Spring 2019 Purdue University

ECE 255: L4

Current Flow in Semiconductors

(Sedra and Smith, 7th Ed., Sec. 3.3)

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Announcements

1) PowerPoint Lecture slides are available at:

   https://nanohub.org/groups/ece255_2019

2) Information on LTspice also available there

3) HW1 due today at 5:00 PM in MSEE 180 dropbox

4) HW2 due **Tuesday**, 1/22

In-class exercise

Take out a sheet of paper, put your name and ID# on it, and solve this problem:

We have a semiconductor with $n_i = 1.0 \times 10^{12}$ cm^{-3} and we determine that p = 1.0×10^{6} cm^{-3}.

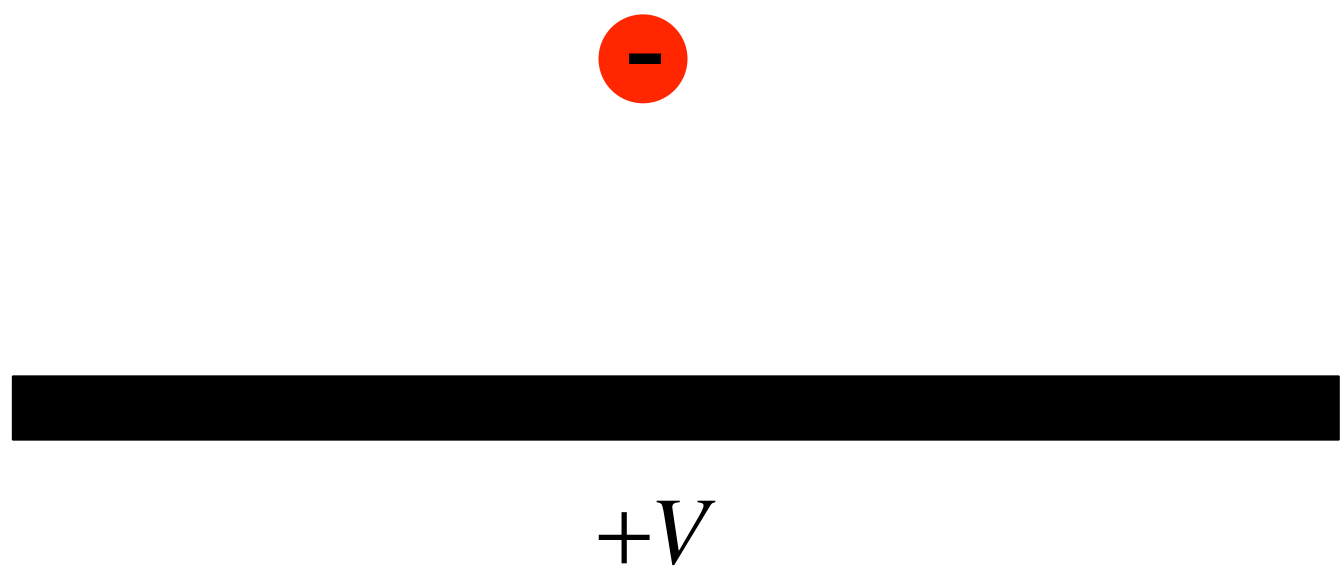
What is the doping density?

Current flow in semiconductors

1) Voltage and electron energy, voltage and electron concentration, electric field, thermal velocity
2) Drift current
3) Diffusion current
4) The drift-diffusion equation
5) The Einstein relation

Voltage and electron energy

$$E = \text{constant} - qV$$



A positive potential **lowers** the energy of an electron.

