Announcements

1) PowerPoint Lecture slides are available at:
   https://nanohub.org/groups/ece255_2019

2) Information on LTspice also available there

3) HW1 due today at 5:00 PM in MSEE 180 dropbox

4) HW2 due Tuesday, 1/22
In-class exercise

Take out a sheet of paper, put your name and ID# on it, and solve this problem:

We have a semiconductor with $n_i = 1.0 \times 10^{12} \text{ cm}^{-3}$ and we determine that $p = 1.0 \times 10^6 \text{ cm}^{-3}$.

What is the doping density?
Current flow in semiconductors

1) Voltage and electron energy, voltage and electron concentration, electric field, thermal velocity
2) Drift current
3) Diffusion current
4) The drift-diffusion equation
5) The Einstein relation
Voltage and electron energy

\[ E = \text{constant} - qV \]

A positive potential lowers the energy of an electron.
Voltage and electron concentration

\[ f(E) \, dE = e^{-E/k_B T} \]

\[ n \propto e^{qV/k_B T} \]

\[ n = n_i e^{qV/k_B T} \]

\[ np = n_i^2 \]

\[ p = n_i e^{-qV/k_B T} \]

\[ n = n_i e^{qV/k_B T} \]

\[ p = n_i e^{-qV/k_B T} \]

(equilibrium)
The electric field is minus the gradient of the electrostatic potential.
\[
\langle E \rangle = \frac{3}{2} k_B T = 0.039 \text{ eV}
\]
Thermal velocity

random thermal motion

\[ \langle v \rangle = 0 \]

\[ v_{rms} \approx 10^7 \text{ cm/s} \]

\[ \langle KE \rangle = \frac{3}{2} k_B T \]

\[ \langle KE \rangle = \frac{1}{2} m^* \langle v^2 \rangle \]

\[ \sqrt{\langle v^2 \rangle} = v_{rms} = \sqrt{\frac{3k_B T}{m^*}} \]
What current flows?

\[ \mathcal{E} \text{ V/cm} \]

\[ I = ? \]

N-type

area \[ A \text{ cm}^2 \]

Lundström: 2019
Newton’s Law

\[ F = m_0 a = \frac{dp}{dt} \]
\[ v(t) \quad x(t) \quad p(t) = m_0 v(t) \]

For electrons (and holes) in semiconductors, the electron rest mass in vacuum, \( m_0 \), must be replaced by an **effective** mass.

\[ F = m^*_n a = \frac{dp}{dt} \]
\[ v(t) \quad x(t) \quad p(t) = m^*_n v(t) \]
Drift velocities in silicon

\[ \frac{dp}{dt} = F_e - \frac{p}{\tau_n} = -qE - \frac{p}{\tau_n} \]

steady-state: \[ p = m_n^* \nu_{dn} = -qE \tau_n \]

\[ \nu_{dn} = -\left( q \frac{\tau_n}{m_n^*} \right) E \]

\[ \nu_{dn} = -\mu_n E \]

\( \mu_n: \) “mobility”
Drift velocities in Si

\[ v_{dn} = -\mu_n E \quad \mu_n = 1360 \frac{\text{cm}^2}{\text{V-s}} \]

\[ v_{dp} = +\mu_p E \quad \mu_p = 480 \frac{\text{cm}^2}{\text{V-s}} \]

pure (undoped) Si at 300 K

Assume a modest strength electric field

\[ E = 100 \frac{\text{V}}{\text{cm}} \]

\[ v_{dn} = -1.4 \times 10^5 \frac{\text{cm}}{\text{s}} \quad v_{dp} = +0.5 \times 10^5 \frac{\text{cm}}{\text{s}} \quad v_{rms} \approx 1 \times 10^7 \frac{\text{cm}}{\text{s}} \]
Drift vs. thermal velocities

1) random walk with a small bias from left to right

2) assume that electrons “drift” to the right at an average velocity, $v_d$

$$v_{dnx} = -\mu_n E_x$$

3) what is $I$?
Drift current (electrons)

\[ \nu_{dn} = -\mu_n E \]

\[ I_n = n(-q)\nu_{dn}A = (+nq\mu_n E)A \quad \text{Amperes} \]

\[ J_n = \frac{I_n}{A} = (nq\mu_n)E \quad \text{A/cm}^2 \]

\[ J_n = \sigma_n E \quad \text{A/cm}^2 \]

\( \sigma_n \): conductivity
Drift current (holes)

\[ \nu_{dp} = +\mu_p E \]

\[ I_p = p(+q)\nu_{dp} A = (pq\mu_p E) A \quad \text{Amperes} \]

\[ J_p = \frac{I_p}{A} = (pq\mu_p E) \frac{A}{\text{cm}^2} \]

\[ J_p = \sigma_p E \frac{A}{\text{cm}^2} \quad \sigma_p: \text{conductivity} \]
Total drift current

\[ J_p = \sigma_p \mathcal{E} \text{ A/cm}^2 \quad J_n = \sigma_n \mathcal{E} \text{ A/cm}^2 \]

\[ J = J_n + J_p = (\sigma_n + \sigma_p) \mathcal{E} = \sigma \mathcal{E} \text{ A/cm}^2 \]

\[ \sigma_p = pq\mu_p \]

\[ \sigma_n = nq\mu_n \]

\[ \sigma = (\sigma_n + \sigma_p) \quad \text{conductivity} \]

\[ \rho = \frac{1}{\sigma} = \frac{1}{(\sigma_n + \sigma_p)} \quad \text{\Omega-cm resistivity} \]
Electron diffusion current

\[ \mathcal{F}_e = -D_n \frac{dn}{dx} \text{ cm}^2 \text{-s} \]

