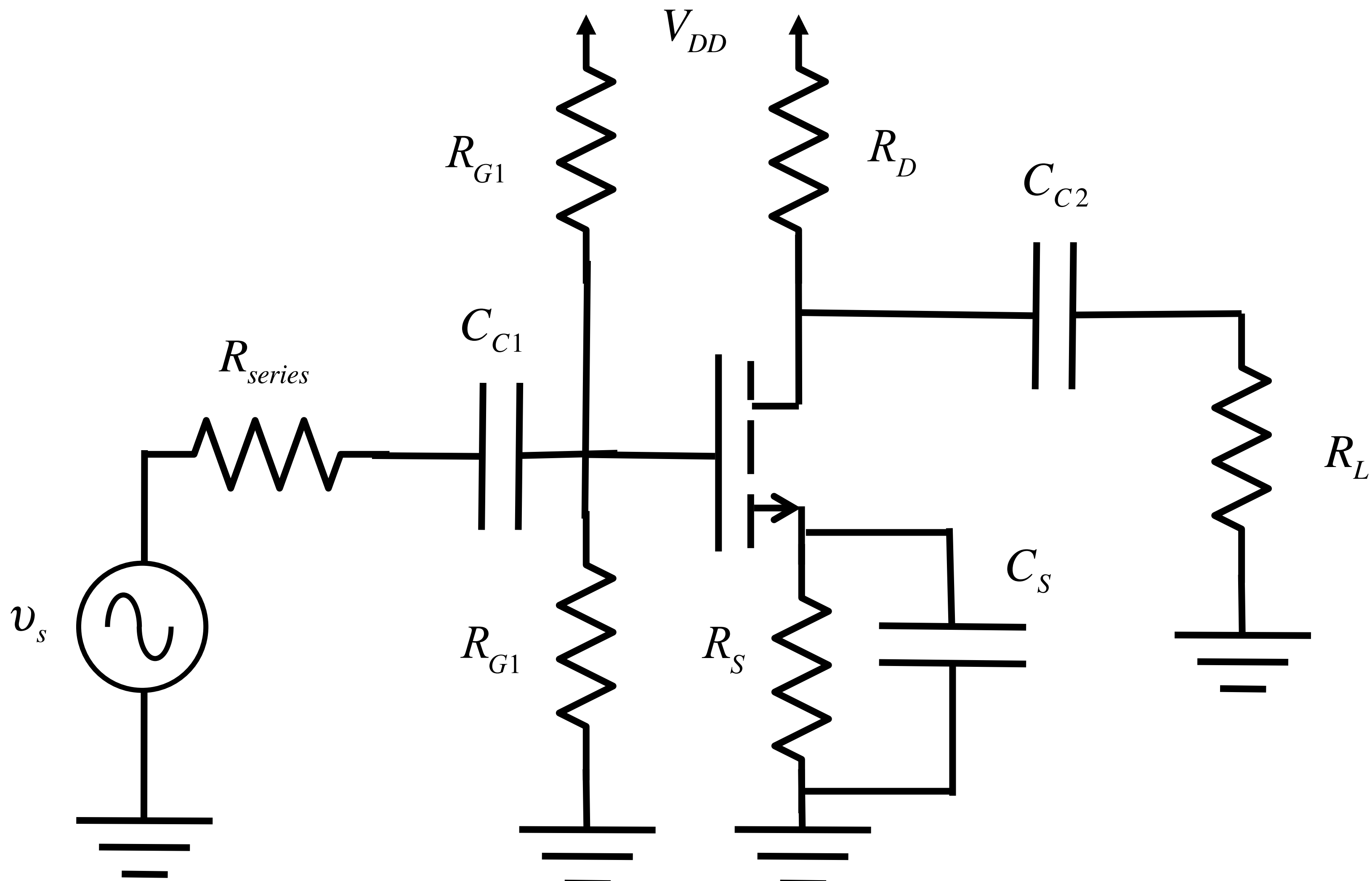
Our CS circuit



Our (simplified) approach

1) Short two of the capacitors (they are supposed to have essentially zero impedance at the frequencies of interest).

2) Evaluate the STC circuit:

$$\omega_{L1} = 1/(C_1 R_{th1})$$

3) Repeat for the other two capacitors.

