ECE 255: L33

High Frequency Response I
(Sedra and Smith, 7th Ed., Sec. 10.2)

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Announcements

HW10 Due 5:00 PM Friday, April 19 in EE-209 dropbox

LTSpice Project III Due 5:00 PM Wed, April 17

LTSpice Help Session to be announced

Practice Final Exam posted

No Office Hours today (Lundstrom)
Outline

1) LF review
2) Low pass filter / STC circuits
3) High freq model for MOSFETs
4) High freq models for BJTs
A Bode plot is a graphical representation of a system's frequency response. It shows the magnitude and phase of a frequency response as a function of frequency on a logarithmic scale. The vertical axis represents the magnitude in decibels (dB), and the horizontal axis represents the frequency on a logarithmic scale (log f).

The plot typically includes the magnitude of the frequency response, denoted as $|A_{v}|$ dB, and the magnitude of the mid-frequency response, denoted as $|A_{v}(\text{mid})|$. The bandwidth (BW) is defined as the frequency range where the magnitude is within 3 dB of its maximum value.

The lower 3 dB point is the frequency at which the magnitude drops to 3 dB below its maximum value, $f_L$. The upper 3 dB point is the frequency at which the magnitude drops to 3 dB below its maximum value, $f_H$.
Short-circuit time constant method

\[ V_i(j\omega) \]

\[ \omega_{L1} = \frac{1}{RthC_1} \]

\[ V_o(j\omega) \]

The corner frequency (the pole) is one over a time constant – just find the RC time constant.

While we compute this corner frequency, we assume that the other C’s are shorted – this produces a STC circuit.

\[ \omega_L \approx \omega_{L_{\text{max}}} \]

\[ \omega_L < \omega_{L1} + \omega_{L2} + \omega_{L3} + \ldots \]
Example: Common Drain Amplifier

\[ \omega_{L1} = \frac{1}{R_{th}C_{C1}} \]

\[ \omega_{L1} = \frac{1}{(R_{series} + R_G)C_{C1}} \]
Example: Common Drain Amplifier

\[ \omega_{L2} = \frac{1}{R_{th}C_{C2}} \]

\[ \omega_{L2} = \frac{1}{\left( R_L + R_S \parallel \left( 1/g_m \right) \right) C_{C2}} \]
Compute the LF corner freq.

\[ R_{G_1 \parallel R_{G_2}} = 243 \text{k} \]

\[ R_{\text{series}} = 1 \text{k} \quad R_S = 1.3 \text{k} \]

\[ R_L = 24 \text{k} \quad g_m = 1 \text{mS} \]

\[ C_{C_1} = 0.1 \mu\text{F} \quad C_{C_1} = 47 \mu\text{F} \]
Compute the LF corner freq.

\[ \omega_{L1} = \frac{1}{(1+243) \times 10^3 \times 0.1 \times 10^{-6}} = 41 \]

\[ \omega_{L2} = \frac{1}{(24+1.3 || 1) \times 10^3 \times 47 \times 10^{-6}} = 0.87 \]

\[ \omega_L \approx \omega_{L1} = 41 = 2\pi f_L \]

\[ f_L = 6.5 \text{ Hz} \]

\[ R_{G1} \parallel R_{G2} = 243 \text{ k} \]

\[ R_{\text{series}} = 1 \text{ k} \quad R_S = 1.3 \text{ k} \]

\[ R_L = 24 \text{ k} \quad g_m = 1 \text{ mS} \]

\[ C_{C1} = 0.1 \mu\text{F} \quad C_{C1} = 47 \mu\text{F} \]

\[ \omega_{L1} = \frac{1}{(R_{\text{series}} + R_G)C_{C1}} \]

\[ \omega_{L2} = \frac{1}{(R_L + R_S \parallel (1/g_m))C_{C2}} \]
Find the Thevenin eq. resistance for $C_1$
Thevenin eq. resistance for $C_1$

$$R_{th} = \frac{r_\pi}{2} \parallel R_1 \parallel \frac{1}{g_m}$$
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Bode plot

\[ |A_v| \text{ dB} \]

\[ -3 \text{ dB} \]

\[ |A_v(\text{mid})| \]

\[ f_L \]

\[ f_H \]

\[ \log f \]

BW

upper 3 dB point
\[ T(j\omega) = \frac{Y_1}{Y_1 + Y_2} = \frac{1}{1/R_1 + (1/R_2 + j\omega C)} \]

\[ T(j\omega) = \frac{T_{mid}}{1 + j\omega/\omega_H} \quad \omega_H = \frac{1}{(R_1 \parallel R_2)C} = \frac{1}{R_{th}C} \]
How would we analyze this circuit?

