Spring 2019 Purdue University

ECE 255: L33

High Frequency Response I

(Sedra and Smith, 7th Ed., Sec. 10.2)

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Announcements

HW10 Due 5:00 PM Friday, April 19 in EE-209 dropbox

LTSpice Project III Due 5:00 PM Wed, April 17

LTSpice Help Session to be announced

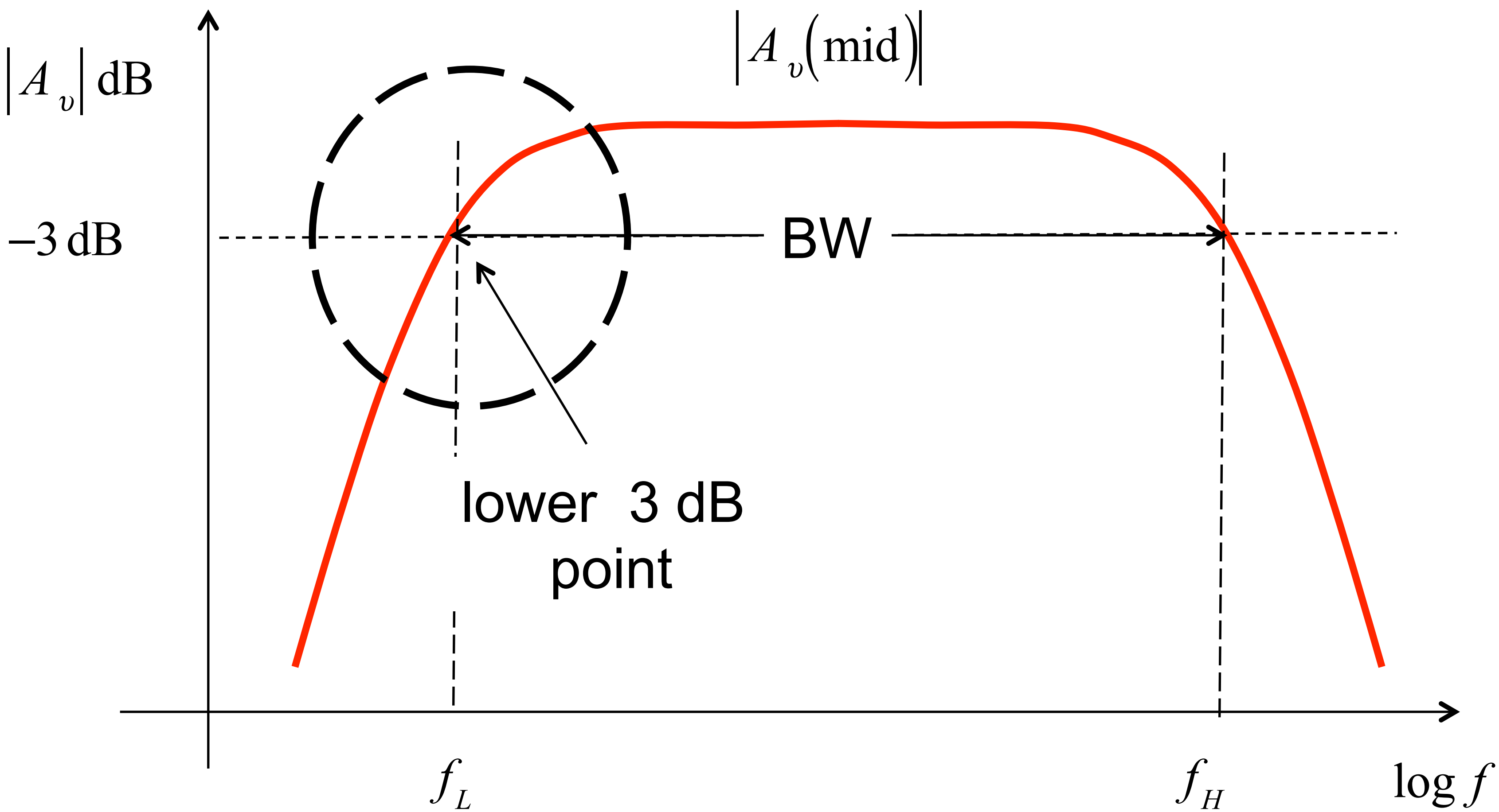
Practice Final Exam posted

No Office Hours today (Lundstrom)

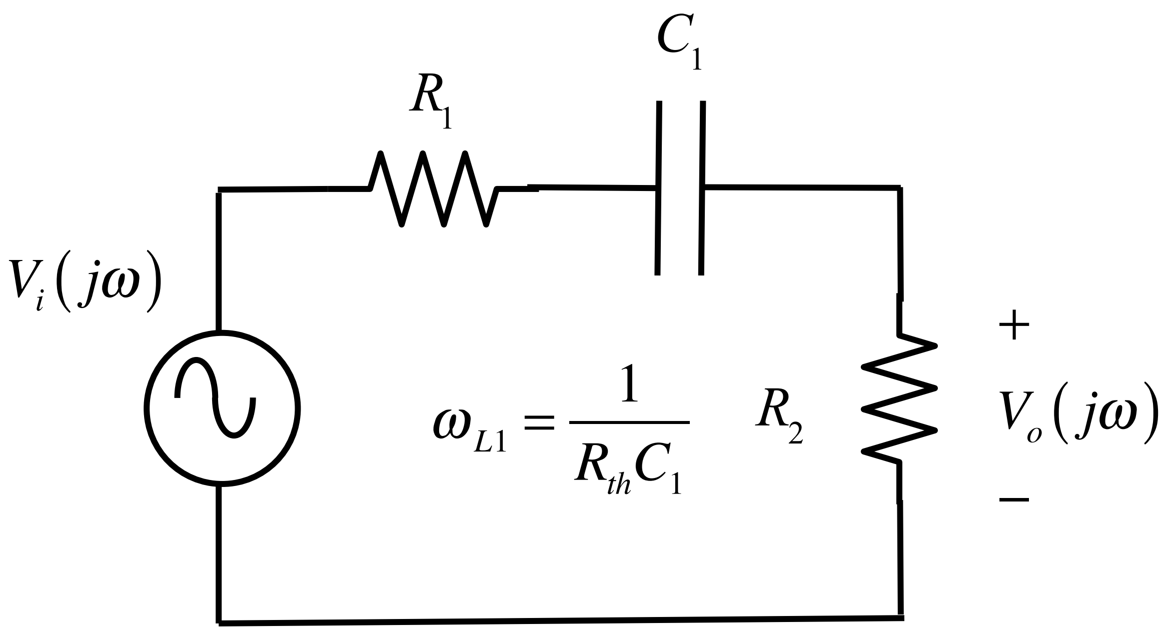
Outline

1) **LF review**
2) Low pass filter / STC circuits
3) High freq model for MOSFETs
4) High freq models for BJTs