4) Estimate the frequency response the amplifier as:

$$|A_v(\omega)| = \frac{A_v(\text{mid})}{\sqrt{1+(\omega_L/\omega)^2}} \qquad \omega_L = \omega_{L1} + \omega_{L2} + \omega_{L3}$$

Our (simplified) approach (ii)

This approach assume that the overall transfer function is the product of the three STC circuits:

$$A_v(\omega) = A_v(\text{mid})\left(\frac{1}{1+\omega_{L1}/j\omega}\right)\left(\frac{1}{1+\omega_{L2}/j\omega}\right)\left(\frac{1}{1+\omega_{L3}/j\omega}\right)$$

$$T(\omega) = T_1(\omega)\times T_2(\omega)\times T_3(\omega)$$

There is no guarantee that this will always work, but the procedure often gives us a good estimate for the corner frequency.

How would we solve the problem correctly?

Exact low frequency response of CS amplifier

$$A_{v_S}(\omega) = A_{v_S}(\text{mid}) \frac{1 + \dfrac{\omega_Z}{j\omega}}{\left(1 + \dfrac{\omega_{C1}}{j\omega}\right)\left(1 + \dfrac{\omega_S}{j\omega}\right)\left(1 + \dfrac{\omega_{C2}}{j\omega}\right)}$$

(See Appendix)

Almost the product of 3 STC responses, but there is an additional zero.

$$\omega_Z = \frac{1}{R_S C_S} << \omega_S = \frac{1}{\left(R_S \parallel 1/g_m\right) C_S}$$

So the zero is not important - it does not affect our conclusions about the LF corner frequency.

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Short-circuit time constant method

1) Short the other capacitors (they are supposed to have essentially zero impedance at the frequencies of interest).

2) Evaluate the STC circuit:

$$\omega_{L1} = 1/\left(C_1 R_{th1}\right)$$

3) Repeat for the other capacitors.

4) Select the largest corner frequency and estimate the frequency response the amplifier as:

$$\left|A_v(\omega)\right| = \frac{A_v(\text{mid})}{\sqrt{1+\left(\omega_L/\omega\right)^2}}$$

