ECE 255: L7

Ideal and Real Diodes
(Sedra and Smith, 4.1-4.2)

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Announcements

No office hours today.
(Remember, you can always email me to set up another time.)

Exam 1: Thursday, Feb. 7, 6:30 PM, LILY 1105.
(Weeks -1- 4 topics, semiconductors, diodes, BJTs)
Ideal and Real Diodes

1) Ideal vs. real diodes
2) Circuit analysis using ideal diodes
3) Circuit analysis using “real” diodes
Ideal vs. real diodes

The exponential model is a reasonably good model for real diodes.

$$I_D = I_S \left( e^{\frac{qV_D}{k_B T}} - 1 \right)$$

$$V_T = k_B T / q$$

$$V_T \left( 300 \text{ K} \right) = 0.026 \text{ V}$$

$$\approx 0.6 - 0.8 \text{ V}$$

(silicon)
“Ideal” diode model

\[ I_D = I_s \left( e^{V_D/V_T} - 1 \right) \]

\[ V_D = 0 \]

\[ V_D < 0 \]

\[ I_D \approx 0.7 \text{V} \]
Constant-voltage-drop model

\[ + V_D - \]

\[ V_D \geq 0.7 \text{ V} \]

\[ V_D < 0.7 \text{ V} \]

\[ I_D = I_S \left( e^{V_D/V_T} - 1 \right) \]

\[ \approx 0.7 \text{ V} \]

Constant voltage drop model

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Constant-voltage-drop model

\[ + V_D - \]

\[ + 0.7 \text{ V} - \]

\[ \text{Ideal diode} \]

\[ \text{Ideal battery} \]

\[ V_D \geq 0.7 \]

\[ + 0.7 - \]

\[ V_D < 0.7 \]
A simple circuit to analyze

\[ V_D = I_D \times R \]

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Let’s make it even simpler

\[ +10 \text{ V} \quad 1\,\text{k}\Omega \quad - \]

\[ V_D \quad I_D \]
Analyze the circuit

\[ V_{th} = 5 \text{ V} \]

\[ R_{th} = 0.5 \text{ k}\Omega \]

\[ I_D \]

1 Equation in 2 unknowns

\[ V_{th} - I_D R_{th} - V_D = 0 \]

Need an equation for \( V_D(I_D) \) or \( I_D(V_D) \)
1) Ideal diode analysis ($V_D = 0$)

$R_{th} = 0.5 \text{ k}\Omega$

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$V_{th} = 5 \text{ V}$

$I_D = \frac{V_{th}}{R_{th}} = 10 \text{ mA}$

($\text{Exact: } I_D = 8.61 \text{ mA}$)
2) Constant-voltage-drop model ($V_D = 0.7$ V)

\[ R_{th} = 0.5 \text{ k}\Omega \]

\[ I_D = \left(5 - 0.7\right) \text{V} / 0.5 \text{ k}\Omega = 8.6 \text{ mA} \]

(Exact: $I_D = 8.61 \text{ mA}$)
In this case, our model was a very good approximation to the actual diode.

\[ I_D = I_S \left( e^{V_D/V_T} - 1 \right) \approx 0.7 \text{ V} \]
Analyzing diode circuits

\[ V_{th} - I_D R_{th} - V_D = 0 \]

1) Ideal diode model \((V_D = 0)\)

2) Constant-diode-drop model \((V_D \sim 0.7)\)

3) Diode equation?

Need another equation for \(V_D(I_D)\) or \(I_D(V_D)\)
Examples (assume the constant-V-drop model)

Find $V_0$ and the two diode currents

a) $V_1 = 10 \text{ V}, V_2 = 0 \text{ V}$
b) $V_1 = 5 \text{ V}, V_2 = 0 \text{ V}$
c) $V_1 = 10 \text{ V}, V_2 = 5 \text{ V}$
d) $V_1 = 10 \text{ V}, V_2 = 10 \text{ V}$

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a) $V_1 = 10\text{ V}, V_2 = 0\text{ V}$
c) $V_1 = 10\text{ V}, V_2 = 5\text{ V}$
d) $V_1 = 10$ V, $V_2 = 10$ V
Examples: Answers