Bode plot



Short-circuit time constant method

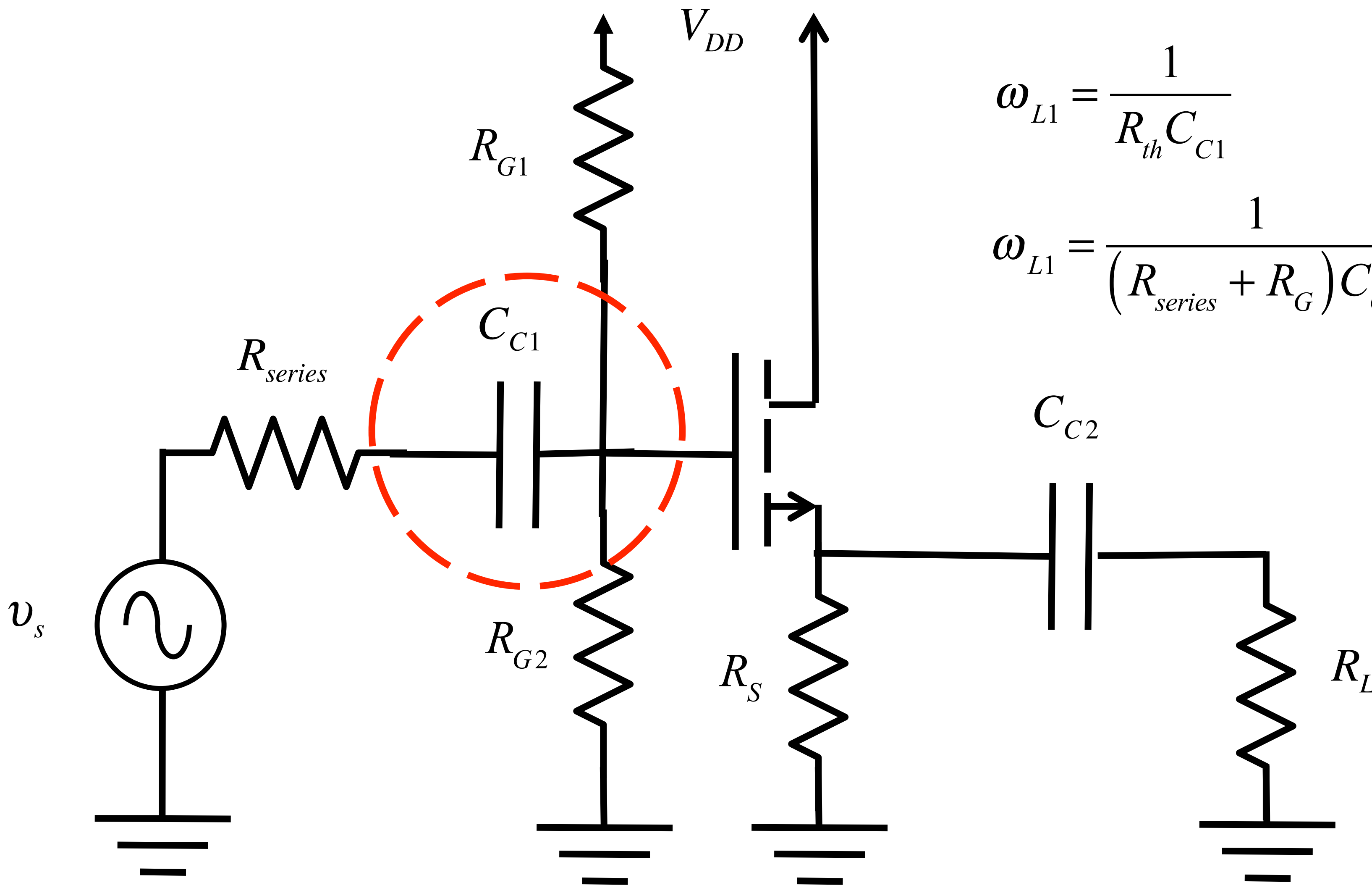


The corner frequency (the pole) is one over a time constant − just find the RC time constant.

While we compute this corner frequency, we assume that the other C's are shorted − this produces a STC circuit.

$$\omega_L \approx \omega_{L\max} \qquad \omega_L < \omega_{L1} + \omega_{L2} + \omega_{L3} + ...$$

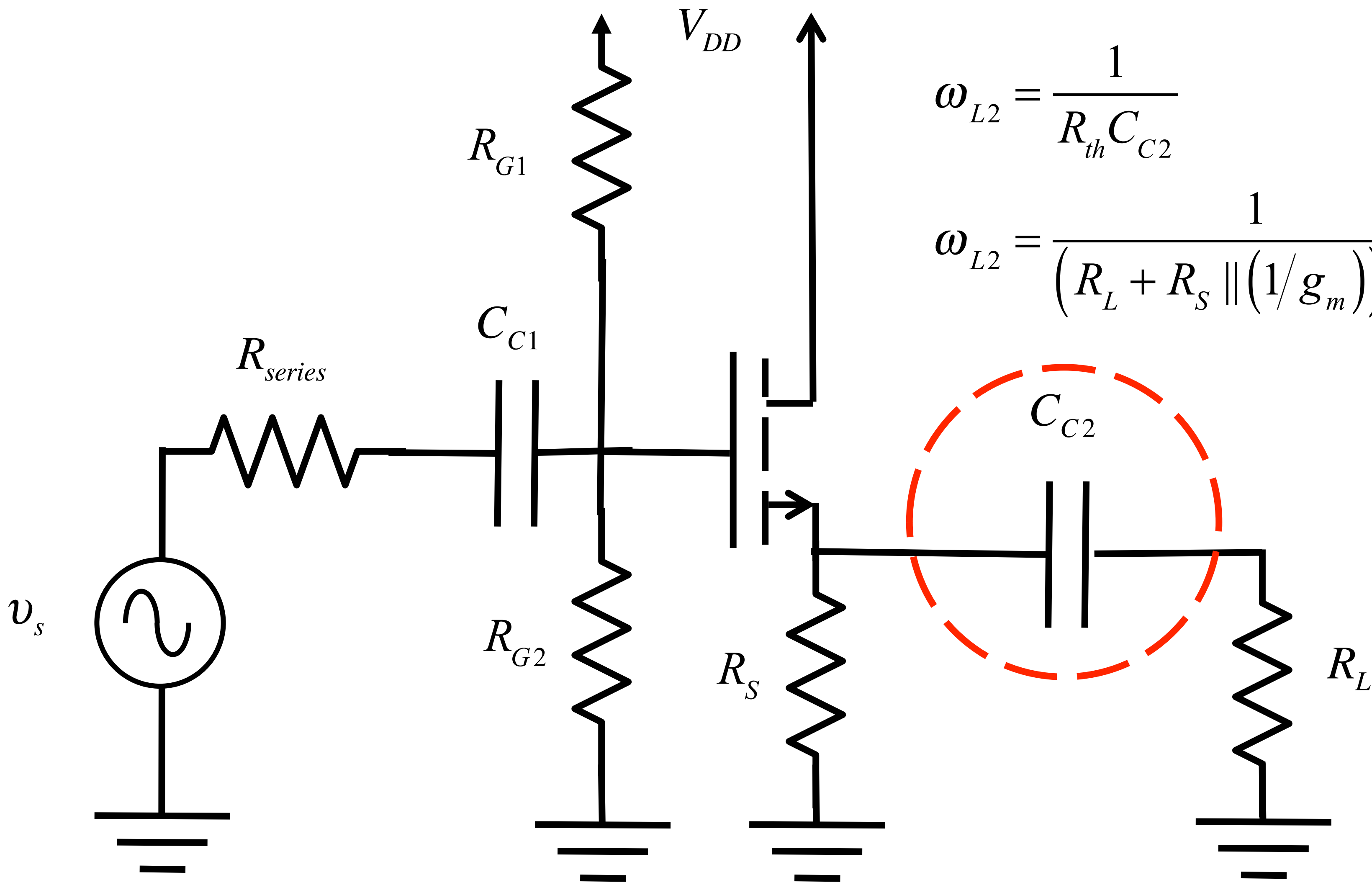
Example: Common Drain Amplifier



$$\omega_{L1} = \frac{1}{R_{th} C_{C1}}$$

$$\omega_{L1} = \frac{1}{\left(R_{series} + R_G\right) C_{C1}}$$

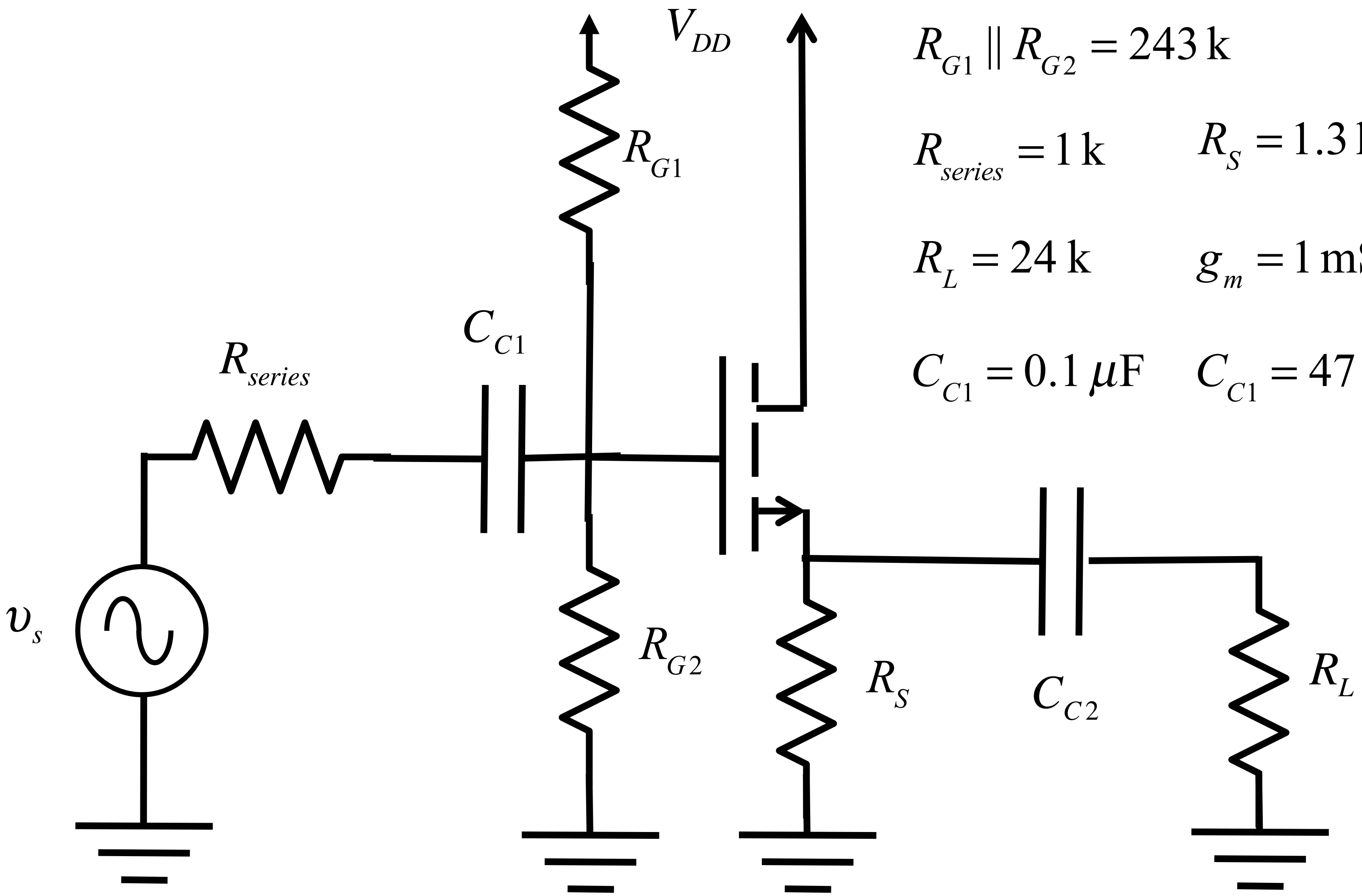
Example: Common Drain Amplifier



$$\omega_{L2} = \frac{1}{R_{th} C_{C2}}$$

$$\omega_{L2} = \frac{1}{\left(R_L + R_S \parallel \left(1/g_m\right)\right) C_{C2}}$$

Compute the LF corner freq.