Voltage and electron concentration

$$f(E)\,dE = e^{-E/k_BT}$$

$$n \propto e^{qV/k_BT}$$

$$n = n_i e^{qV/k_BT}$$

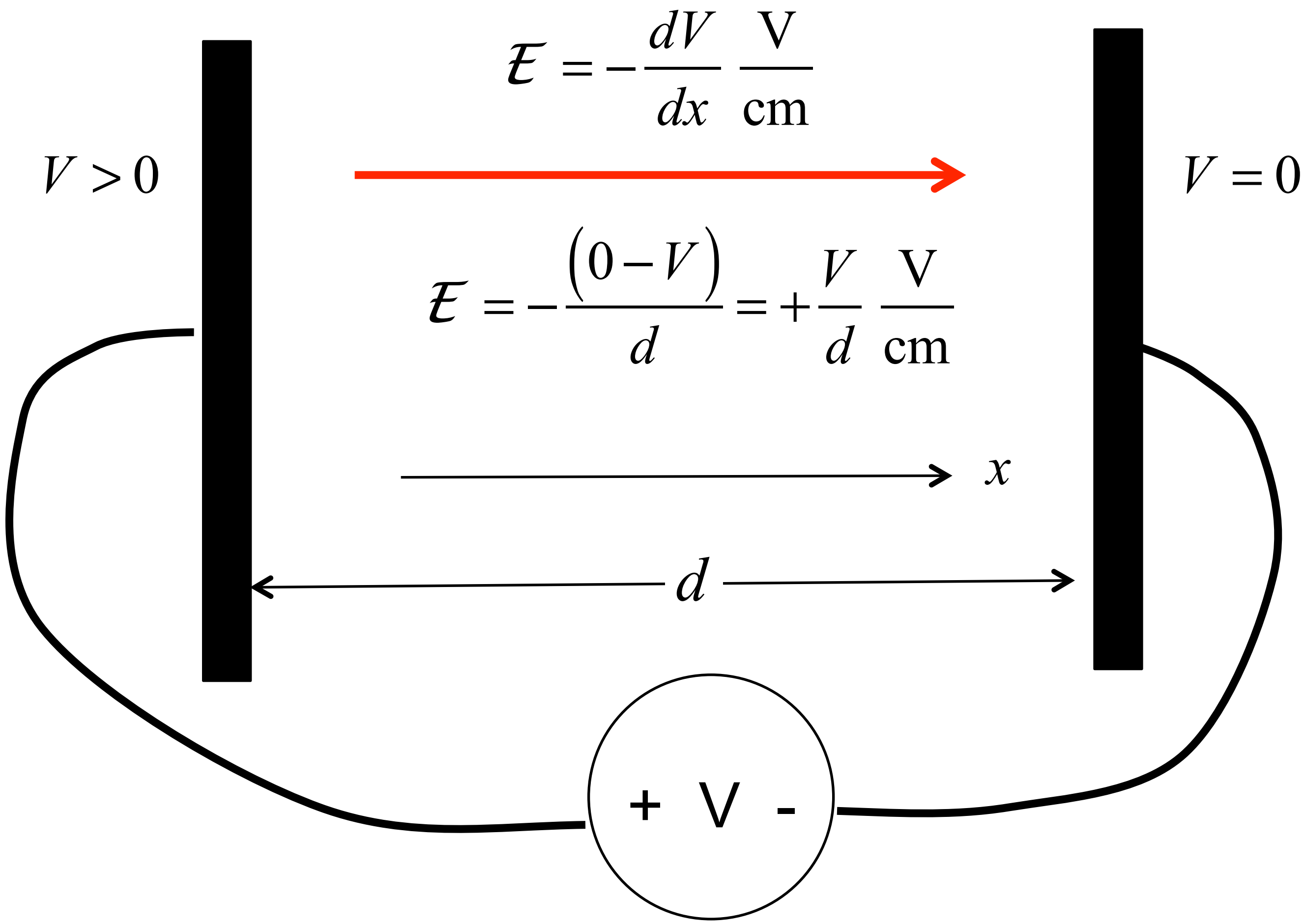
$$np = n_i^2$$

$$p = n_i e^{-qV/k_BT}$$

$$\boxed{\begin{gathered} n = n_i e^{qV/k_BT} \\ p = n_i e^{-qV/k_BT} \end{gathered}}$$

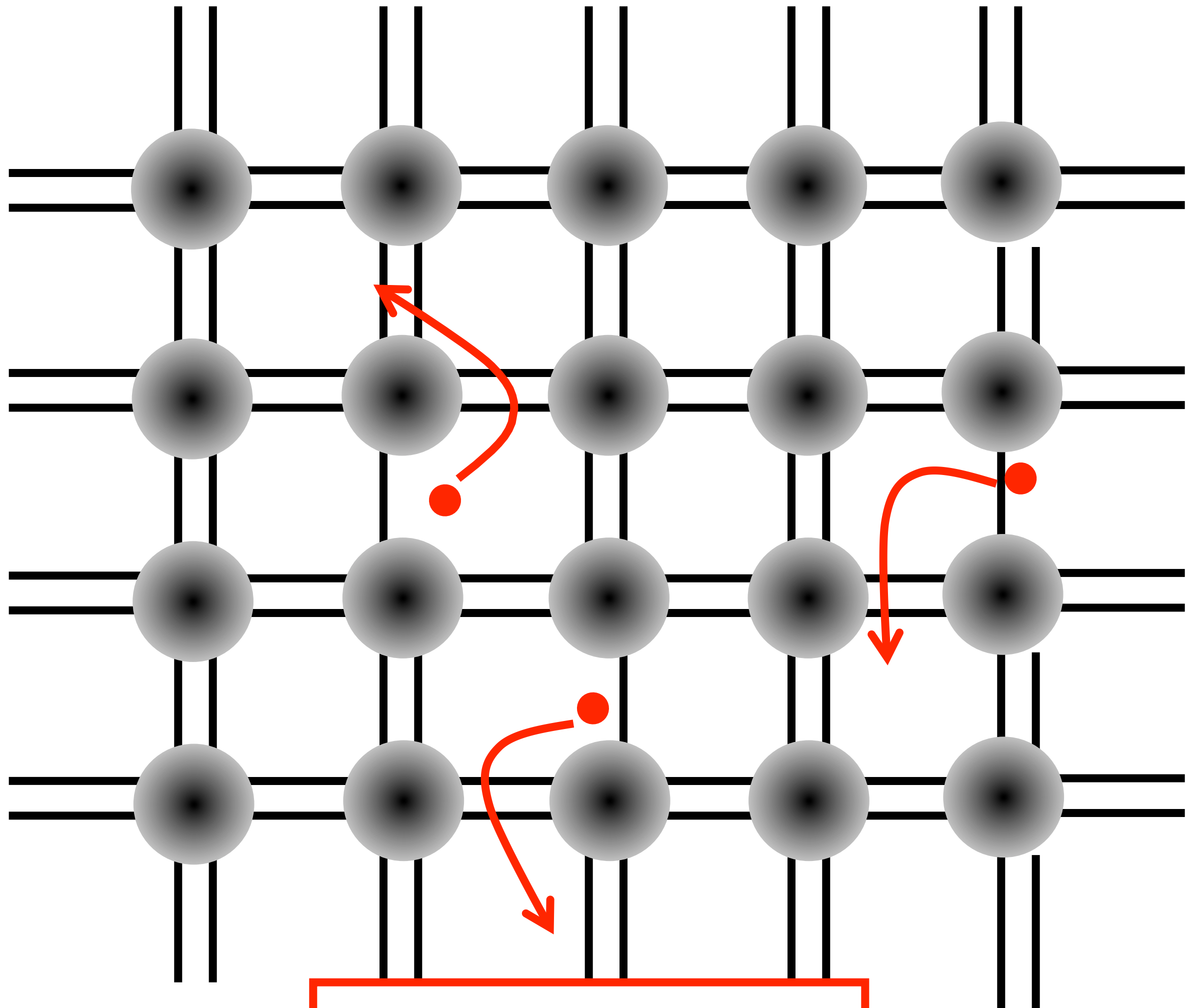
(equilibrium)

Electric field and voltage



The electric field is minus the gradient of the electrostatic potential.

