ECE 255: L8

Modeling Diodes
(Sedra and Smith, 4.3-4.4)

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Lundstrom: 2019
Announcements

Exam 1: Thursday, Feb. 7, 6:30 PM, LILY 1105.
(Weeks -1- 4 topics, semiconductors, diodes, BJTs. i.e. HW1 – HW4.)

Remember: Do the assigned reading before class!
Quiz problem

Assume ideal diodes and find V and all diode currents
Guess:

D1 (FB)  Show why this does not work
D2 (FB)
D3 (FB)
Quiz problem

Guess:

D1 (FB)
D2 (FB)
D3 (FB)

Find:
IR1 = (10-5)/8.2K=0.61 mA
IR2 = (5-0)/12K =0.42 mA
Implies
ID2 < 0
D2 is NOT FB

Show why this does not work
Quiz problem

Guess:

D1 (FB)
D2 (RB)
D3 (FB)

Show that this does work.
Quiz problem

Guess:

D1 (FB)
D2 (RB)
D3 (FB)

Find:

IR1 = (10 – 0) / 20.2 K = 0.495 mA
V = 10 – 0.495x8.2 = 5.94
VD2 < 0 RB
IR3 = (0-(-5))/10 = 0.5 mA
ID3 = IR3 – IR1 – 0.005 mA

Show that this does work.
1) Model parameters
2) Exponential model
3) Small signal model
4) Zener diode model
5) Zener diode applications
Three diode models

- $V_D$ +

Ideal diode

Constant-voltage-drop model

$\approx 0.6 - 0.8 \text{ V (silicon)}$

Exponential model

$I_D = I_S \left( e^{V_D/V_T} - 1 \right)$

Big idea: Complicated devices can be “simply” modeled.
### Model parameters

1) **Ideal diode model**

No model parameters

2) **Constant-voltage-drop model**

One model parameter

\[ V \approx +0.6 - 0.8 \text{V} \]

3) **Exponential model**

One model parameter

\[ I_D = I_S \left( e^{V_D/V_T} - 1 \right) \]
Question: Given a diode for which we want to model with the exponential equation, how do we determine $I_S$?

Answer: Do a measurement at 300 K:

\[ I_D(V_D = 0.7 \, V) = 5 \, mA \]

What is $I_S$?
Computing $I_S$

\[ I_D = I_S \left( e^{V_D/V_T} - 1 \right) \approx I_S e^{V_D/V_T} \quad I_S = I_D e^{-V_D/V_T} \]

\[ V_T = k_B T / q = 0.0259 \text{ V} \]

\[ I_S = 5 \times 10^{-3} e^{-0.7/0.026} \quad I_S = 9.15 \times 10^{-15} \text{ A} \]

But this would not be considered good engineering judgment. Why?

How could we do better?
Determining model parameters

Given a diode for which we want to model with the constant-voltage drop model, how do we determine $V_D$?

First, estimate the current that will result in the circuit.

e.g. $I_D \approx 10 \, \mu A$

Then do a measurement, or compute $V_D$ if we know $I_S$. 
Determining model parameters

Given: \( I_s = 9.15 \times 10^{-15} \text{ A} \)

The expected current in the circuit is about 10 microamps.

Find \( V_D \):

\[
I_D = I_s (e^{V_D/V_T} - 1) \approx I_s e^{V_D/V_T}
\]

\[
V_D = V_T \ln \left( \frac{I_D}{I_s} \right)
\]

\[
V_D = 0.026 \ln \left( \frac{10^{-5}}{9.15 \times 10^{-15}} \right) = 0.54 \text{ V}
\]

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1) Model parameters
2) Exponential model
3) Small signal model
4) Zener diode model
5) Zener diode applications
A question

How much do we need to **increase** $V_D$ to **increase** the current by 10X?

