Spring 2019 Purdue University

ECE 255: L8

Modeling Diodes

(Sedra and Smith, 4.3-4.4)

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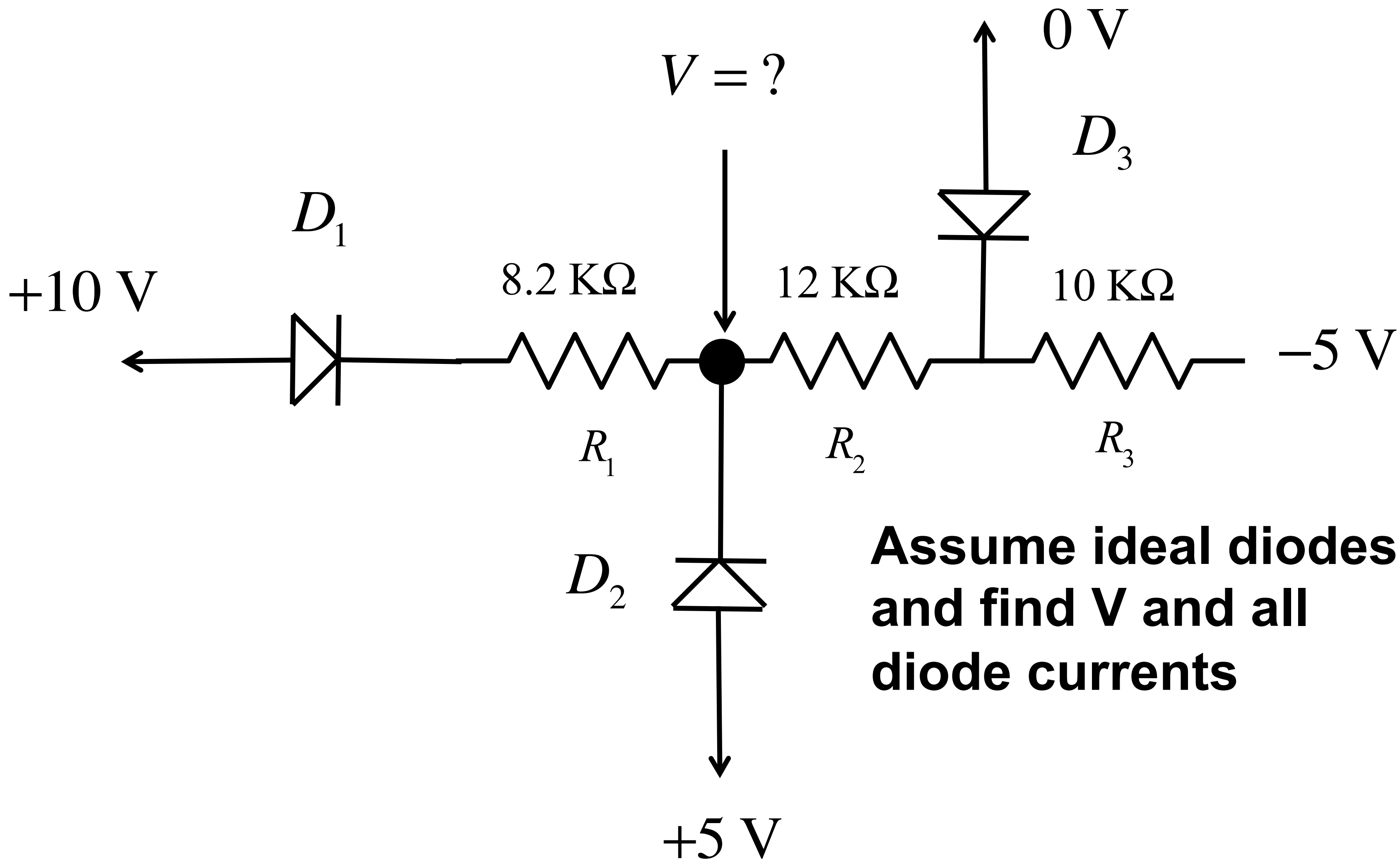
Announcements

Exam 1: Thursday, Feb. 7, 6:30 PM, LILY 1105.
(Weeks -1- 4 topics, semiconductors, diodes, BJTs. i.e. HW1 – HW4.)

Remember: Do the assigned reading before class!

Quiz problem



Assume ideal diodes and find V and all diode currents

Quiz problem

Guess:

D1 (FB) **Show why this does not work**
D2 (FB)
D3 (FB)

Quiz problem

Guess:

D1 (FB)
D2 (FB)
D3 (FB)

Show why this does not work

Find:
IR1 = (10-5)/8.2K=0.61 mA
IR2 = (5-0)/12K =0.42 mA
Implies
ID2 < 0
D2 is NOT FB

Quiz problem

Guess:

D1 (FB)
D2 (RB)
D3 (FB)

Show that this does work.