$R_{G1} \| R_{G2} = 243\,\text{k}$

$R_{series} = 1\,\text{k}$ $R_S = 1.3\,\text{k}$

$R_L = 24\,\text{k}$ $g_m = 1\,\text{mS}$

$C_{C1} = 0.1\,\mu\text{F}$ $C_{C1} = 47\,\mu\text{F}$

Compute the LF corner freq.

$$\omega_{L1} = \frac{1}{(1+243)\times 10^3 \times 0.1\times 10^{-6}} = 41$$

$$\omega_{L2} = \frac{1}{(24+1.3\parallel 1)\times 10^3 \times 47\times 10^{-6}} = 0.87$$

$$\omega_L \approx \omega_{L1} = 41 = 2\pi f_L$$

$$f_L = 6.5\ \text{Hz}$$

$$R_{G1} \parallel R_{G2} = 243\ \text{k}$$

$$R_{series} = 1\ \text{k} \qquad R_S = 1.3\ \text{k}$$

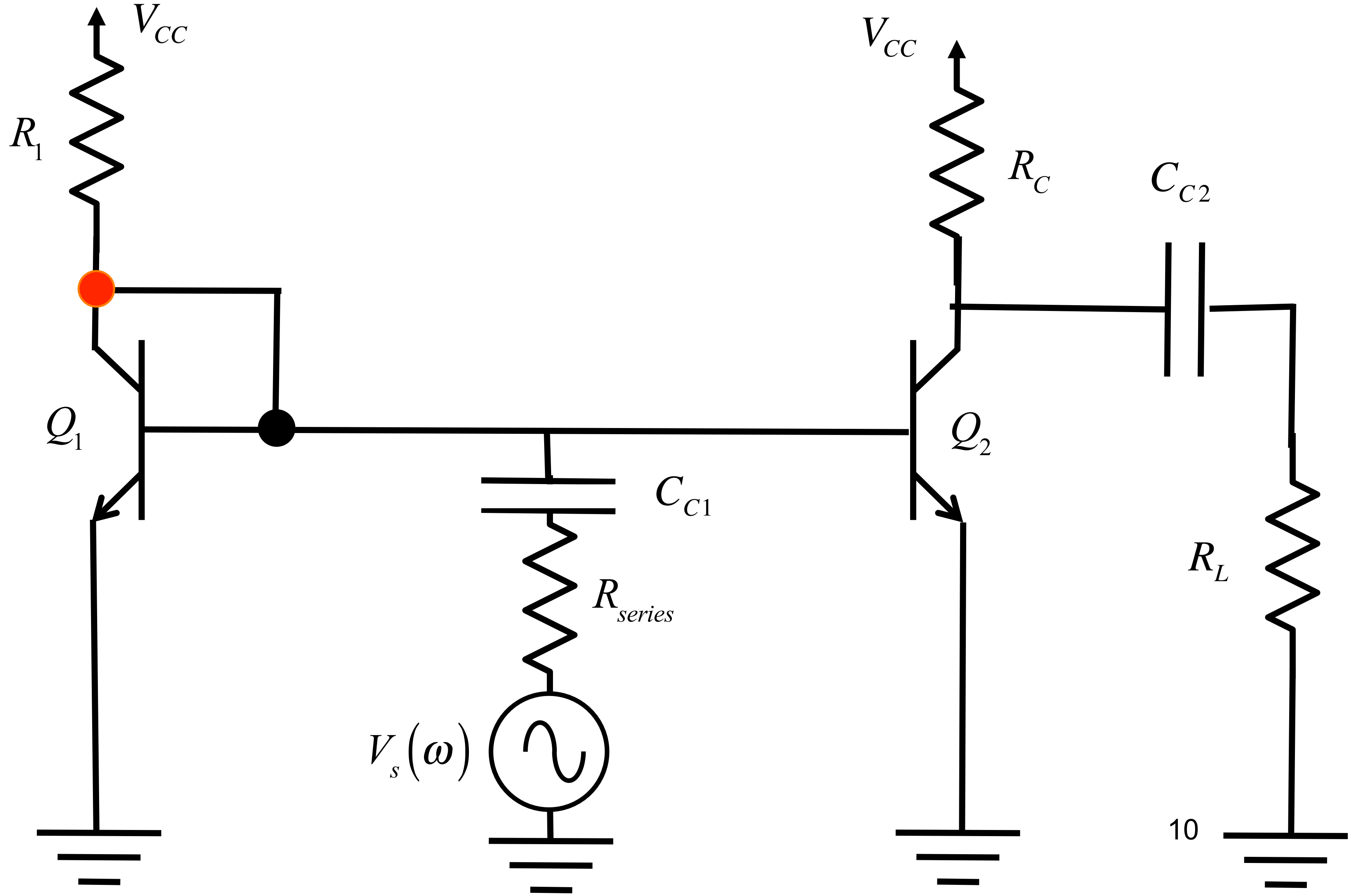
$$R_L = 24\ \text{k} \qquad g_m = 1\ \text{mS}$$

$$C_{C1} = 0.1\ \mu\text{F} \qquad C_{C1} = 47\ \mu\text{F}$$

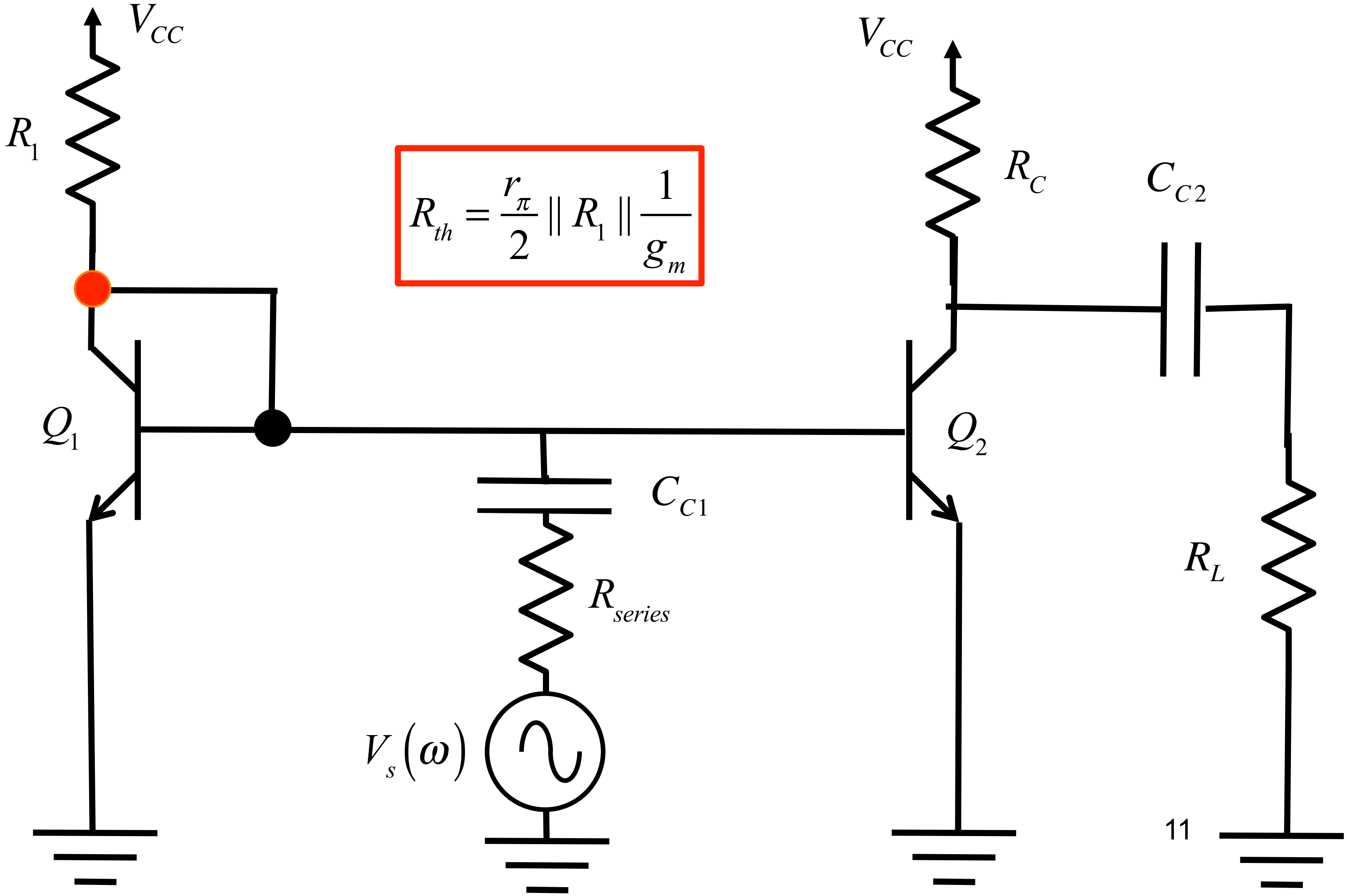
$$\omega_{L1} = \frac{1}{(R_{series} + R_G)C_{C1}}$$

$$\omega_{L2} = \frac{1}{\left(R_L + R_S \parallel \left(1/g_m\right)\right)C_{C2}}$$