1) \[ \omega_L = \frac{1}{(R_1 + R_2)C_1} \]  
   \[ \text{C}_2 \text{ open} \]

2) \[ \omega_H = \frac{1}{(R_1 \parallel R_2)C_2} \]  
   \[ \text{C}_1 \text{ short} \]
Bode plot

\[ |A_v| \text{ dB} \]

\[ |A_v(\text{mid})| \]

\[ f_L \] \hspace{2cm} \[ f_H \]

-3 dB

BW

Small caps open-circuited.

Large caps short-circuited.

Other large caps short-circuited

Other small caps open-circuited
Outline

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The answer

\[ C_{gd} \]

\[ C_{gs} \]
The hf hybrid pi model

\[ C_{gd} \]

\[ C_{gs} \]

\[ g_m v_{gs} \]

\[ r_o \]

\[ v_{ds} \]
MOSFET under low $V_{DS}$

\[ C = \frac{K_r \epsilon_0}{t} \]

\[ C_{gs} = \frac{W L}{2} \frac{K_{ox} \epsilon_0}{t_{ox}} = C_{gd} \]
MOSFET under high $V_{DS}$

$V_{GS} > V_T$

pinch-off

$C_{gs} = \frac{2}{3} \frac{W L K_{ox} \varepsilon_0}{t_{ox}}$

$C_{gd} \approx 0$
Gate-drain overlap/fringing field capacitance

\[ C_{gs} = \frac{2}{3} \frac{W L K_{\text{ox}} \varepsilon_0}{t_{\text{ox}}} \]

\[ C_{gd} > 0 \]

Miller capacitance!

Two plates separated by distance.
Characterizing a MOSFET

Short circuit current gain vs. frequency

$I_o(\omega)$
The hf hybrid pi model

\[ I_2 + I_o = g_m V_{gs} \]
\[ I_o = (g_m - j\omega C_{gd}) V_{gs} \]
\[ \frac{I_o}{I_i} = \frac{g_m}{j\omega (C_{gs} + C_{gd})} \left( 1 - j\omega C_{gd} / g_m \right) \]
\[ I_o = g_m V_{gs} - I_2 \]
\[ V_{gs} = I_i \frac{1}{j\omega (C_{gs} + C_{gd})} \]
\[ \frac{I_o}{I_i} \approx \frac{g_m}{j\omega (C_{gs} + C_{gd})} \]
\[ I_2 = V_{gs} j\omega C_{gd} \]
Characterizing a MOSFET

Short circuit current gain vs. frequency

\[ \beta(\omega) = \frac{I_o}{I_i} \approx \frac{g_m}{j\omega(C_{gs} + C_{gd})} \]

\[ \beta(\omega) = \frac{\omega_T}{j\omega} \]

\[ \omega_T = \frac{g_m}{C_{gs} + C_{gd}} \]

\[ \omega_T = \frac{g_m}{C_{tot}} \]

\[ I_o(\omega) \]
Gain-Bandwidth product

\[ \beta(\omega) = \frac{\omega_T}{j\omega} \]

\[ |\beta(\omega)| = \frac{\omega_T}{\omega} \]

\[ |\beta(\omega_T)| = 1 \]

20 dB per decade

\[ \omega = \omega_T \quad \log_{10} \omega \]
Gain-Bandwidth product

\[ \omega_T = \frac{g_m}{(C_{gs} + C_{gd})} = 2\pi f_T \]

\( f_T \) is an important figure of merit for a transistor.

\[ f_T (\text{max}) = \frac{1}{2\pi t_t} \]

\[ t_t = \frac{L}{\langle v \rangle} \]