Quiz problem

Guess:

D1 (FB)
D2 (RB)
D3 (FB)

Show that this does work.

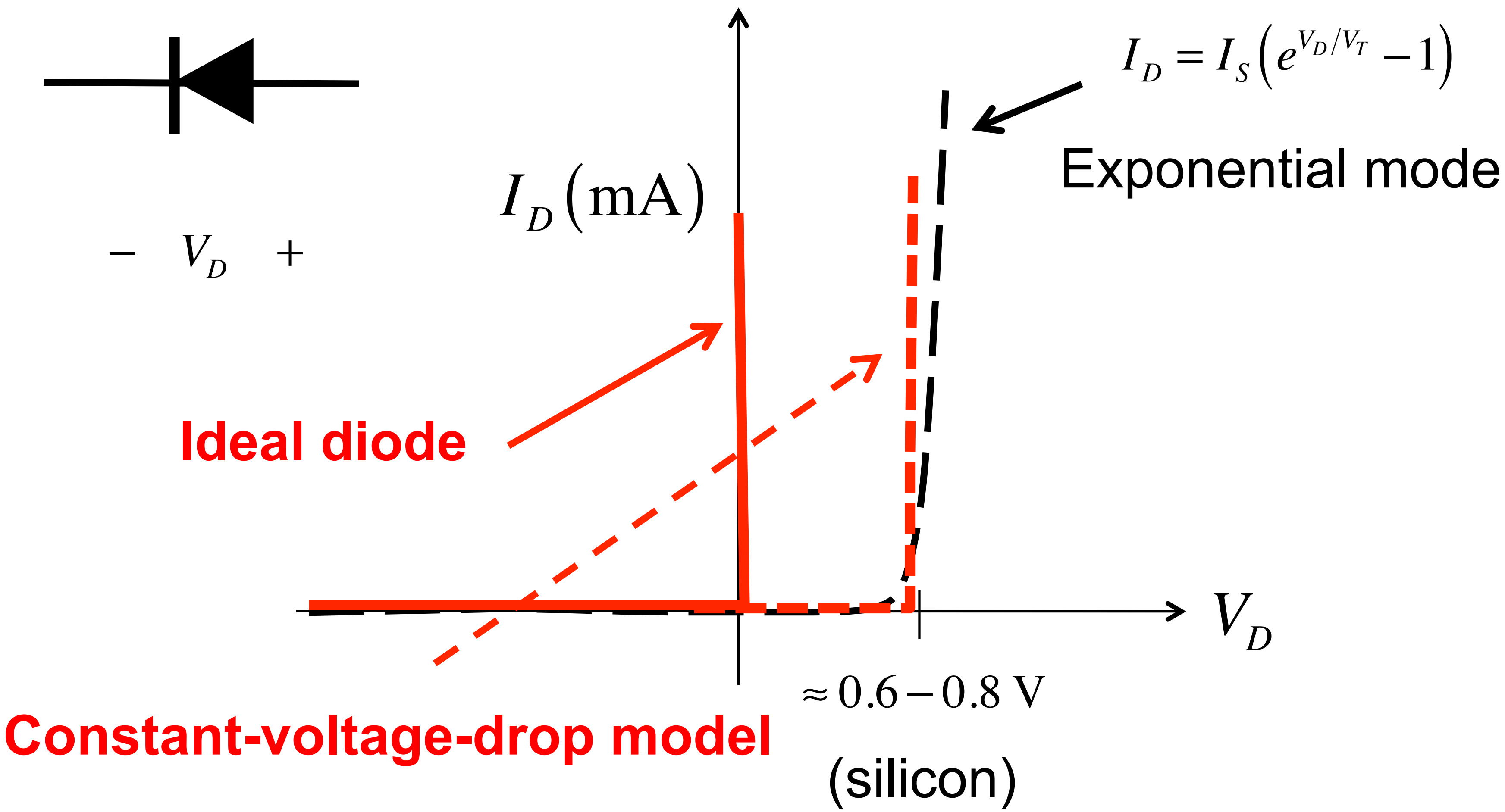
Find:

IR1 = (10 − 0) / 20.2 K = 0.495 mA
V = 10 − 0.495x8.2 = 5.94
VD2 < 0 RB
IR3 = (0-(-5))/10 = 0.5 mA
ID3 = IR3 − IR1 − 0.005 mA

Modeling diodes

1) Model parameters
2) Exponential model
3) Small signal model
4) Zener diode model
5) Zener diode applications

Three diode models

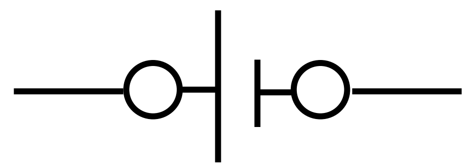


Big idea: Complicated devices can be "simply" modeled.