Find the Thevenin eq. resistance for C_1



Thevenin eq. resistance for C_1



$$R_{th} = \frac{r_\pi}{2} \parallel R_1 \parallel \frac{1}{g_m}$$

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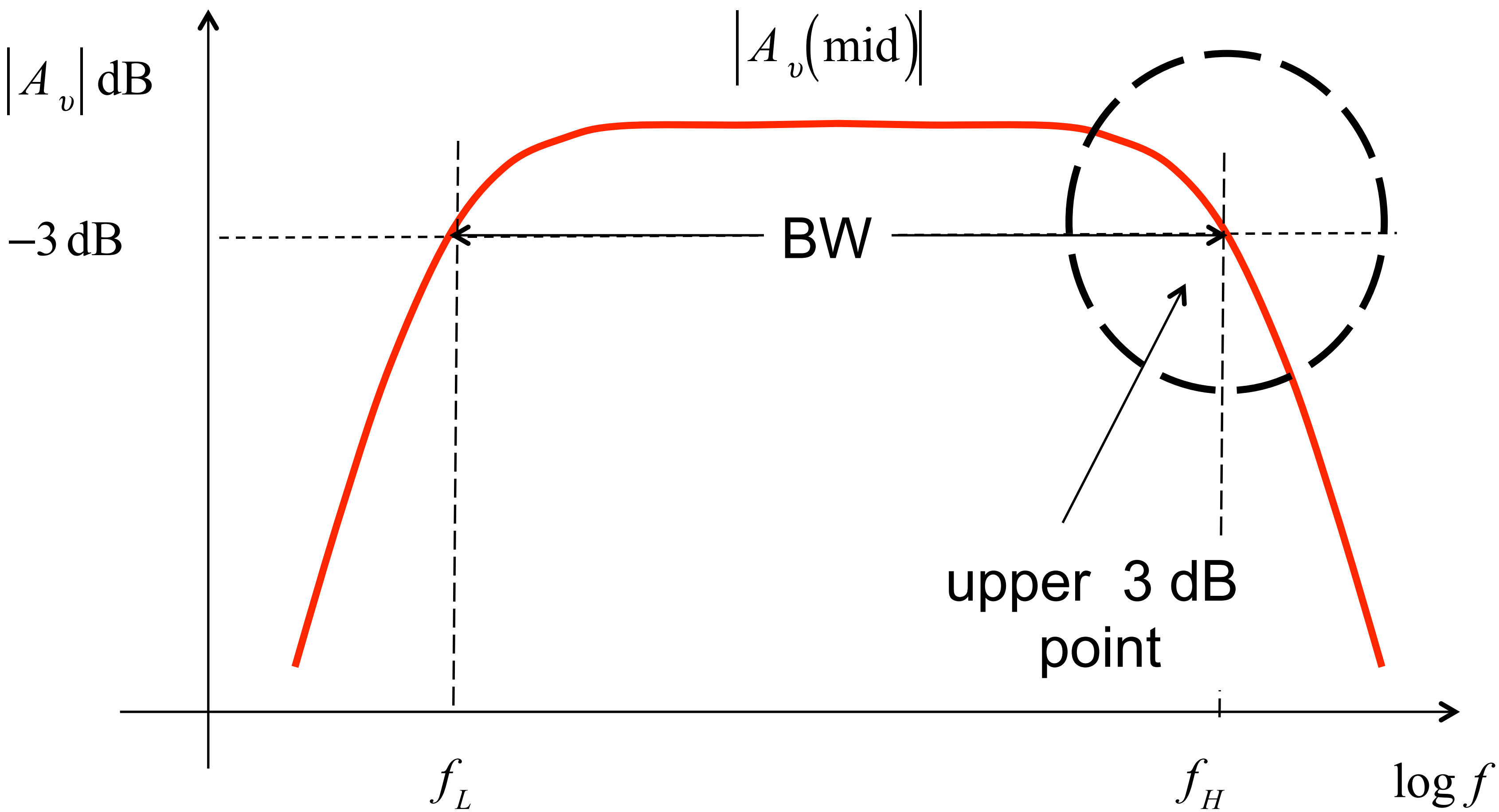
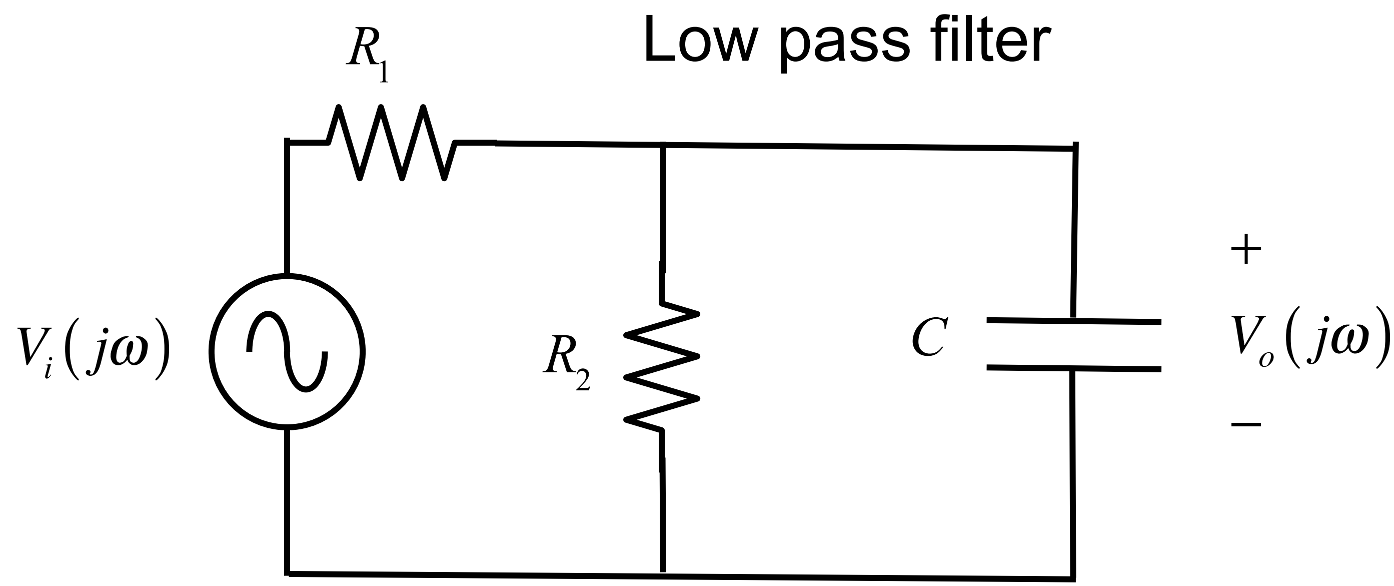
Bode plot



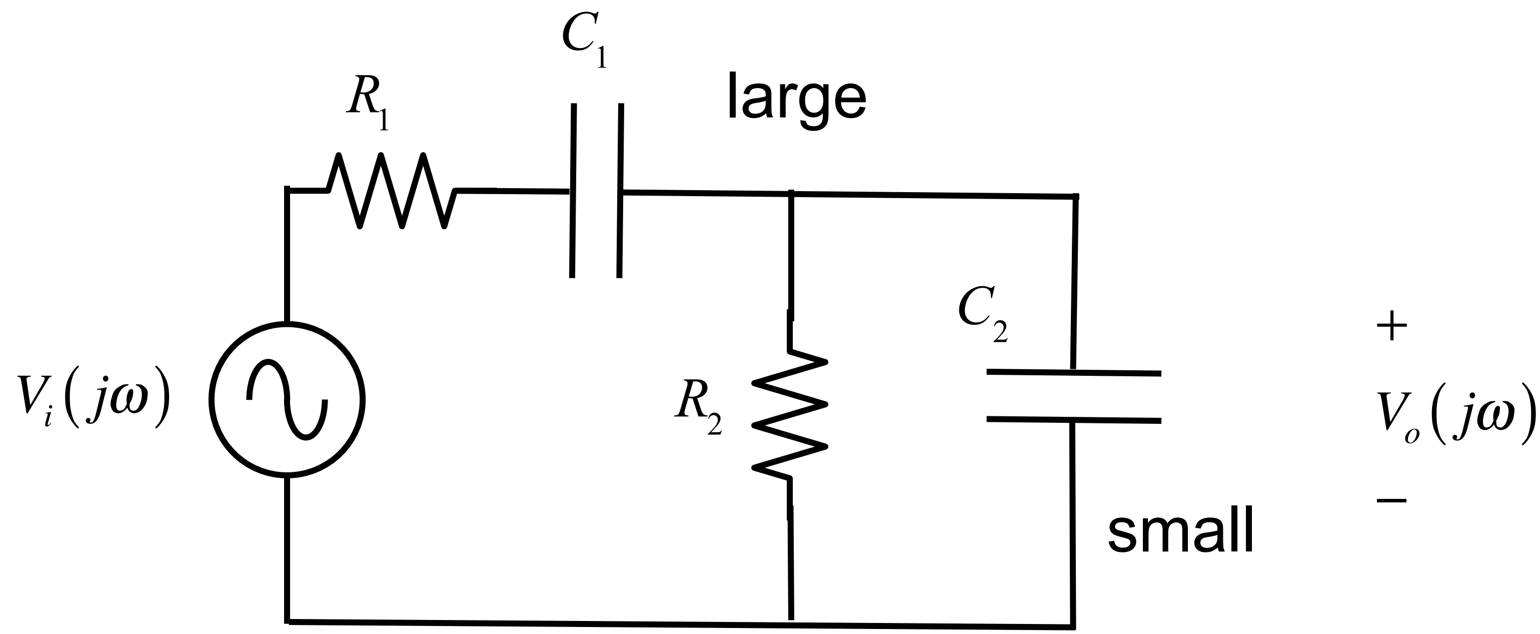
STC



$$T(j\omega) = \frac{Y_1}{Y_1 + Y_2} = \frac{1/R_1}{1/R_1 + \left(1/R_2 + j\omega C\right)}$$

$$T(j\omega) = \frac{T_{mid}}{1 + j\omega/\omega_H} \qquad \omega_H = \frac{1}{\left(R_1 \parallel R_2\right)C} = \frac{1}{R_{th}C}$$