$$\omega_L = \omega_{L1} + \omega_{L2} + \omega_{L3}$$

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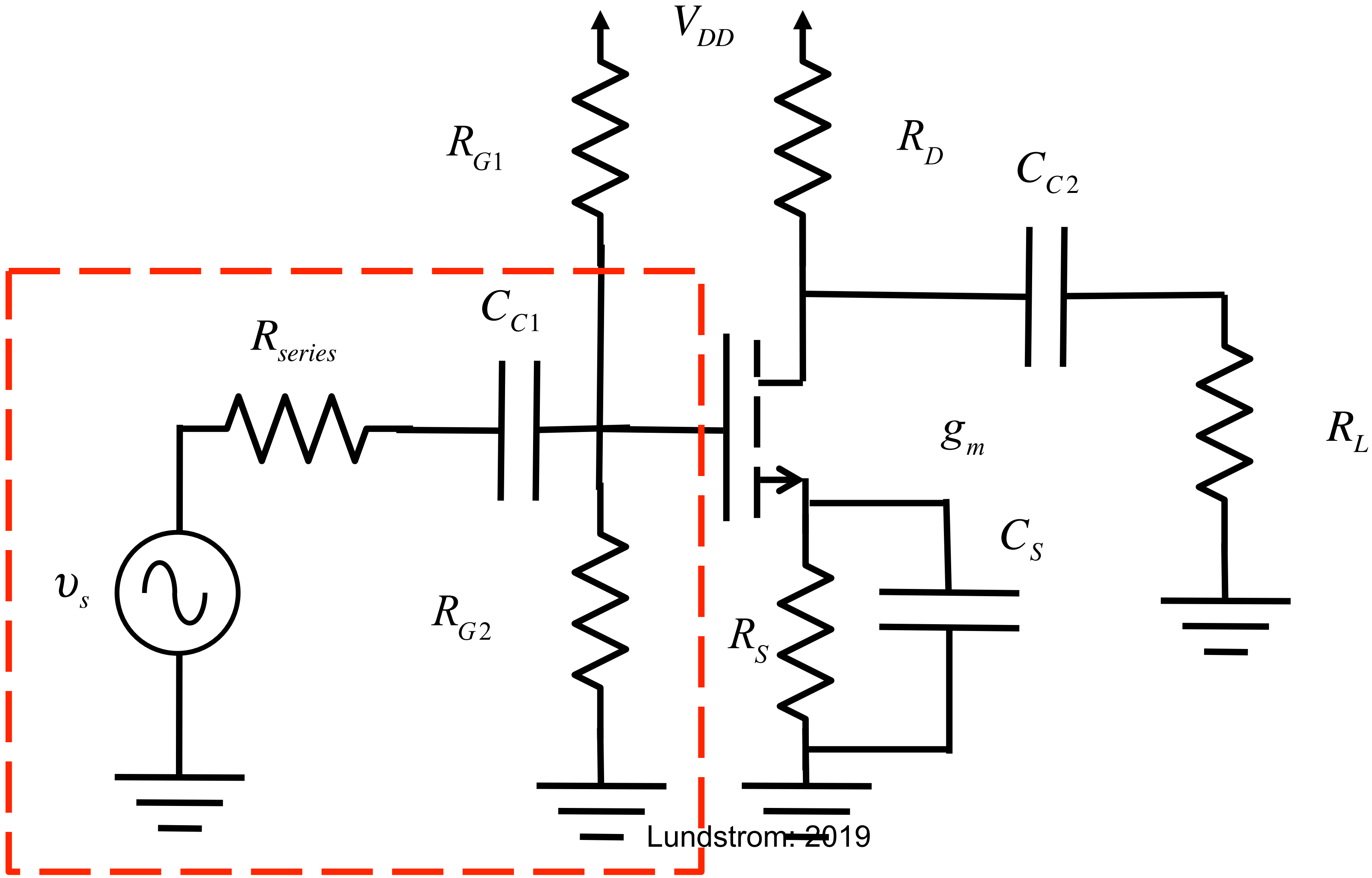
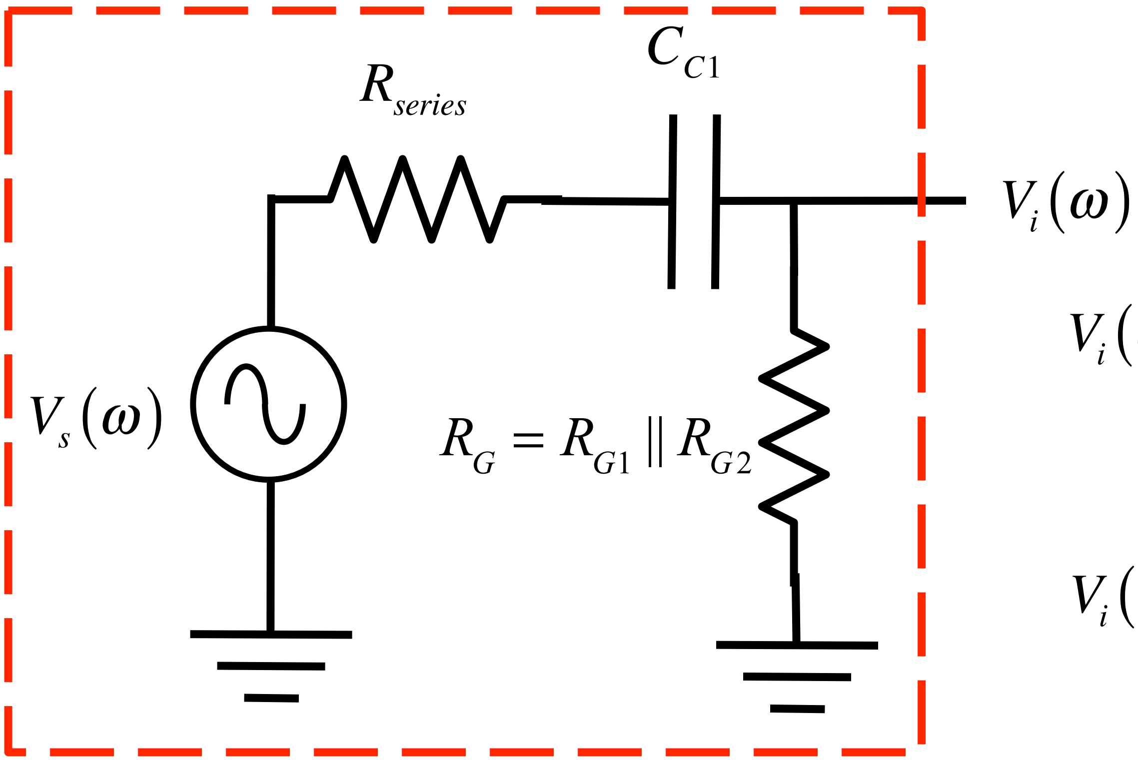
Exact low frequency response of CS amplifier