Find $V_0$ and the two diode currents

\[ \begin{align*}
V_1 & = 10 \text{ V}, V_2 = 0 \text{ V} \\
V_1 & = 5 \text{ V}, V_2 = 0 \text{ V} \\
V_1 & = 10 \text{ V}, V_2 = 5 \text{ V} \\
V_1 & = 10 \text{ V}, V_2 = 10 \text{ V}
\end{align*} \]

\[ \begin{align*}
I_{D1} & = 0.93 \text{ mA}, I_{D2} = 0 \text{ mA} \quad V_0 = 8.8 \text{ V} \\
I_{D1} & = 0.43 \text{ mA}, I_{D2} = 0 \text{ mA} \quad V_0 = 4.1 \text{ V} \\
I_{D1} & = 0.93 \text{ mA}, I_{D2} = 0 \text{ mA} \quad V_0 = 8.8 \text{ V} \\
I_{D1} & = 0.48 \text{ mA}, I_{D2} = 0.48 \text{ mA} \quad V_0 = 9.1 \text{ V}
\end{align*} \]
Analyze the circuit (exponential model)

\[ V_{th} - I_D R_{th} - V_D = 0 \]

\[ I_D = I_S \left( e^{V_D/V_T} - 1 \right) \]

\[ V_T = k_B T / q \]

Need another equation for \( V_D(I_D) \) or \( I_D(V_D) \)
Analyze the circuit (exponential model)

\[ V_{th} - I_D R_{th} - V_D = 0 \]  \hspace{1cm} (1)

\[ I_D = I_S \left( e^{V_D/V_T} - 1 \right) \]  \hspace{1cm} (2)

\[ I_D = \frac{(V_{th} - V_D)}{R_{th}} \]

\[ I_D = I_S e^{V_D/V_T} \]

\[ I_S e^{V_D/V_T} = \frac{(V_{th} - V_D)}{R_{th}} \]
**Graphical approach**

**Circuit:**

\[ I_D = \frac{(V_{th} - V_D)}{R_{th}} \]

“load line”

**Diode:**

\[ I_D = I_S e^{qV_D/k_B T} \]

\[ I_D \approx I_S e^{V_D/V_T} \]

\[ I_D = \frac{(V_{th} - V_D)}{R_{th}} \]

\[ V_{th} \approx 0.7 \text{ V} \]

Q (operating point)

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For our circuit...

\[ V_{th} = 5 \text{ V} \]

\[ R_{th} = 0.5 \text{ k}\Omega \]

\[ I_D \approx I_S e^{V_D/V_T} \]

\[ I_D = \frac{(V_{th} - V_D)}{R_{th}} \]

\[ \approx 0.72 \text{ V} \]
Analyze the circuit (exponential model)

\[ V_{th} - I_D R_{th} - V_D = 0 \]  

\[ I_D = I_S \left( e^{V_D/V_T} - 1 \right) \]  

\[ I_D = \left( V_{th} - V_D \right) / R_{th} \]  

\[ I_D = I_S e^{V_D/V_T} \] 

\[ I_S e^{V_D/V_T} = \left( V_{th} - V_D \right) / R_{th} \]
Numerical solution

\[ V_{th} - \left( I_S e^{V_D/V_T} \right) R_{th} - V_D = 0 \]

\[ 5 - \left( I_S e^{V_D/0.026} \right) 500 - V_D = 0 \]

\[ I_S = 10^{14} \, A \]

\[ 5 - \left( 10^{-14} e^{V_D/0.026} \right) 500 - V_D = 0 \]

Guess \( V_D \)

Compute LHS

Is LHS = 0?

Try new guess and repeat

\[ V_D \quad \text{LHS} \]

\begin{align*}
0.7 & \quad +1.837 \\
0.72 & \quad -1.036 \\
0.71 & \quad 0.671 \\
0.713 & \quad 0.226 \\
0.714 & \quad 0.065 \\
0.715 & \quad -0.101 \\
\textbf{0.7145} & \quad \textbf{-0.017} \\
\end{align*}

\[ I_D = 10^{-14} e^{0.7145/0.026} = 8.61 \, mA \]
The ideal diode is a very simple, sometimes good enough model for a diode.

The constant-voltage-drop model is better, usually acceptable for first-pass, analysis by hand. The one model parameter, $V_D$, is generally between 0.6 – 0.8 V for Si diodes.

The exponential model is the best. It is nonlinear which makes it harder to use for hand analysis. The one model parameter, $I_S$, needs to be specified.
Real and ideal diodes

1) Ideal vs. “real” diodes
2) Circuit analysis using ideal (or C-V-D) diodes
3) Circuit analysis using real diodes