Model parameters

1) Ideal diode model — **No model parameters**

2) Constant-voltage-drop model — **One model parameter**



$$\approx +\,0.6-0.8\text{V}\,-$$

3) Exponential model — **One model parameter**

$$I_D = I_S\left(e^{V_D/V_T} - 1\right)$$

Determining model parameters

Question: Given a diode for which we want to model with the exponential equation, how do we determine I_S?

Answer: Do a measurement at 300 K:

e.g. $I_D\left(V_D = 0.7 \text{ V}\right) = 5 \text{ mA}$

What is I_S?

Computing I_S

$$I_D = I_S\left(e^{V_D/V_T} - 1\right) \approx I_S e^{V_D/V_T}$$

$$I_S = I_D e^{-V_D/V_T}$$

$$V_T = k_B T/q = 0.0259 \text{ V}$$

$$I_S = 5\times 10^{-3} e^{-0.7/0.026}$$

$$I_S = 9.15\times 10^{-15} \text{ A}$$ ✓

But this would not be considered good engineering judgment. Why?

How could we do better?

Determining model parameters

Given a diode for which we want to model with the constant-voltage drop model, how do we determine V_D?

First, estimate the current that will result in the circuit.

e.g. $I_D \approx 10\ \mu\text{A}$

Then do a measurement, or compute V_D if we know I_S.

Determining model parameters

Given: $I_S = 9.15 \times 10^{-15}\ \mathrm{A}$

The expected current in the circuit is about 10 microamps.

Find V_D: $$I_D = I_S\left(e^{V_D/V_T} - 1\right) \approx I_S e^{V_D/V_T}$$

$$V_D = V_T \ln\left(\frac{I_D}{I_S}\right)$$

$$V_D = 0.026 \ln\left(\frac{10^{-5}}{9.15 \times 10^{-15}}\right) = 0.54\ \mathrm{V}$$ ✓

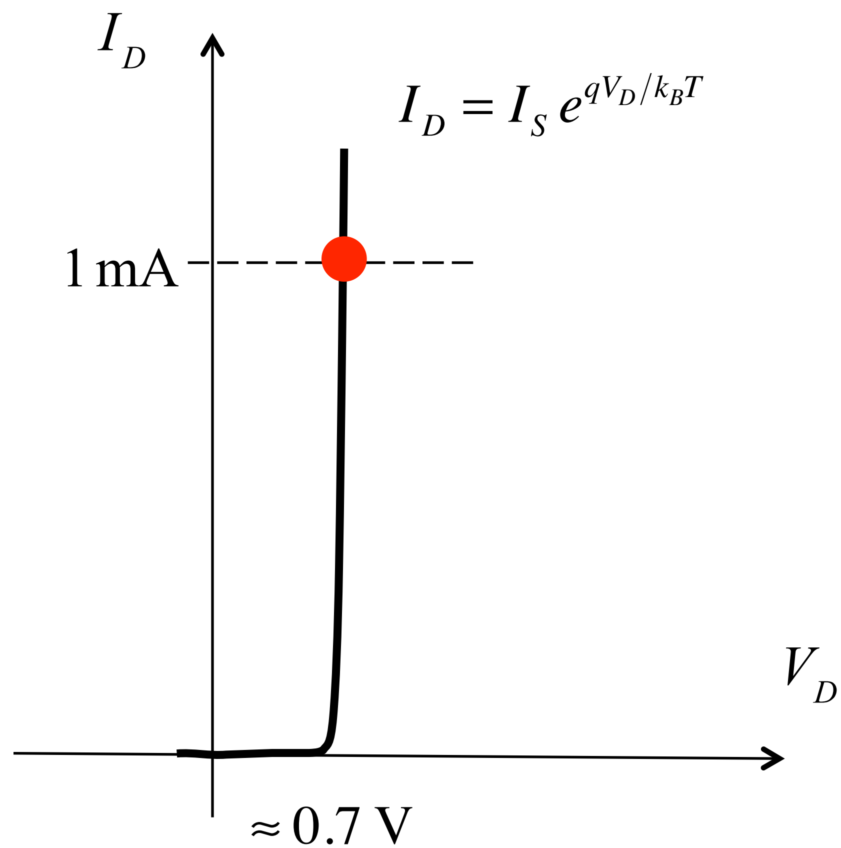
Ideal and Real Diodes

1) Model parameters
2) **Exponential model**
3) Small signal model
4) Zener diode model
5) Zener diode applications

A question

How much do we need to **increase** V_D to **increase** the current by 10X?