\( D_n \text{ cm}^2 / \text{s} \)

(diffusion coefficient)

(Adolph Fick, 1855)

\[ J_n = -q \mathcal{F}_e = +qD_n \frac{dn}{dx} \text{ A/cm}^2 \]

diffusion current
Hole diffusion current

\[ J_p = +q \mathcal{F}_h = -qD_p \frac{dp}{dx} \text{ A/cm}^2 \]

(diffusion current)

\[ \mathcal{F}_h = -D_p \frac{dp}{dx} \frac{#}{\text{cm}^2\text{-s}} \]

(diffusion coefficient)

(Adolph Fick, 1855)
Electron drift-diffusion equation

\[ J_{n \text{-drift}} = -nq\nu_{dn} = +nq\mu_n E \text{ A/cm}^2 \]

drift current

\[ J_{n \text{-diff}} = -qF_e = +qD_n \frac{dn}{dx} \text{ A/cm}^2 \]

diffusion current

\[ J_n = J_{n \text{-drift}} + J_{n \text{-diff}} = nq\mu_n E + qD_n \frac{dn}{dx} \]
Einstein relation

\[ J_n = n q \mu_n \mathcal{E} + q D_n \frac{dn}{dx} \]

equilibrium: \[ J_n = 0 \]

\[ n = n_i e^{qV/k_B T} \]

\[ \mathcal{E} = -\frac{dV}{dx} \]

\[ 0 = n_i e^{qV/k_B T} q \mu_n \left( -\frac{dV}{dx} \right) + q D_n \frac{d}{dx} \left( n_i e^{qV/k_B T} \right) \]

\[ \frac{D_n}{\mu_n} = \frac{k_B T}{q} \]

(Einstein relation)
Diffusion coefficients in Si

\[ \mu_n = 1360 \text{ cm}^2 \text{V}^{-1}\text{s} \]

\[ \mu_p = 480 \text{ cm}^2 \text{V}^{-1}\text{s} \]

pure (undoped) Si at 300 K

\[ \frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = \frac{k_B T}{q} = 0.026 \text{ V} \]

\[ D_n = \frac{k_B T}{q} \mu_n = 35.4 \text{ cm}^2 \text{s}^{-1} \]

\[ D_p = \frac{k_B T}{q} \mu_p = 12.5 \text{ cm}^2 \text{s}^{-1} \]
Drift-diffusion equations

\[ \vec{J}_p = pq\mu_p \vec{E} - qD_p \vec{\nabla}p \]

\text{current} = \text{drift current} + \text{diffusion current}

\[ \vec{J}_n = nq\mu_n \vec{E} + qD_n \vec{\nabla}n \]

\text{total current} = \text{electron current} + \text{hole current}

\[ \vec{J} = \vec{J}_p + \vec{J}_n \]

\[ \mu_p = \frac{q\tau_p}{m_p^*} \quad \mu_n = \frac{q\tau_n}{m_n^*} \]

\[ D_p/\mu_p = D_n/\mu_n = k_B T/q \]

(Einstein, 1905)
The unknowns

3 unknowns

\[ p(\vec{r}), n(\vec{r}), V(\vec{r}) \]

We need to formulate 3 equations in 3 unknowns.
First equation: Continuity equation for holes

\[ \frac{\partial p}{\partial t} = \text{in-flow} - \text{out-flow} + G - R \]

\[ \frac{\partial (qp)}{\partial t} = -\nabla \cdot \vec{J}_p + q \left( G_p - R_p \right) \]
Gauss’s Law

\[ \vec{D} = \varepsilon_0 \vec{E} \]

\[ \vec{D} = \kappa_s \varepsilon_0 \vec{E} \]

\[ \oint \vec{D} \cdot d\vec{S} = Q \]

\[ \nabla \cdot \vec{D} = \rho(x) \]
“The Semiconductor Equations”

\[
\frac{\partial p}{\partial t} = -\nabla \cdot \left( \frac{\vec{J}_p}{q} \right) + G_p - R_p \\
\frac{\partial n}{\partial t} = -\nabla \cdot \left( \frac{\vec{J}_n}{-q} \right) + G_n - R_n \\
\nabla \cdot (\kappa \varepsilon_0 \vec{E}) = \rho 
\]

Three equations in three unknowns:

\[ p(\vec{r}), n(\vec{r}), V(\vec{r}) \]

\[
\vec{J}_p = pq\mu_p \vec{E} - qD_p \vec{\nabla}p \\
\vec{J}_n = nq\mu_n \vec{E} + qD_n \vec{\nabla}n \\
\rho = q(p - n + N_D^+ - N_A^-) \\
\vec{E}(\vec{r}) = -\nabla V(\vec{r})
\]
Dopants cannot diffuse
Holes do diffuse
What stops hole diffusion?

\[ J_{p\text{-diff}} = -qD_p \frac{dp}{dx} \]
Summary

1) Electrostatic potentials (voltage) lower electron energy.
2) The electric field is minus the gradient of the electrostatic potential.
3) The random thermal velocity is large.

Electric fields produce drift velocities and drift currents.

Concentration gradients produce diffusion currents.

In general, there are both drift and diffusion currents.

Mobility and diffusion coefficient are related by the Einstein relation.
Current flow in semiconductors

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