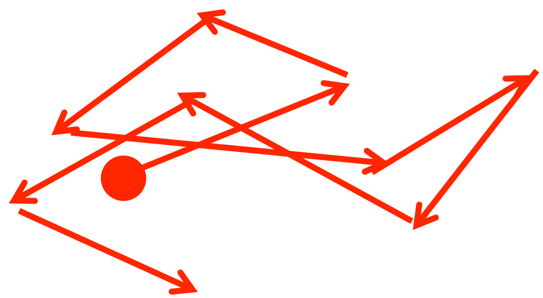
Thermal velocity



$$\langle E \rangle = \frac{3}{2} k_B T = 0.039 \text{ eV}$$

Thermal velocity

random thermal motion



$$\langle \upsilon \rangle = 0$$

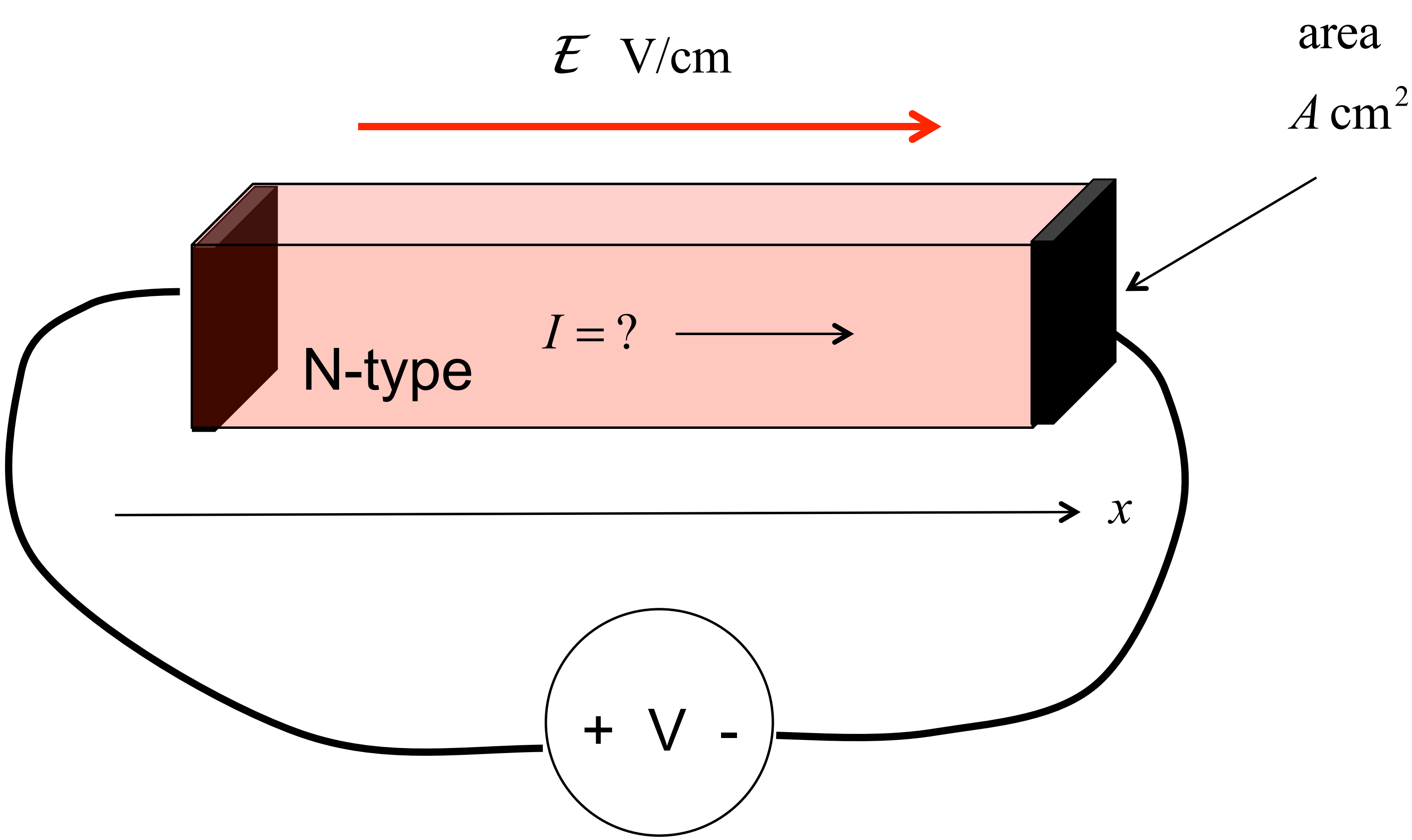
$$\upsilon_{rms} \approx 10^7 \text{ cm/s}$$

$$\langle KE \rangle = \frac{3}{2} k_B T$$

$$\langle KE \rangle = \frac{1}{2} m_n^* \langle \upsilon^2 \rangle$$

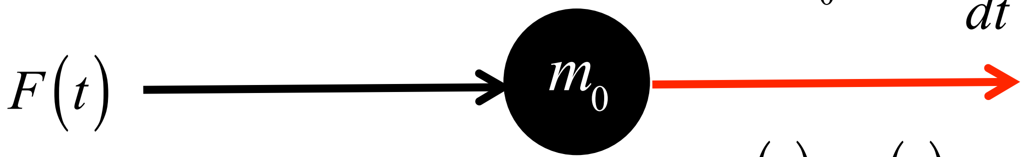
$$\sqrt{\langle \upsilon^2 \rangle} = \upsilon_{rms} = \sqrt{\frac{3k_B T}{m_n^*}}$$

What current flows?