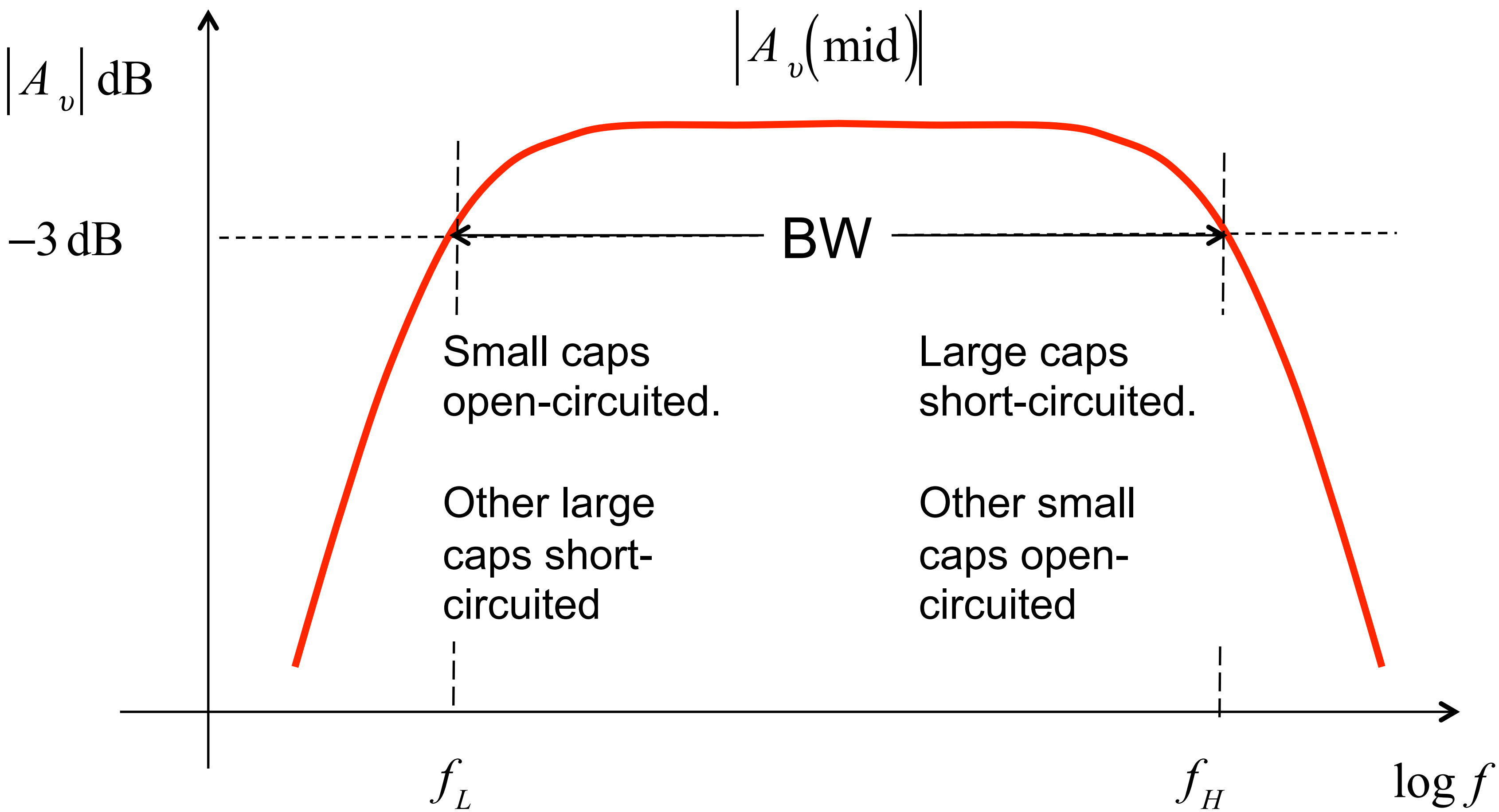
How would we analyze **this** circuit?



1) $$\omega_L = \frac{1}{(R_1 + R_2)C_1} \qquad C_2 \quad \text{open}$$

2) $$\omega_H = \frac{1}{(R_1 \parallel R_2)C_2} \qquad C_1 \quad \text{short}$$

Bode plot



$|A_v|$ dB
$|A_v(\text{mid})|$
-3 dB
BW
Small caps open-circuited.
Other large caps short-circuited
Large caps short-circuited.
Other small caps open-circuited
f_L
f_H
$\log f$

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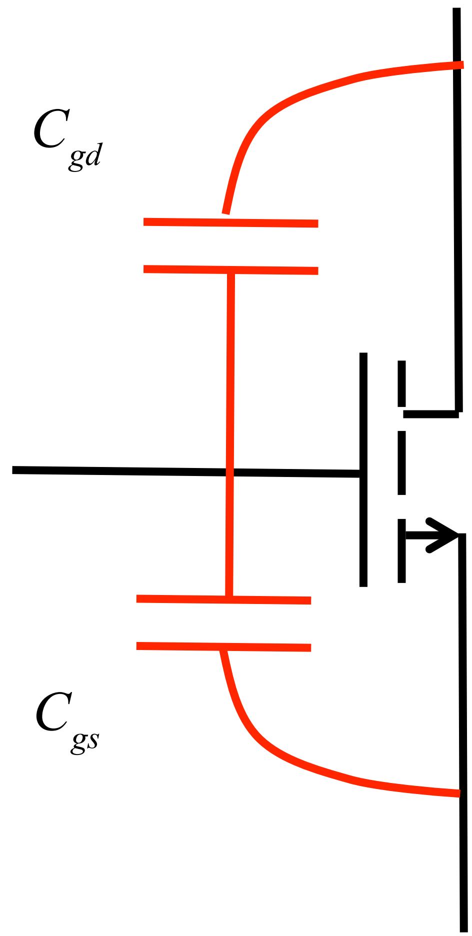
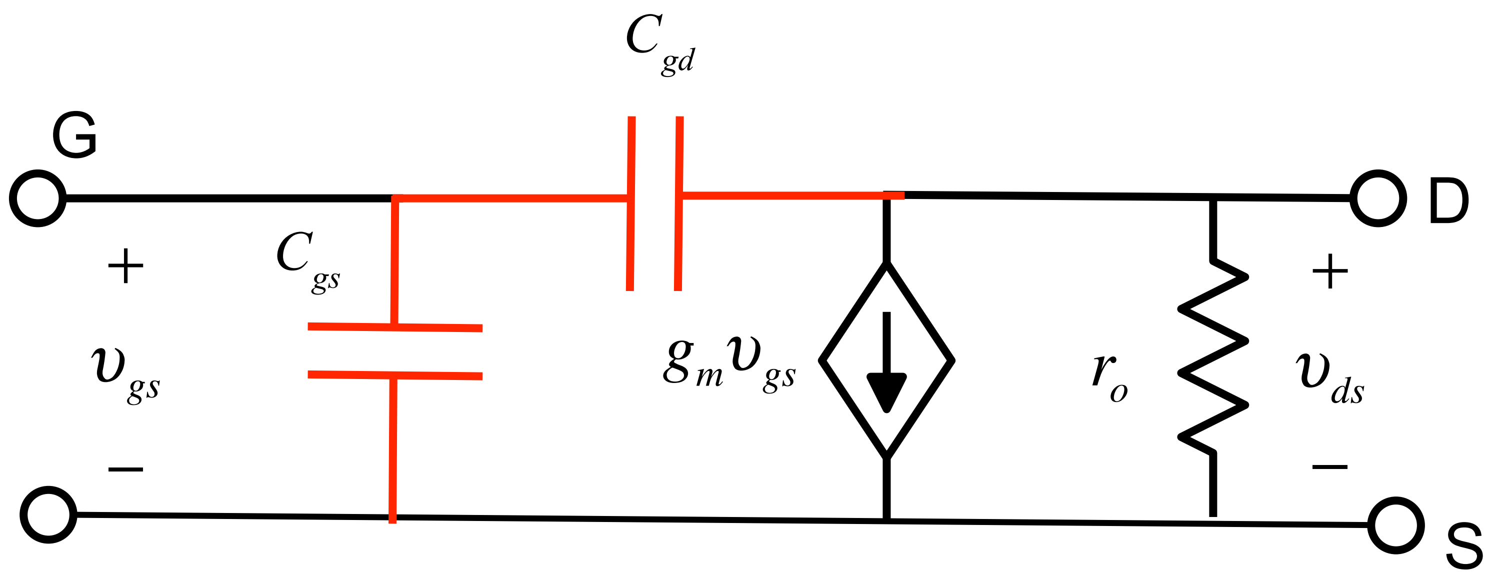
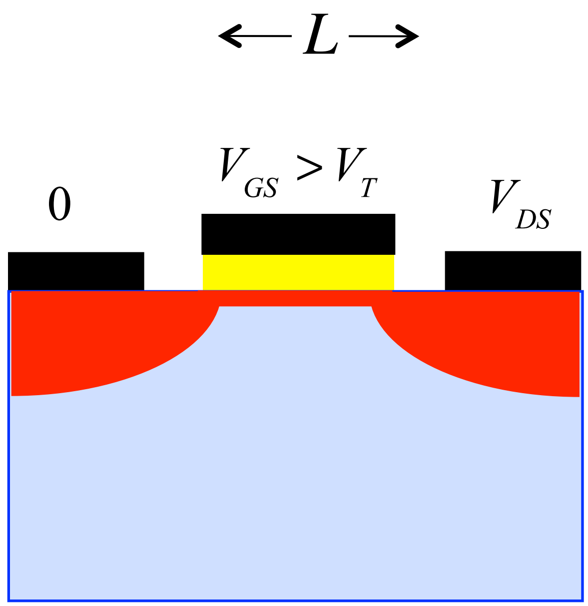
The answer