How much do we need to **decrease** $V_D$ to **decrease** the current by 10X?

\[ I_D = I_S e^{qV_D/k_BT} \]

\[ \approx 0.7 \text{ V} \]

\[ 1 \text{ mA} \]
The answer

\[ I_{D1} = I_S e^{V_{D1}/V_T} \]

\[ I_{D2} = I_S e^{V_{D2}/V_T} \]

\[ \frac{I_{D1}}{I_{D2}} = e^{(V_{D1} - V_{D2})/V_T} \]

\[ \Delta V_D = V_T \ln(10) \]

\[ \Delta V_D = 60 \text{ mV/decade} \]
Temperature sensitivity

\[ I_{D1} = I_s e^{V_{D1}/V_T} \]

\[ I_s \propto n_i^2 \propto e^{-E_G/k_B T} \]

How much does the voltage need to change to keep the current constant if T increases by 1 deg C?

**Answer:** \(~ -2 \text{ mV/degC}~\)
Modeling diodes

1) Model parameters
2) Exponential model
3) **Small signal model**
4) Zener diode model
5) Zener diode applications
Small signal analysis

Notation:
Small letter, capitol subscript means the total time varying quantity.

\[ V_{th} = 5 \text{ V} \]
\[ R_{th} = 0.5 \text{ k}\Omega \]
\[ R_{th} = i_D(t) \]
\[ \nu_D(t) = I_S \left( e^{\frac{v_D(t)}{V_T}} - 1 \right) + C's \]

Large signal:
Small signal:
Small signal analysis by equations

\[ R_{th} = 0.5 \text{ k}\Omega \]

\[ i_D(t) = I_S e^{\frac{V_D}{k_B T}} = I_S e^{\frac{V_D}{V_T}} \]

\[ V_T = k_B T / q = 0.026 \text{ V (300 K)} \]

\[ i_D(t) = I_S e^{\frac{V_D(t)}{V_T}} \]

\[ v_D(t) = V_D + v_d(t) \]

\[ i_D(t) = I_S e^{\frac{V_D + v_d}{V_T}} \]

\[ = I_S e^{\frac{V_D}{V_T}} e^{\frac{v_d}{V_T}} \]

\[ = I_D e^{\frac{v_d}{V_T}} \]

\[ \approx I_D \left( 1 + \frac{v_d}{V_T} \right) \]

\[ e^x \approx 1 + x \quad x \ll 1 \]

\[ v_d \ll V_T = 0.026 \]

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For small signals, the diode behaves as a resistor. The value of the resistor is determined by the dc current.

\[ R_{th} = 0.5 \, \text{k}\Omega \]

\[ i_D(t) \approx I_D \left(1 + \frac{v_d}{V_T}\right) \]

\[ i_D(t) \approx I_D + i_d(t) \]

\[ i_d = \frac{v_d}{V_T/I_D} = \frac{v_d}{r_d} \]

\[ r_d = \frac{V_T}{I_D} \]
Analysis procedure

1) Kill the ac source, and analyze the circuit to determine the dc current.

Answer:

\[ I_D = 8.61 \text{ mA} \quad V_D = 0.7145 \text{ V} \]

2) Determine the value of the small signal resistor.

\[ r_d = \frac{V_T}{I_D} = \frac{0.026}{8.61 \times 10^{-3}} \]

\[ r_d = 3.02 \text{ } \Omega \]
3) Kill the dc source, and replace the diode by its ac model (resistor).

4) Ac circuit analysis.

\[ v_d = \left( \frac{r_d}{r_d + R_{th}} \right) 5\cos \omega t \text{ mV} \]

\[ v_d = \left( \frac{3.02}{503} \right) 5\cos \omega t \text{ mV} \]

\[ v_d = 0.03\cos \omega t \text{ mV} \]

\[ v_d = 30\cos \omega t \mu \text{V} \]
Analysis procedure

4) ac circuit analysis (cont.)

\[ v_d = 30 \cos \omega t \ \mu V \]

\[ i_d = \frac{v_d}{r_d} = \frac{30 \cos \omega t \ \mu V}{3.02 \ \Omega} = 10 \cos \omega t \ \mu A \]
Analysis procedure

5) Add dc and ac.