Exact low frequency response of CS amplifier

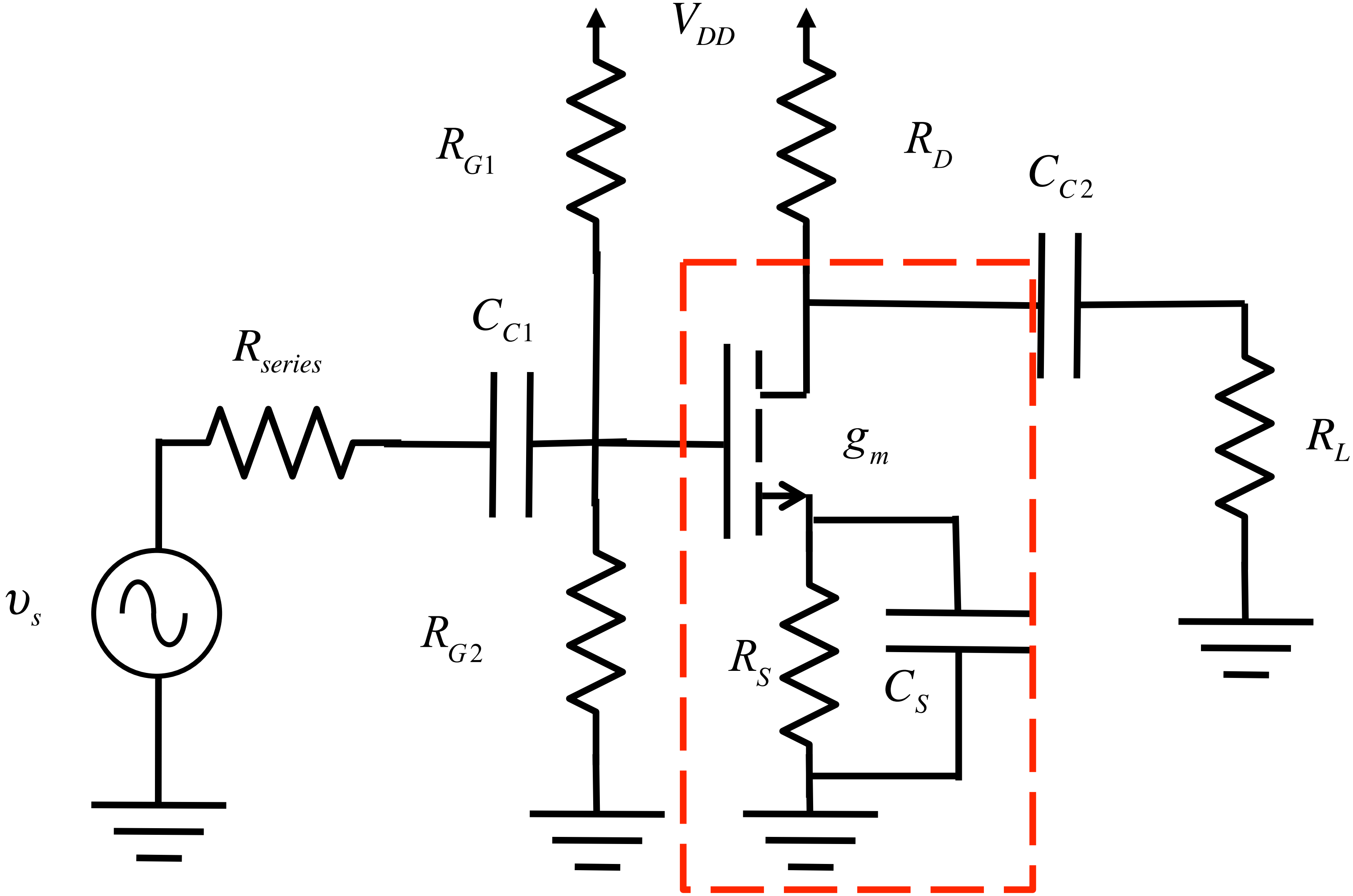


$$V_i(\omega) = \frac{R_G}{R_G + R_{series} + \dfrac{1}{j\omega C_{C1}}} V_s(\omega)$$