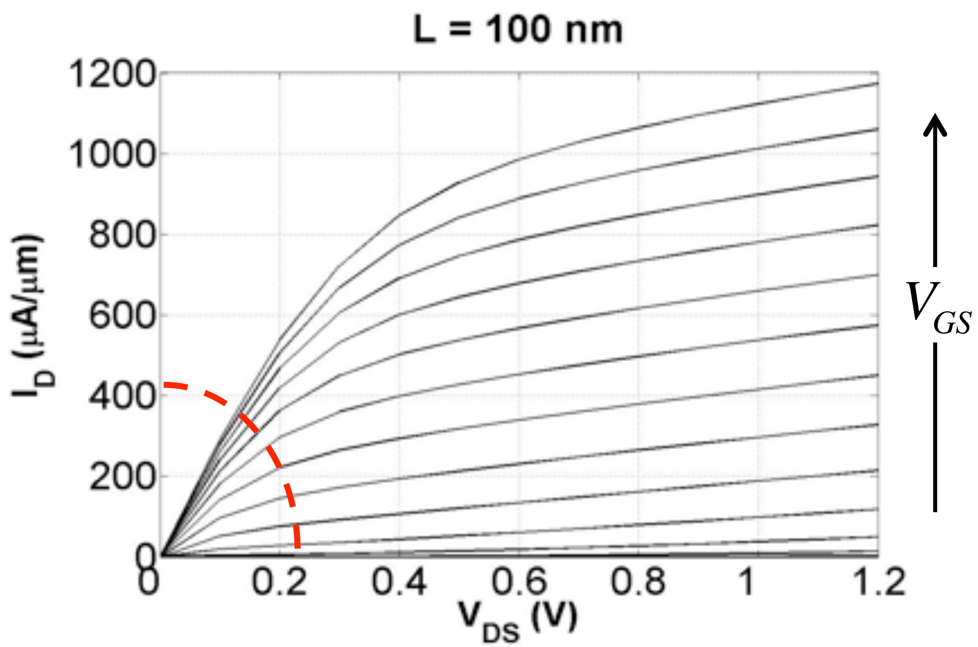


The hf hybrid pi model



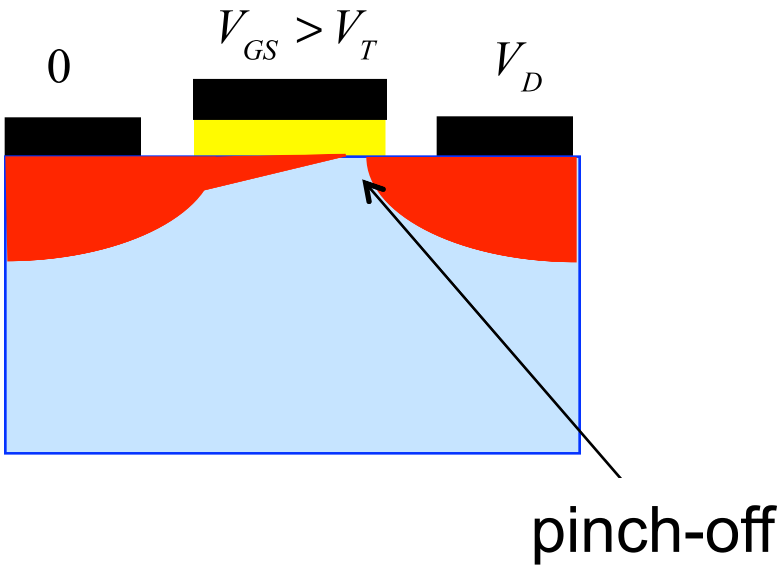
MOSFET under low V_{DS}

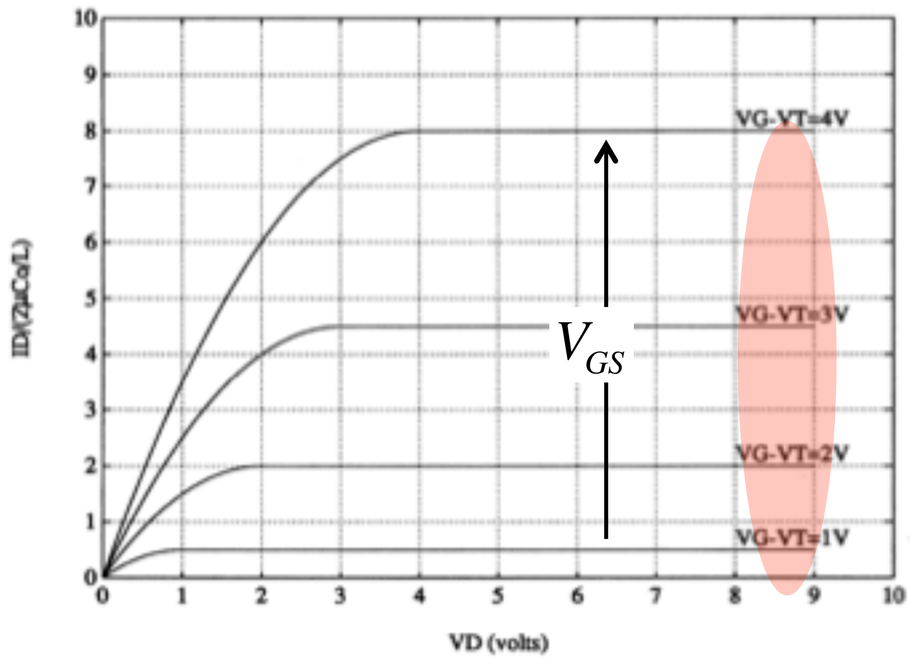




$$C \equiv \frac{K_r \varepsilon_0}{t} \qquad C_{gs} = \frac{WL}{2} \frac{K_{ox} \varepsilon_0}{t_{ox}} = C_{gd}$$