\[ i_D(t) = I_D + i_d(t) \]
\[ i_D(t) = 8.61 \text{ mA} + 10 \cos \omega t \mu \text{A} \]

\[ v_D(t) = V_D + v_d(t) \]
\[ v_D(t) = 0.7145 \text{ V} + 30 \cos \omega t \mu \text{V} \]
Graphical picture

\[ I_D \approx I_S e^{qV_D/k_BT} \]

8.61 mA

\[ g_d = \frac{dI_D}{dV_D} \bigg|_Q = \frac{\Delta I_D}{\Delta V_D} = \frac{i_d}{v_d} = \frac{1}{r_d} \]

\[ r_d = \left( \frac{dI_D}{dV_D} \bigg|_Q \right)^{-1} = 3.02 \, \Omega \]

Set DC bias to get the \( r_d \) needed.
Modeling Diodes

1) Model parameters
2) Exponential model
3) Small signal model
4) **Zener diode model**
5) Zener diode applications
Recall: ideal voltage source

\[ V_0 \]

\[ I \]

\[ V \]

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Recall: real voltage source

\[ V = V_0 + IR_0 \]
Diode IV: reverse breakdown

\[ I_D (\text{mA}) = I_S \left( e^{\frac{V_D}{V_T}} - 1 \right) \]

- Zener (negative temp coeff)
- Avalanche (positive temp coeff.)

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Diode IV: reverse breakdown

slope
$\frac{1}{r_z}$

$I_D (mA)$

$V_D$

$I_{ZK}$

“knee current”

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Modeling the Zener diode

Model parameters:

\[ I_Z > I_{ZK} \]

\[ V_Z > V_{ZK} \]

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Modeling diodes

1) Model parameters
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5) Zener diode applications
"battery regulator"
At the nominal input voltage, what is $V_L$ when $I_L = 0$?

**Diagram Details:**

- Input voltage: $60 \pm 1 \text{ V}$
- Zener diode: $I_Z$ (forward current), $V_Z = 50 \text{ V}$ at $I_Z = 50 \text{ mA}$
- Load resistor: $R_L = 20 \Omega$
- Load current: $I_L$
- Load voltage: $V_L$
- Zener current: $I_{ZK} = 0.3 \text{ mA}$

**Equations:**

- $V_L = \frac{I_L}{R_L} \cdot R_L$
- $I_{ZK} = 0.3 \text{ mA}$
- $V_Z = 50 \text{ V}$ at $I_Z = 50 \text{ mA}$
Zener diode model

$I_Z$

$V_{Z0}$

$r_z$

$V_z$

\[ V_Z = 50 \text{ V at } I_Z = 50 \text{ mA} \]

\[ r_z = 20 \text{ } \Omega \quad I_{ZK} = 0.3 \text{ mA} \]

\[ V_Z = V_{Z0} + I_Z r_z \]

\[ 50 = V_{Z0} + 50 \text{ mA} \times 20 \text{ } \Omega \]

\[ = V_{Z0} + 1 \]

$V_{Z0} = 49 \text{ V}$  ✔
At the nominal input voltage, what is $V_L$ when $I_L = 0$?

\[
I_z = \frac{60 - 49}{1 \, \text{k} + 0.02 \, \text{k}} = 10.8 \, \text{mA}
\]

\[
V_L = 49 + 10.8 \times 0.02 = 49.2
\]
What is the change in output voltage for a +/- 1 V change in input voltage?
What is the change in output voltage if the load draws 2 mA of current?

\[ \Delta V_L = r_z \Delta I_z = 0.020 \times (-2) = -0.040 \]

\[ \frac{\Delta V_L}{\Delta I_L} = -0.020 \frac{V}{A} = -20 \frac{mV}{mA} \]
Question: What value of load resistor produces 2 mA of load current?
What maximum load current can be drawn if the regulator is to operate properly?
Model have **model parameters**. To get good results, we need a good model and accurate model parameters.

We discussed three types of diode models: i) ideal, ii) constant-voltage-drop, and iii) mathematical (exponential).

**Small signal model parameters depend on the DC bias.**

The small signal diode model is a resistor (+ junction and diffusion capacitances).

A good Zener diode in breakdown is like a good battery.
Modeling diodes

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