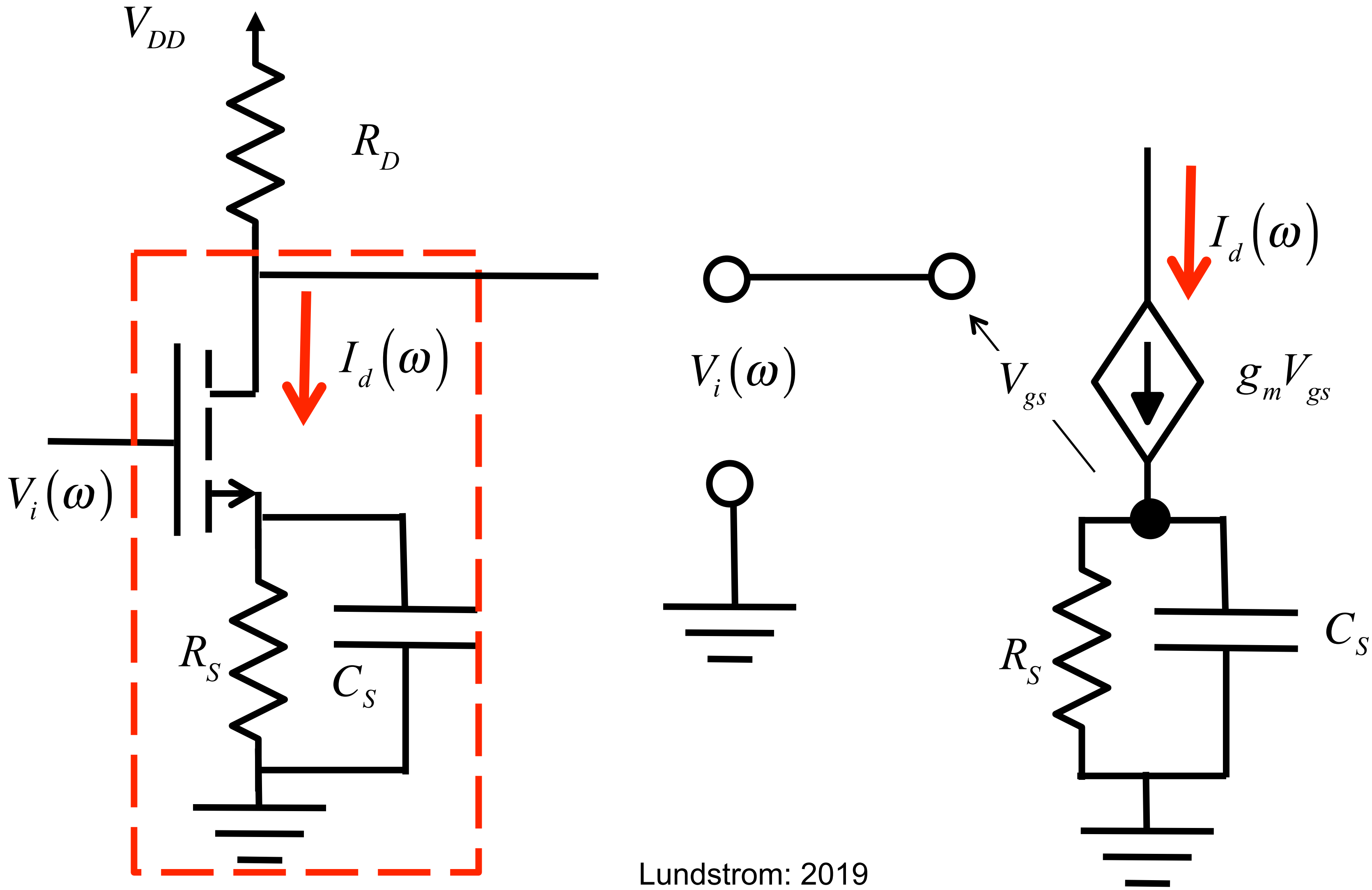
$$V_i(\omega) = \left(\frac{R_G}{R_G + R_{series}} \right) \frac{1}{1 + \dfrac{\omega_{C1}}{j\omega}} V_s(\omega)$$