Newton’s Law

$$F = m_0 a \;= \frac{dp}{dt}$$



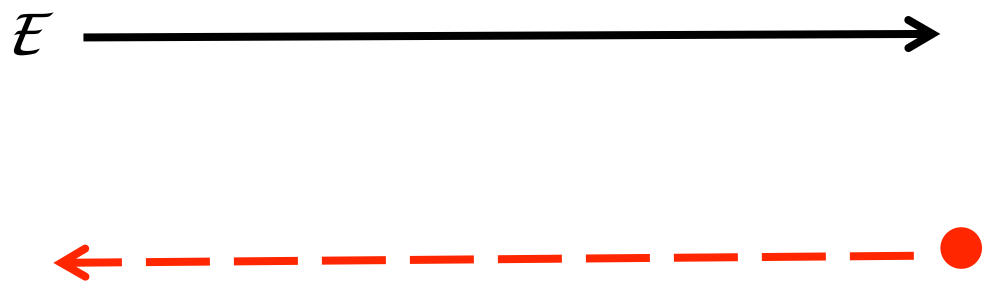
$$\upsilon(t) \quad x(t) \quad p(t) = m_0 \upsilon(t)$$

For electrons (and holes) in semiconductors, the electron rest mass in vacuum, *m_0*, must be replaced by an **effective mass**.

$$F = m_n^* a \;= \frac{dp}{dt}$$

$$\upsilon(t) \quad x(t) \quad p(t) = m_n^* \upsilon(t)$$

Drift velocities in silicon



$$F_e = -q\mathcal{E}$$

$$\frac{dp}{dt} = F_e - \frac{p}{\tau_n} = -q\mathcal{E} - \frac{p}{\tau_n}$$

steady-state:

$$p = m_n^* \upsilon_{dn} = -q\mathcal{E}\tau_n$$

$$\upsilon_{dn} = -\left(\frac{q\tau_n}{m_n^*}\right)\mathcal{E}$$

$$\boxed{\upsilon_{dn} = -\mu_n \mathcal{E}}$$

μ_n: “mobility”

Drift velocities in Si

$$\upsilon_{dn} = -\mu_n \mathcal{E} \qquad \mu_n = 1360 \ \frac{\text{cm}^2}{\text{V-s}}$$

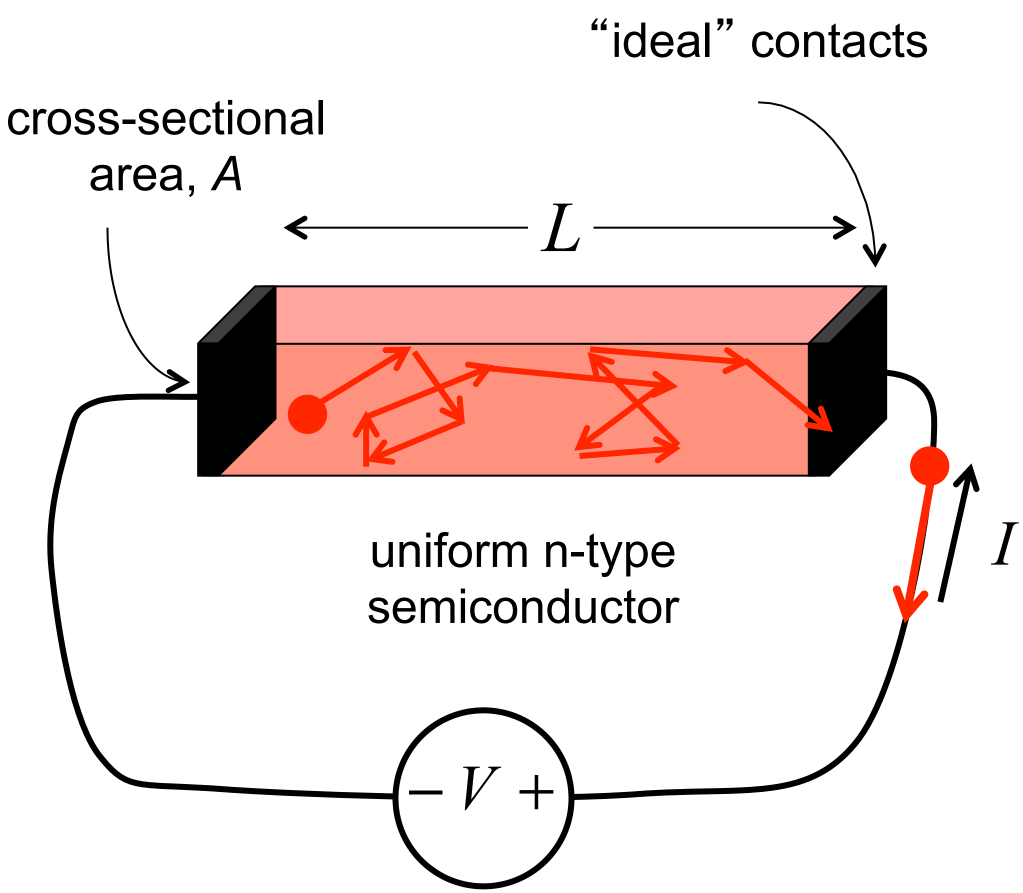
$$\upsilon_{dp} = +\mu_p \mathcal{E} \qquad \mu_p = 480 \ \frac{\text{cm}^2}{\text{V-s}}$$

pure (undoped) Si at 300 K

Assume a modest strength electric field $\mathcal{E} = 100 \ \frac{\text{V}}{\text{cm}}$

$$\upsilon_{dn} = -1.4 \times 10^5 \ \frac{\text{cm}}{\text{s}} \qquad \upsilon_{dp} = +0.5 \times 10^5 \ \frac{\text{cm}}{\text{s}} \qquad \upsilon_{rms} \approx 1 \times 10^7 \ \frac{\text{cm}}{\text{s}}$$

Drift vs. thermal velocities



1) random walk with a small bias from left to right

2) assume that electrons "drift" to the right at an average velocity, υ_d

$$\upsilon_{dnx} = -\mu_n \mathcal{E}_x$$

3) what is *I* ?