How much do we need to **decrease** V_D to **decrease** the current by 10X?



The answer

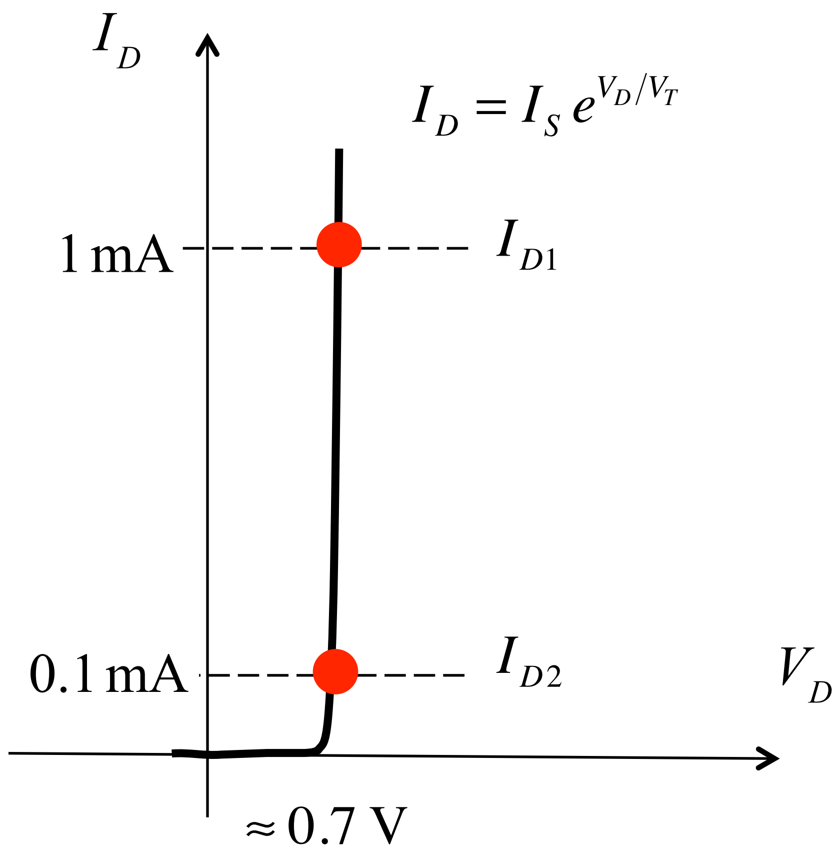
$$I_{D1} = I_S e^{V_{D1}/V_T}$$

$$I_{D2} = I_S e^{V_{D2}/V_T}$$

$$\frac{I_{D1}}{I_{D2}} = e^{(V_{D1}-V_{D2})/V_T}$$

$$\Delta V_D = V_T \ln(10)$$

$$\Delta V_D = 60 \text{ mV/decade}$$



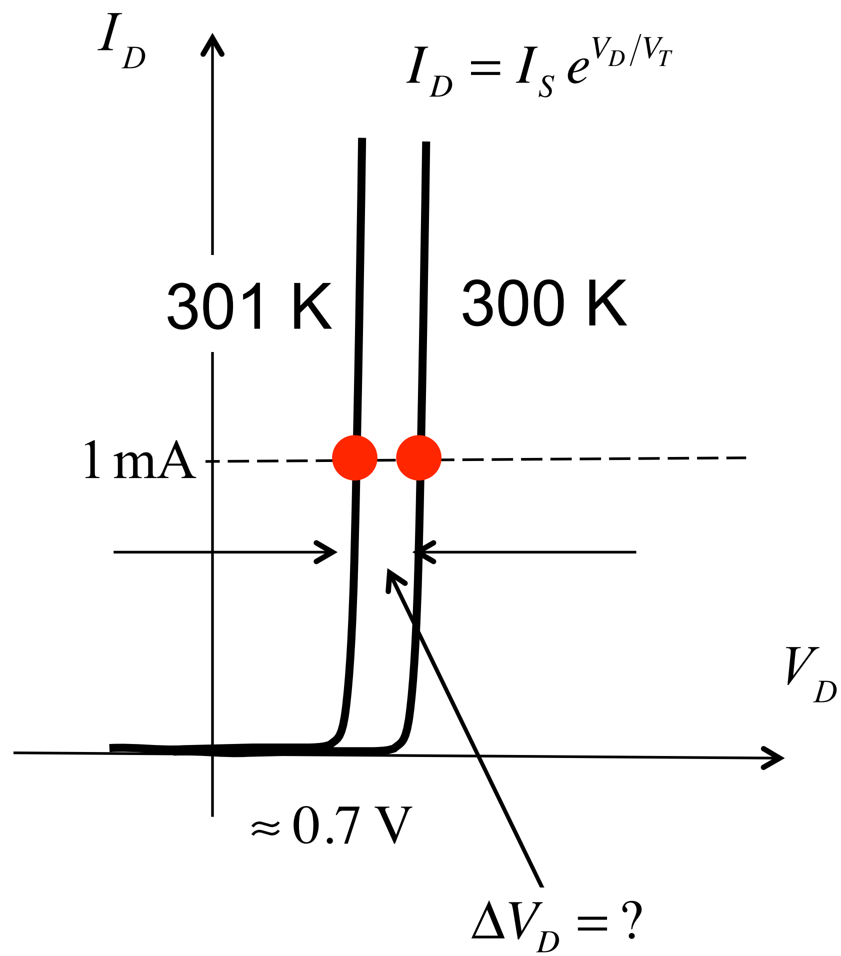
Temperature sensitivity

$$I_{D1} = I_S \, e^{V_{D1}/V_T}$$