MOSFET under high V_{DS}





$$C_{gs} = \frac{2}{3}\frac{WL\,K_{ox}\varepsilon_0}{t_{ox}}$$

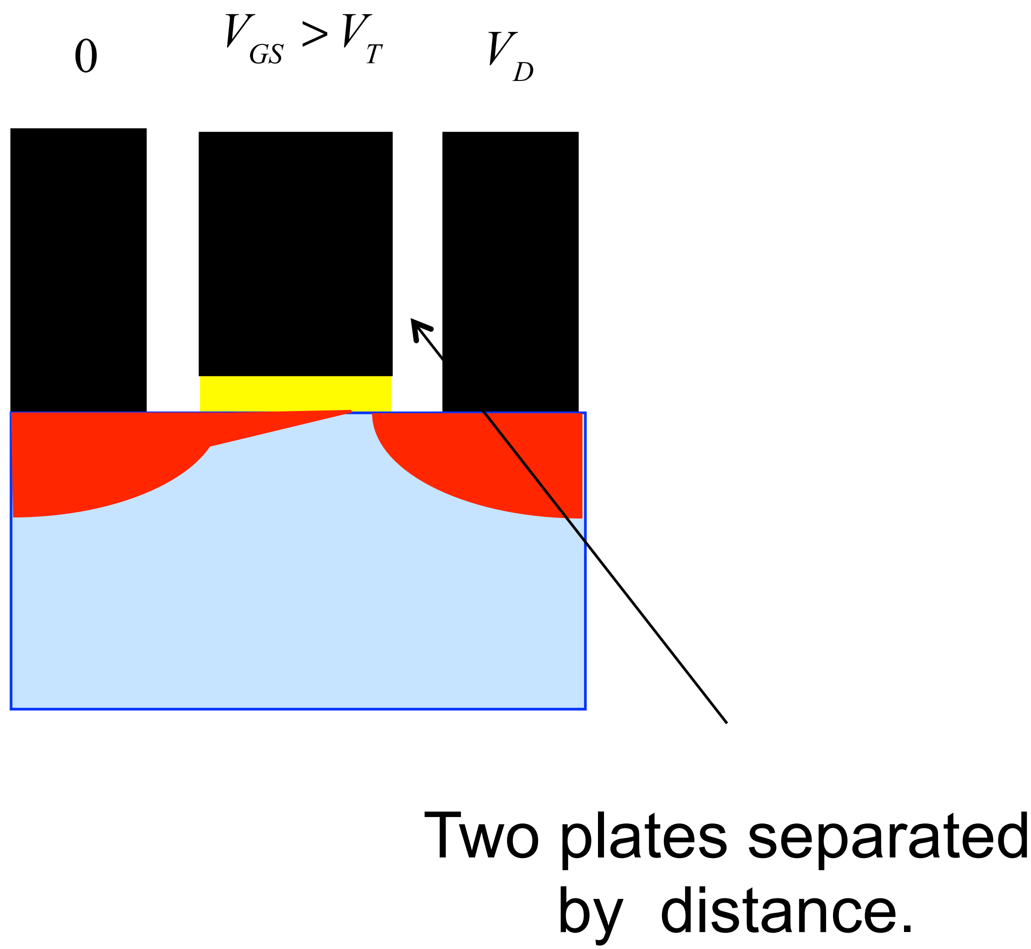
$$C_{gd} \approx 0$$

Gate-drain overlap/fringing field capacitance

$$C_{gs} = \frac{2}{3}\frac{WL\,K_{ox}\varepsilon_0}{t_{ox}}$$

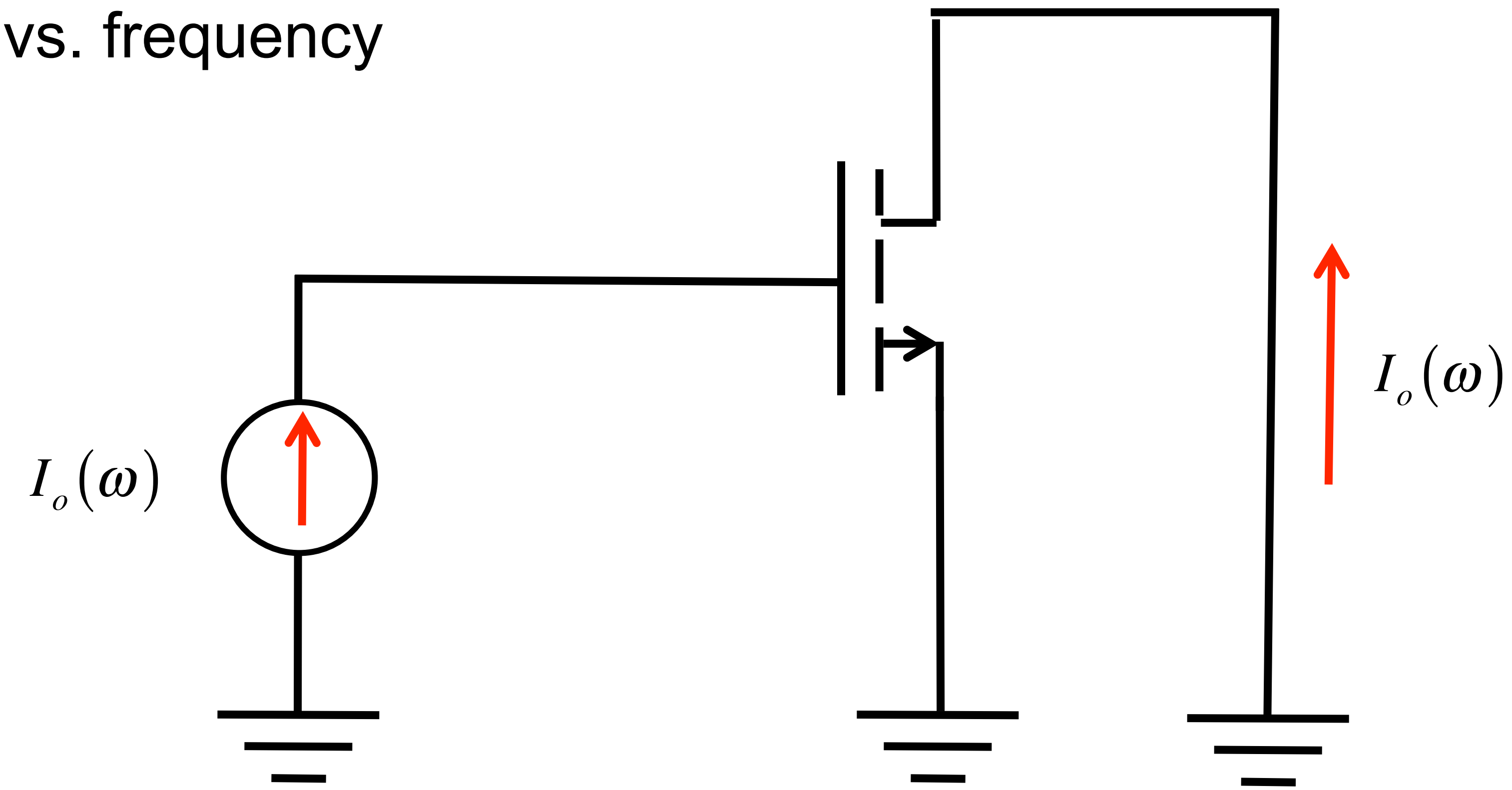
$$C_{gd} > 0$$

Miller capacitance!

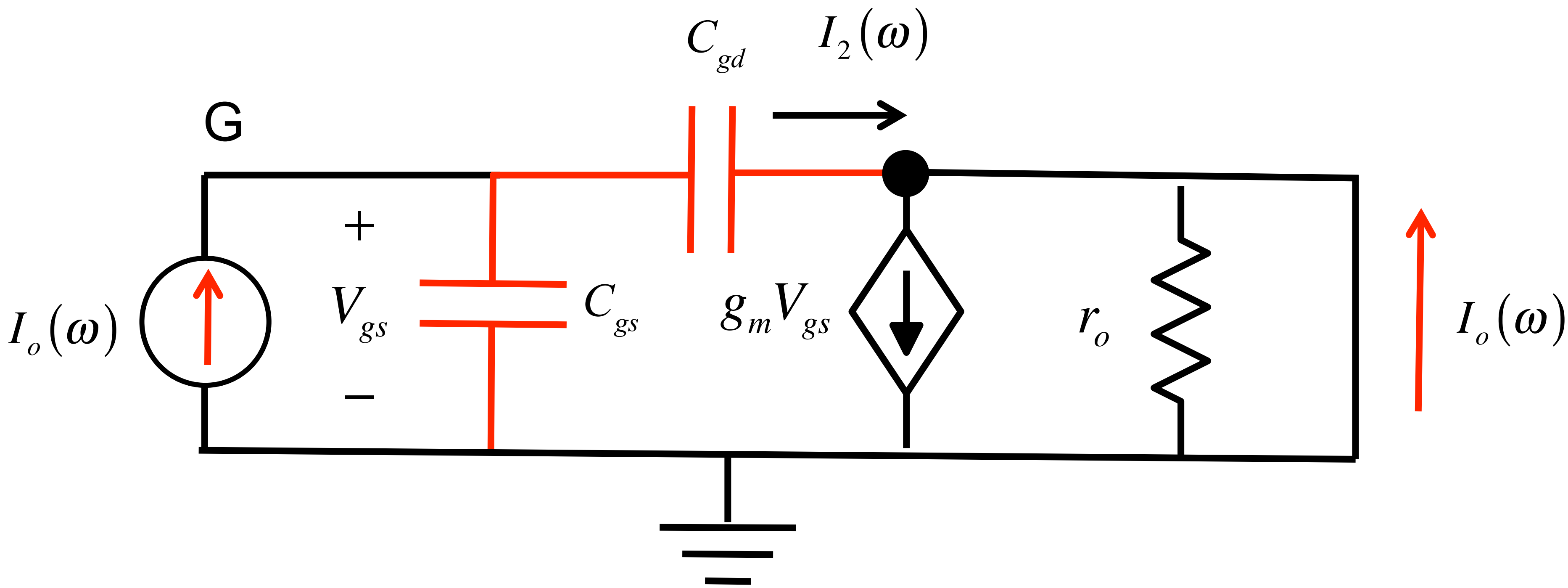


Characterizing a MOSFET

Short circuit current gain vs. frequency



The hf hybrid pi model



$$I_2 + I_o = g_m V_{gs}$$

$$I_o = \left(g_m - j\omega C_{gd}\right)V_{gs}$$

$$\frac{I_o}{I_i} = \frac{g_m}{j\omega\left(C_{gs} + C_{gd}\right)}\left(1 - j\omega C_{gd}/g_m\right)$$