Drift current (electrons)

$$\upsilon_{dn} = -\mu_n \mathcal{E}$$

$$I_n = n(-q)\upsilon_{dn} A = (+nq\mu_n \mathcal{E})A \quad \text{Amperes}$$

$$J_n = \frac{I_n}{A} = (nq\mu_n)\mathcal{E} \ \text{A/cm}^2$$

$$\boxed{J_n = \sigma_n \mathcal{E} \ \text{A/cm}^2}$$

σ_n: conductivity

Drift current (holes)

$$\upsilon_{dp} = +\mu_p \mathcal{E}$$

$$I_p = p(+q)\upsilon_{dp} A = \left(+pq\mu_p \mathcal{E}\right) A \quad \text{Amperes}$$

$$J_p = \frac{I_p}{A} = \left(pq\mu_p\right)\mathcal{E} \ \text{A/cm}^2$$

$$\boxed{J_p = \sigma_p \mathcal{E} \ \text{A/cm}^2}$$

σ_p: conductivity

Total drift current

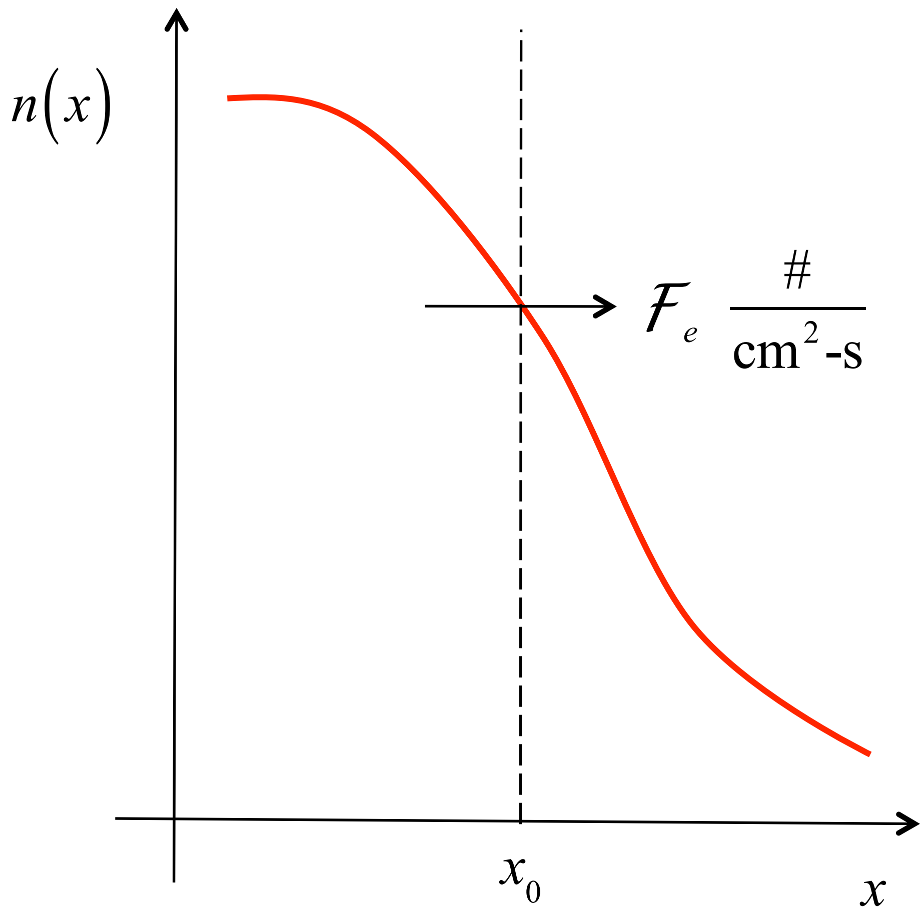
$$J_p = \sigma_p \mathcal{E} \text{ A/cm}^2 \qquad J_n = \sigma_n \mathcal{E} \text{ A/cm}^2$$

$$J = J_n + J_p = \left(\sigma_n + \sigma_p\right)\mathcal{E} = \sigma \mathcal{E} \quad \text{A/cm}^2$$

$$\sigma_p = pq\mu_p \qquad \sigma = \left(\sigma_n + \sigma_p\right) \qquad \text{conductivity}$$

$$\sigma_n = nq\mu_n \qquad \rho = \frac{1}{\sigma} = \frac{1}{\left(\sigma_n + \sigma_p\right)} \ \Omega\text{-cm} \qquad \text{resistivity}$$