$$\omega_{C1} = \frac{1}{\left(R_G + R_{series}\right) C_{C1}}$$

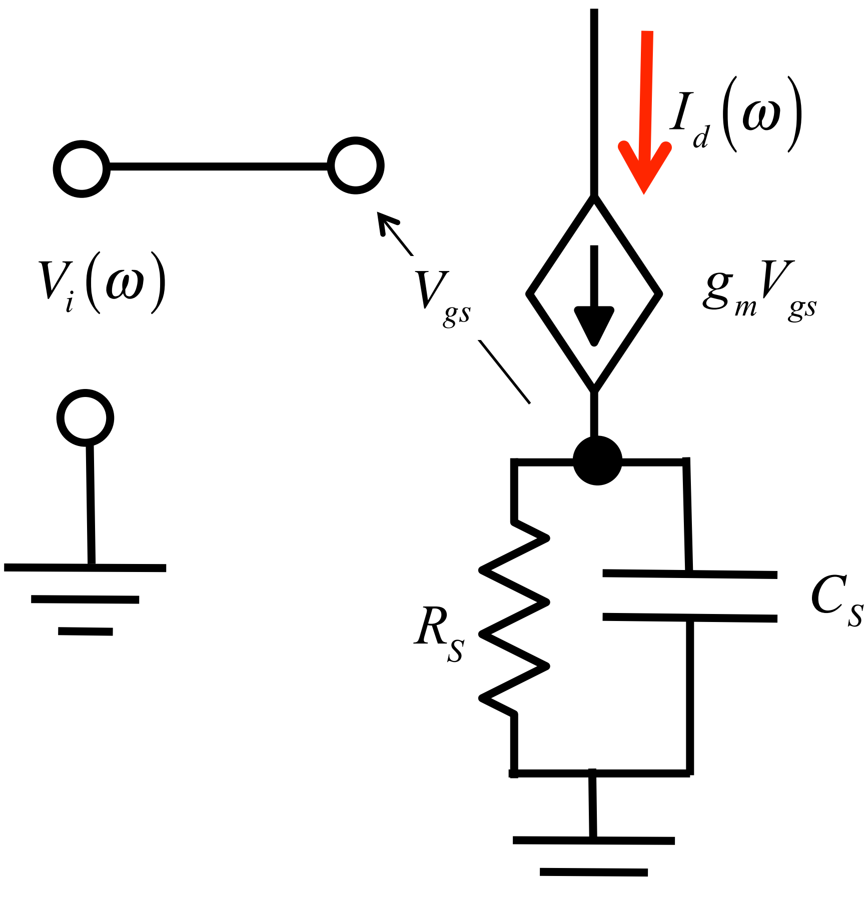
Exact low frequency response of CS amplifier