$$I_o = g_m V_{gs} - I_2$$

$$V_{gs} = I_i \frac{1}{j\omega\left(C_{gs} + C_{gd}\right)}$$

$$\frac{I_o}{I_i} \approx \frac{g_m}{j\omega\left(C_{gs} + C_{gd}\right)}$$

$$I_2 = V_{gs}\ j\omega C_{gd}$$

Characterizing a MOSFET

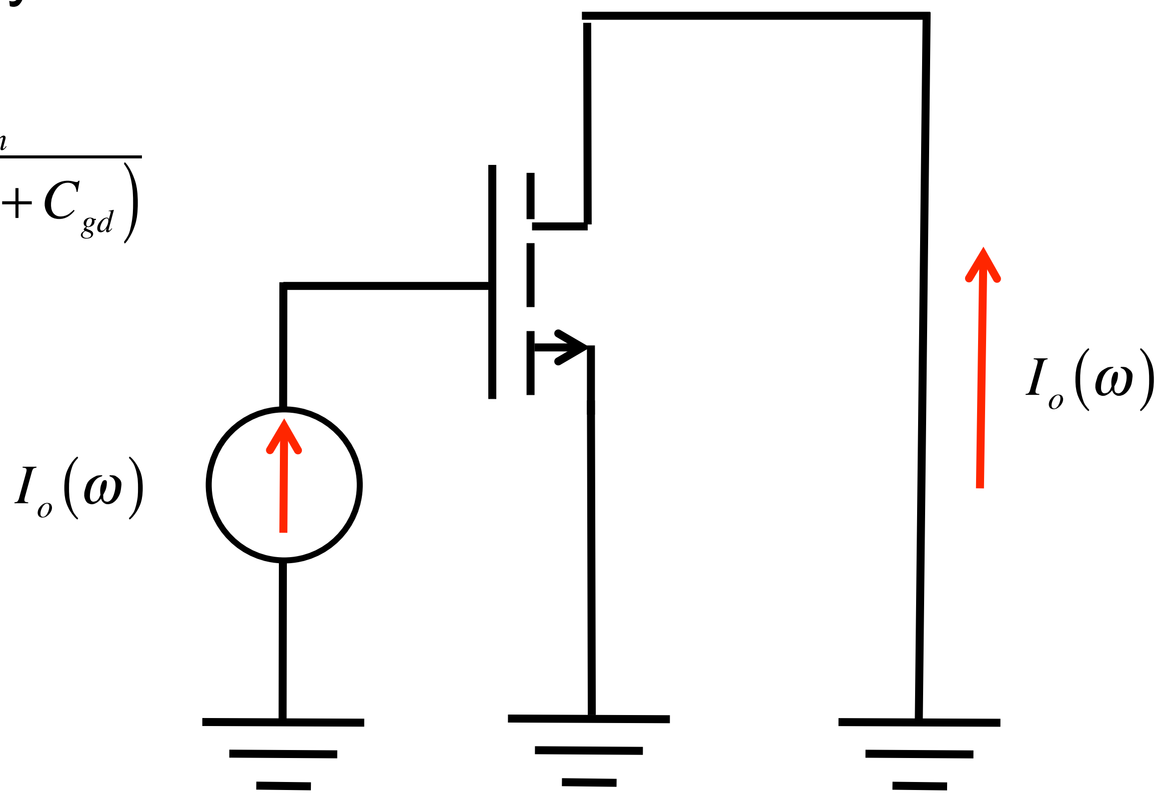
Short circuit current gain vs. frequency

$$\beta(\omega) = \frac{I_o}{I_i} \approx \frac{g_m}{j\omega\left(C_{gs} + C_{gd}\right)}$$

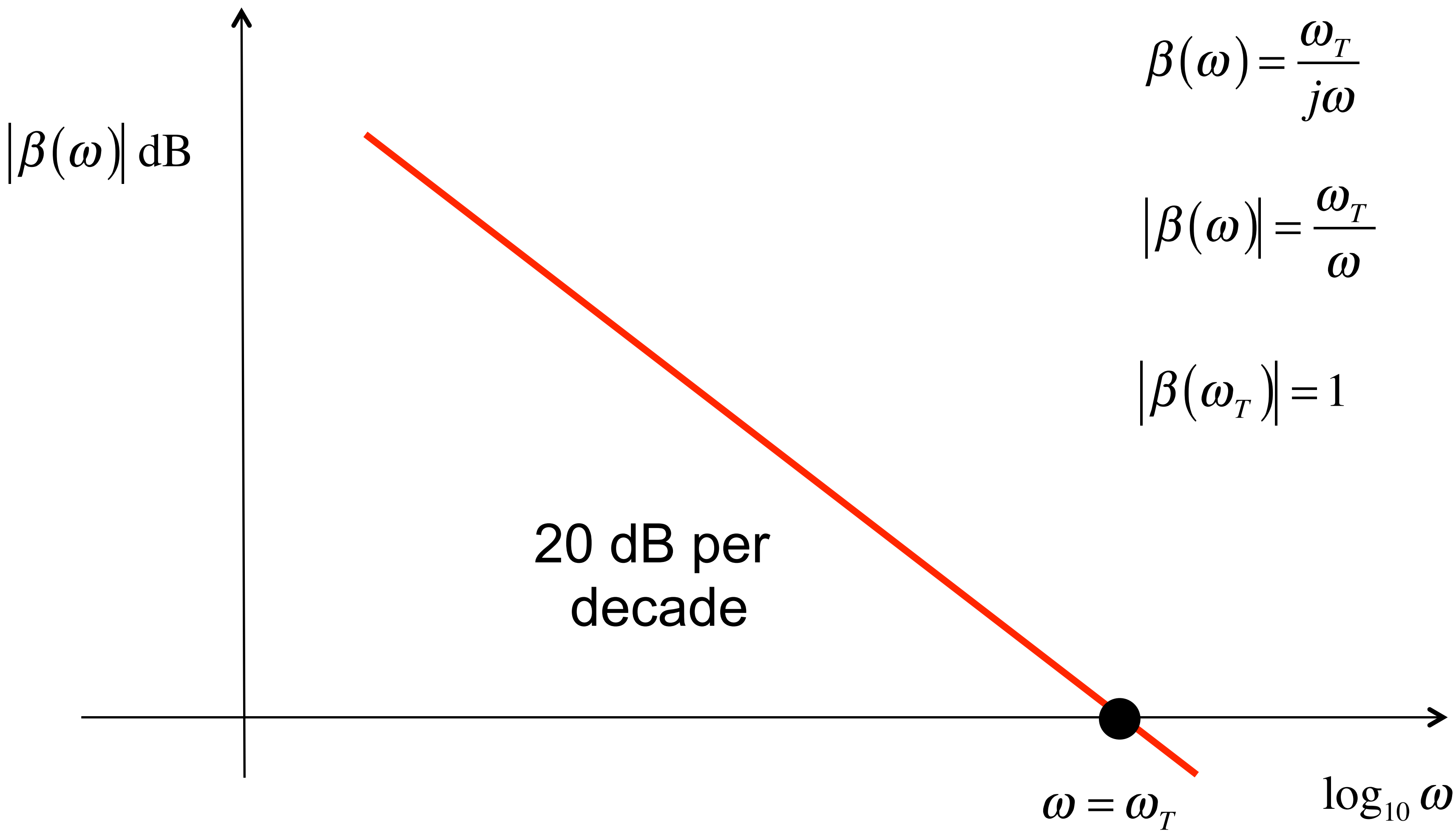
$$\beta(\omega) = \frac{\omega_T}{j\omega}$$

$$\omega_T = \frac{g_m}{\left(C_{gs} + C_{gd}\right)}$$

$$\boxed{\omega_T = \frac{g_m}{C_{tot}}}$$



Gain-Bandwidth product

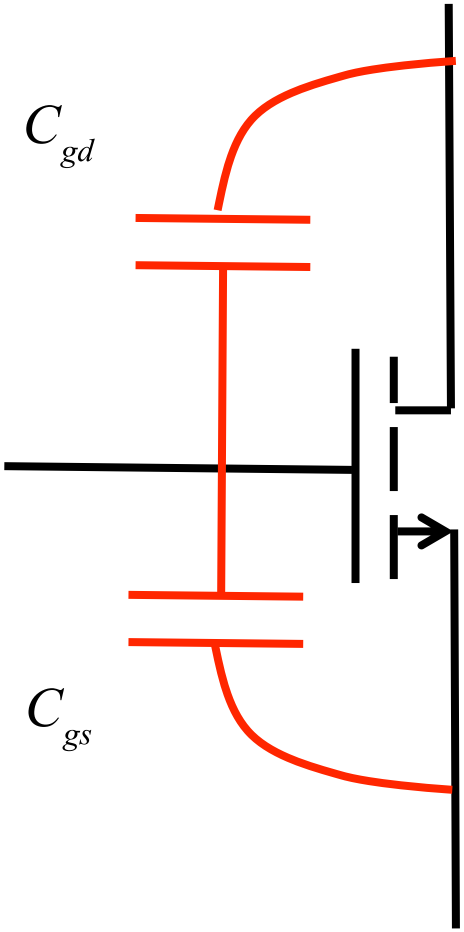


$$\beta(\omega) = \frac{\omega_T}{j\omega}$$

$$|\beta(\omega)| = \frac{\omega_T}{\omega}$$

$$|\beta(\omega_T)| = 1$$

Gain-Bandwidth product



$$\omega_T = \frac{g_m}{\left(C_{gs} + C_{gd}\right)} = 2\pi f_T$$

f_T is an important figure of merit for a transistor.

$$f_T\left(\max\right) = \frac{1}{2\pi t_t}$$

$$t_t = \frac{L}{\langle \upsilon \rangle}$$