Electron diffusion current



$$\mathcal{F}_e = -D_n \frac{dn}{dx} \qquad \frac{\#}{\text{cm}^2\text{-s}}$$

$$D_n \qquad \text{cm}^2/\text{s}$$

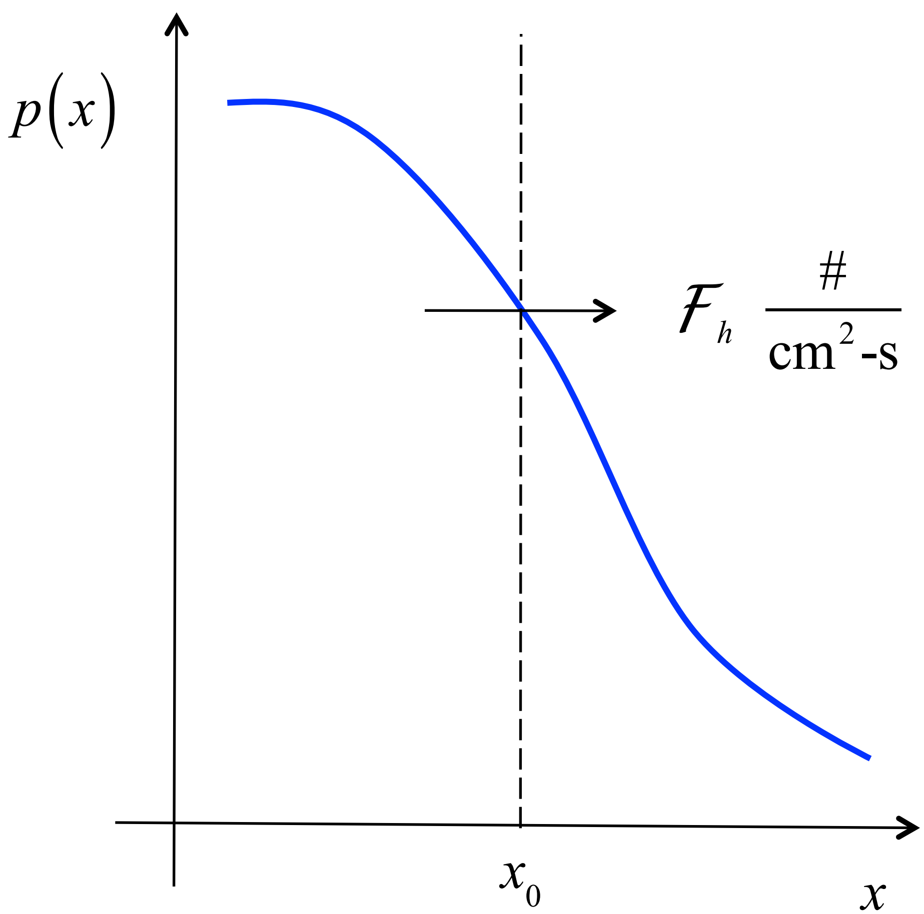
(diffusion coefficient)

(Adolph Fick, 1855)

$$J_n = -q\mathcal{F}_e = +qD_n \frac{dn}{dx} \ \text{A/cm}^2$$

diffusion current

Hole diffusion current



$$\mathcal{F}_h = -D_p \frac{dp}{dx} \qquad \frac{\#}{\text{cm}^2\text{-s}}$$

$$D_p \qquad \text{cm}^2/\text{s}$$

(diffusion coefficient)

(Adolph Fick, 1855)

$$J_p = +q\mathcal{F}_h = -qD_p \frac{dp}{dx} \ \text{A/cm}^2$$

diffusion current

Electron drift-diffusion equation

$$J_{n-drift} = -nq\upsilon_{dn} = +nq\mu_n \mathcal{E} \text{ A/cm}^2$$

drift current

$$J_{n-diff} = -q\mathcal{F}_e = +qD_n \frac{dn}{dx} \text{ A/cm}^2$$

diffusion current

$$\boxed{J_n = J_{n-drift} + J_{n-diff} = nq\mu_n \mathcal{E} + qD_n \frac{dn}{dx}}$$

Einstein relation

$$J_n = nq\mu_n\mathcal{E} + qD_n\frac{dn}{dx}$$

equilibrium: $J_n = 0$

$$n = n_i e^{qV/k_BT}$$

$$\mathcal{E} = -\frac{dV}{dx}$$

$$0 = n_i e^{qV/k_BT} q\mu_n\left(-\frac{dV}{dx}\right) + qD_n\frac{d\left(n_i e^{qV/k_BT}\right)}{dx}$$

$$\boxed{\frac{D_n}{\mu_n} = \frac{k_BT}{q}}$$

(Einstein relation)

Diffusion coefficients in Si

$$\mu_n = 1360 \ \frac{\text{cm}^2}{\text{V-s}}$$

$$\mu_p = 480 \ \frac{\text{cm}^2}{\text{V-s}}$$

pure (undoped) Si at 300 K

$$\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = \frac{k_B T}{q} = 0.026 \text{ V}$$

$$D_n = \frac{k_B T}{q} \mu_n = 35.4 \ \frac{\text{cm}^2}{\text{s}} \qquad D_p = \frac{k_B T}{q} \mu_p = 12.5 \ \frac{\text{cm}^2}{\text{s}}$$

Drift-diffusion equations

$$\vec{J}_p = pq\mu_p\vec{\mathcal{E}} - qD_p\vec{\nabla}p$$

current = drift current + diffusion current

$$\vec{J}_n = nq\mu_n\vec{\mathcal{E}} + qD_n\vec{\nabla}n$$

total current = electron current + hole current

$$\vec{J} = \vec{J}_p + \vec{J}_n$$

$$\mu_p = \frac{q\tau_p}{m_p^*} \qquad \mu_n = \frac{q\tau_n}{m_n^*} \qquad D_p/\mu_p = D_n/\mu_n = k_BT/q$$

(Einstein, 1905)