Exact low frequency response of CS amplifier



Exact low frequency response of CS amplifier



$$I_d = g_m V_{gs}$$

$$V_{gs} = V_i - I_d Z_S$$

$$I_d = \frac{g_m V_i}{1 + g_m Z_S}$$

$$I_d = g_m V_i \frac{Y_S}{Y_S + g_m}$$

$$Y_S = \frac{1}{R_S} + j\omega C_S$$

Exact low frequency response of CS amplifier

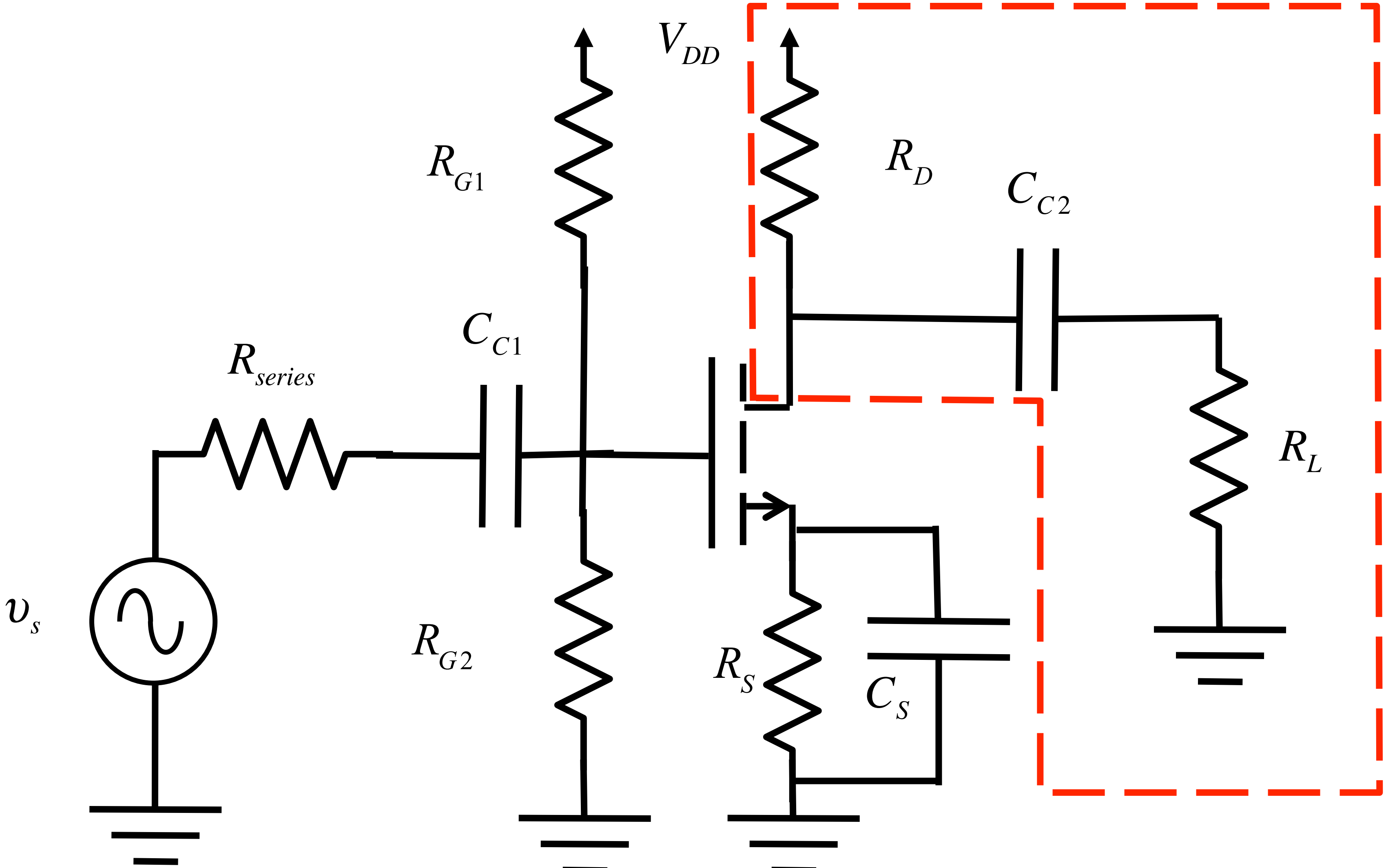
$$I_d = g_m V_i \frac{Y_S}{Y_S + g_m} \qquad Y_S = \frac{1}{R_S} + j\omega C_S$$

After some work, we find:

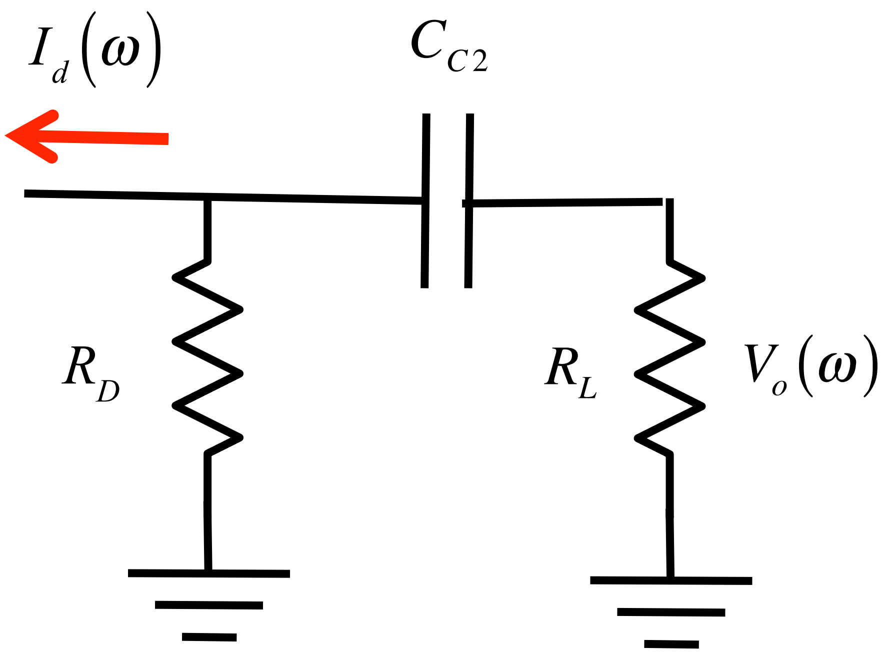
$$I_d = g_m V_i \frac{1 + \dfrac{\omega_Z}{j\omega}}{1 + \dfrac{\omega_S}{j\omega}} \qquad \omega_S = \frac{1}{\left(R_S \parallel 1/g_m\right) C_S} \qquad \omega_Z = \frac{1}{R_S C_S}$$

In addition to the expected pole, we find a zero.

Exact low frequency response of CS amplifier



Putting it all together



$$I_{R_L}(\omega) = I_d(\omega)\frac{R_D}{R_D + \left(R_L + \frac{1}{j\omega C_{C2}}\right)}$$

$$V_o(\omega) = I_{R_L}(\omega) R_L$$

$$V_o(\omega) = -\left(R_D \parallel R_L\right)\frac{1}{1 + \frac{\omega_{C2}}{j\omega}} I_d(\omega)$$

We find the expected pole.

Exact low frequency response of CS amplifier

$$V_o(\omega) = -\left(R_D \parallel R_L\right)\frac{1}{1+\frac{\omega_{C2}}{j\omega}} I_d(\omega)$$

$$I_d(\omega) = g_m \frac{1+\frac{\omega_Z}{j\omega}}{1+\frac{\omega_S}{j\omega}} V_i(\omega)$$

$$V_i(\omega) = \left(\frac{R_G}{R_G + R_{series}}\right)\frac{1}{1+\frac{\omega_{C1}}{j\omega}} V_s(\omega)$$

$$\frac{V_o(\omega)}{V_i(\omega)} = A_{\upsilon_S}(\omega) = \left(\frac{R_G}{R_G + R_{series}}\right) g_m R_L \parallel R_D \times \frac{1+\frac{\omega_Z}{j\omega}}{\left(1+\frac{\omega_{C1}}{j\omega}\right)\left(1+\frac{\omega_S}{j\omega}\right)\left(1+\frac{\omega_{C2}}{j\omega}\right)}$$

Exact low frequency response of CS amplifier

$$A_{\upsilon_S}(\omega) = A_{\upsilon_S}(\text{mid}) \frac{1+\dfrac{\omega_Z}{j\omega}}{\left(1+\dfrac{\omega_{C1}}{j\omega}\right)\left(1+\dfrac{\omega_S}{j\omega}\right)\left(1+\dfrac{\omega_{C2}}{j\omega}\right)}$$

$$A_{\upsilon_S}(\text{mid}) = -\left(\frac{R_G}{R_G + R_{series}}\right) g_m \left(R_L \parallel R_D\right)$$

Almost the product of 3 STC responses, but there is an additional zero.

$$\omega_Z = \frac{1}{R_S C_S} << \omega_S = \frac{1}{\left(R_S \parallel 1/g_m\right) C_S}$$

So the zero is not important - it does not affect our conclusions about the LF corner frequency.