The unknowns

3 unknowns

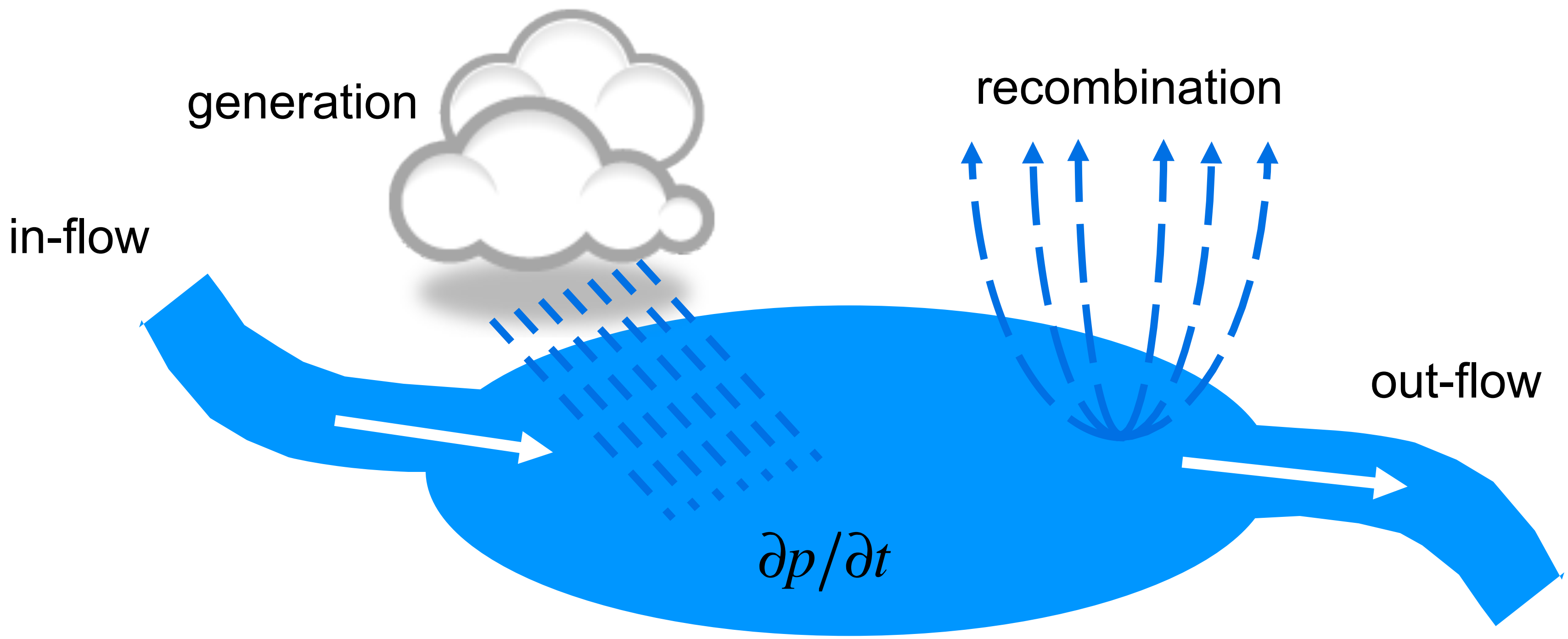
$$p(\vec{r}), n(\vec{r}), V(\vec{r})$$

We need to formulate 3 equations in 3 unknowns.

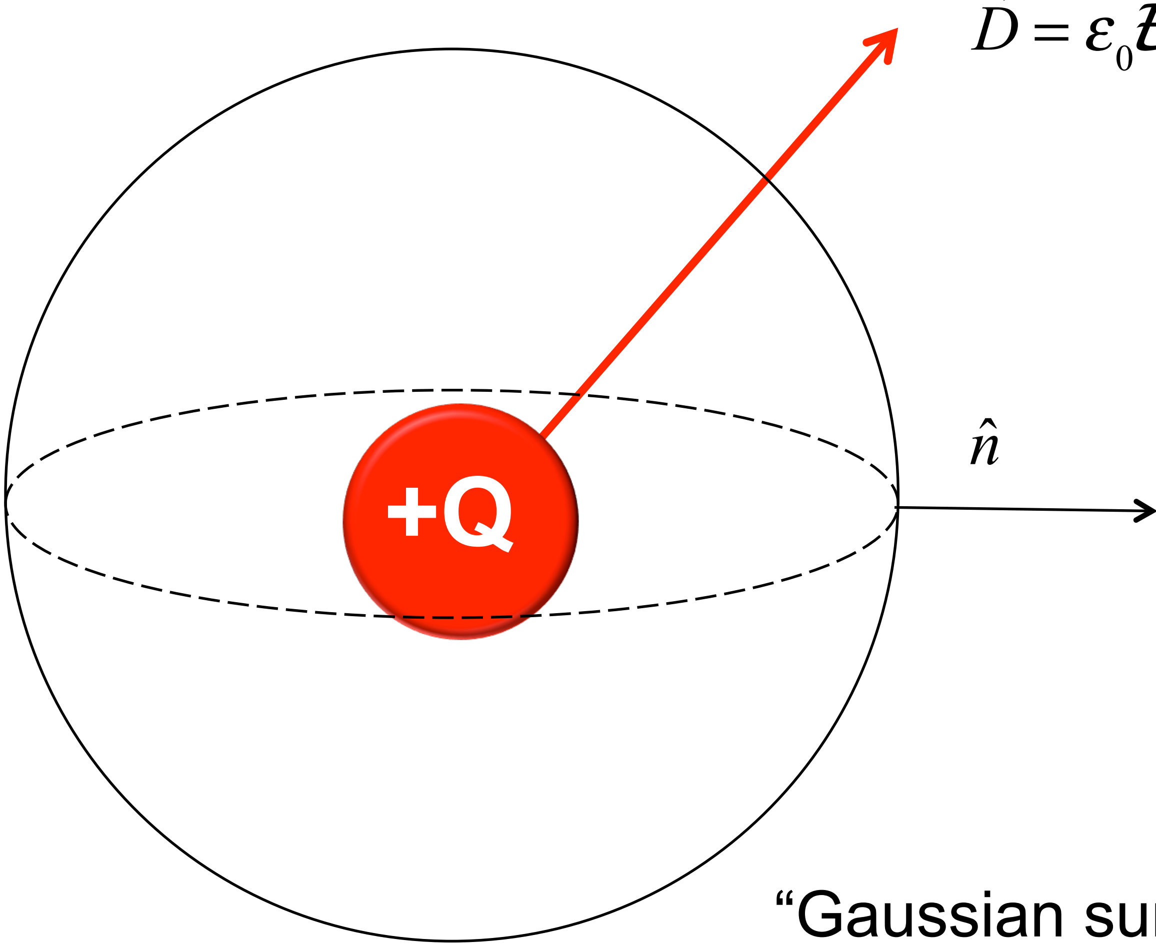
First equation: Continuity equation for holes

$$\frac{\partial p}{\partial t} = \text{in-flow - out-flow} + G - R$$

$$\frac{\partial (qp)}{\partial t} = -\nabla \bullet \vec{J}_p + q\left(G_p - R_p\right)$$



Gauss's Law



$$\vec{D} = \varepsilon_0 \vec{\mathcal{E}} \qquad \vec{D} = \kappa_S \varepsilon_0 \vec{\mathcal{E}}$$

$\hat{n}$

$$\boxed{\begin{aligned} &\oint \vec{D} \bullet d\vec{S} = Q \\ &\nabla \bullet \vec{D} = \rho(x) \end{aligned}}$$

"Gaussian surface"

"The Semiconductor Equations"

$$\frac{\partial p}{\partial t} = -\nabla \bullet \left(\frac{\vec{J}_p}{q} \right) + G_p - R_p$$

$$\frac{\partial n}{\partial t} = -\nabla \bullet \left(\frac{\vec{J}_n}{-q} \right) + G_n - R_n$$

$$\nabla \bullet \left(\kappa_S \varepsilon_0 \vec{\mathcal{E}} \right) = \rho$$

Three equations in three unknowns:

$$p(\vec{r}),\ n(\vec{r}),\ V(\vec{r})$$

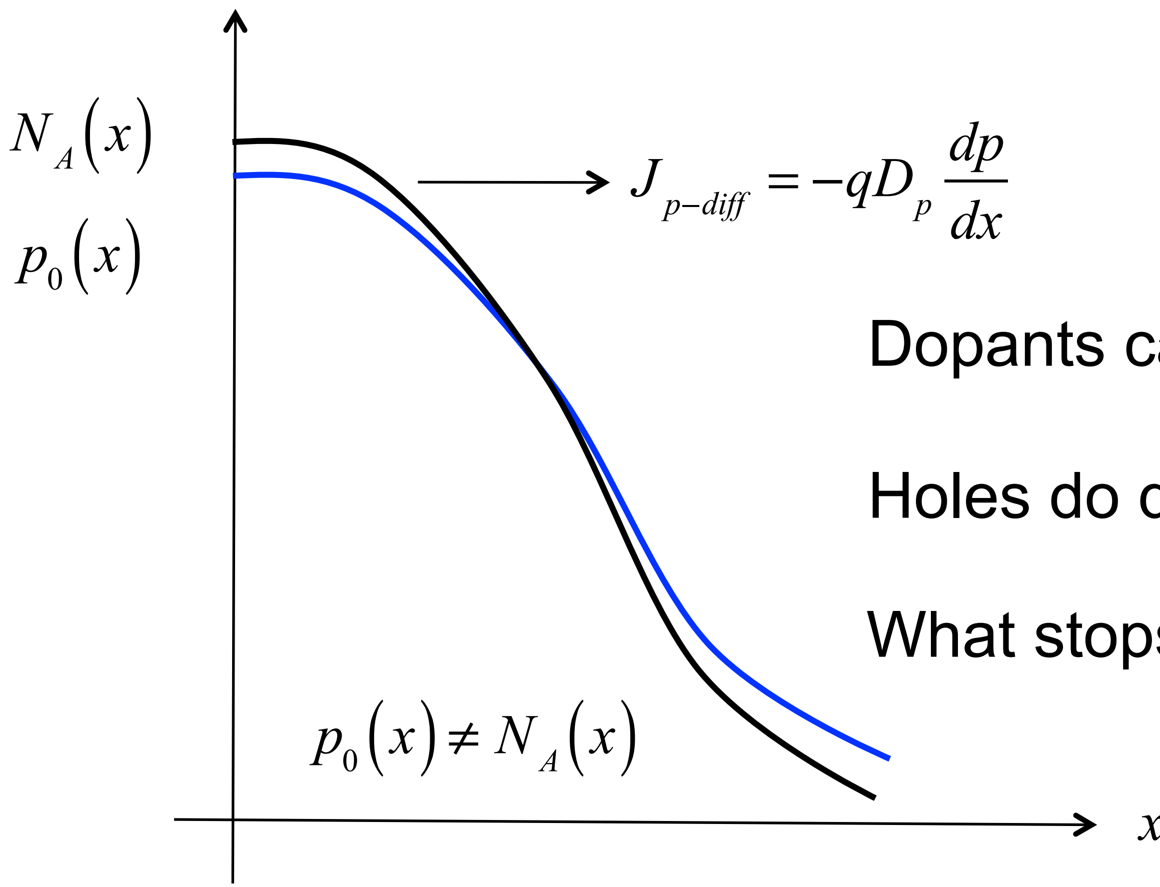
$$\vec{J}_p = pq\mu_p \vec{\mathcal{E}} - qD_p \vec{\nabla} p$$

$$\vec{J}_n = nq\mu_n \vec{\mathcal{E}} + qD_n \vec{\nabla} n$$

$$\rho = q\left(p - n + N_D^+ - N_A^- \right)$$

$$\vec{\mathcal{E}}(\vec{r}) = -\nabla V(\vec{r})$$

Question



Dopants cannot diffuse

Holes do diffuse

What stops hole diffusion?

Summary

1) Electrostatic potentials (voltage) lower electron energy energies. 2) The electric field is minus the gradient of the electrostatic potential. 3) The random thermal velocity is large.

Electric fields produce drift velocities and drift currents.

Concentration gradients produce diffusion currents

In general, there are both drift and diffusion currents.

Mobility and diffusion coefficient are related by the Einstein relation.

Current flow in semiconductors

1) Voltage and electron energy, voltage and electron concentration, electric field, thermal velocity
2) Drift current
3) Diffusion current
4) The drift-diffusion equation
5) The Einstein relation