$$I_S \propto n_i^2 \propto e^{-E_G/k_B T}$$

How much does the voltage need to change to keep the current constant if T increases by 1 deg C?

Answer: ~ -2 mV/degC



Modeling diodes

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3) **Small signal model**
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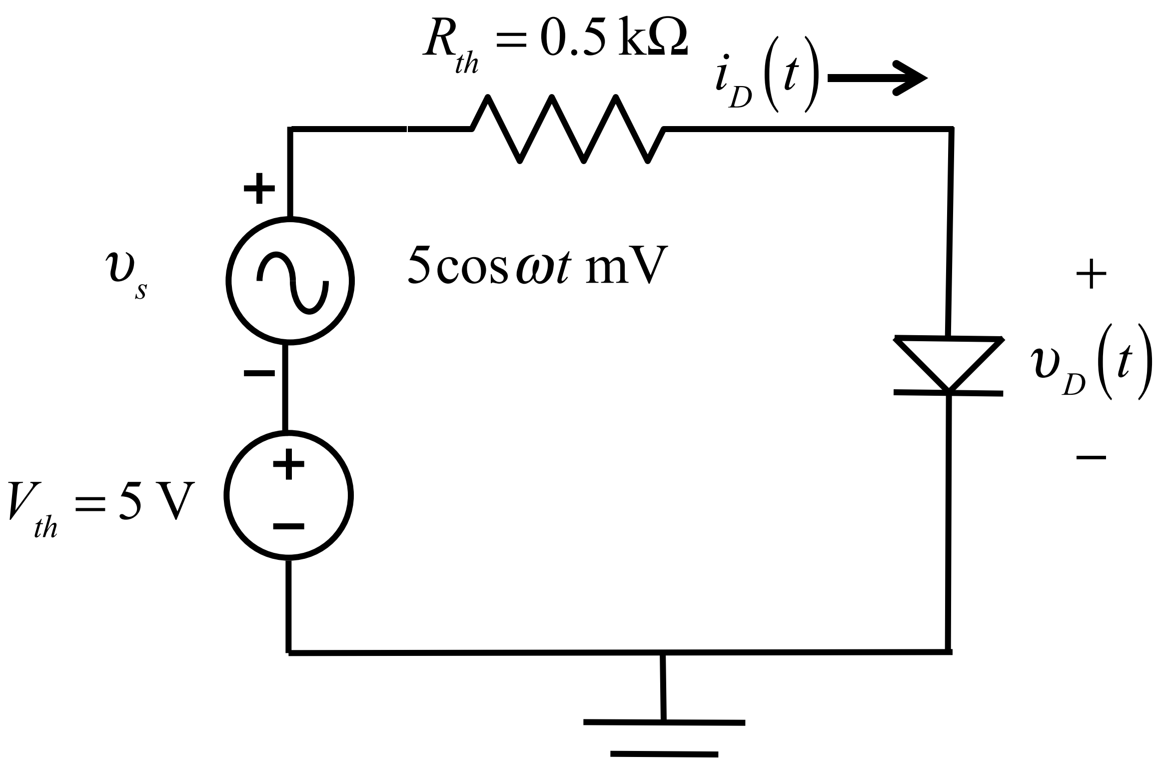
Small signal analysis



Notation:

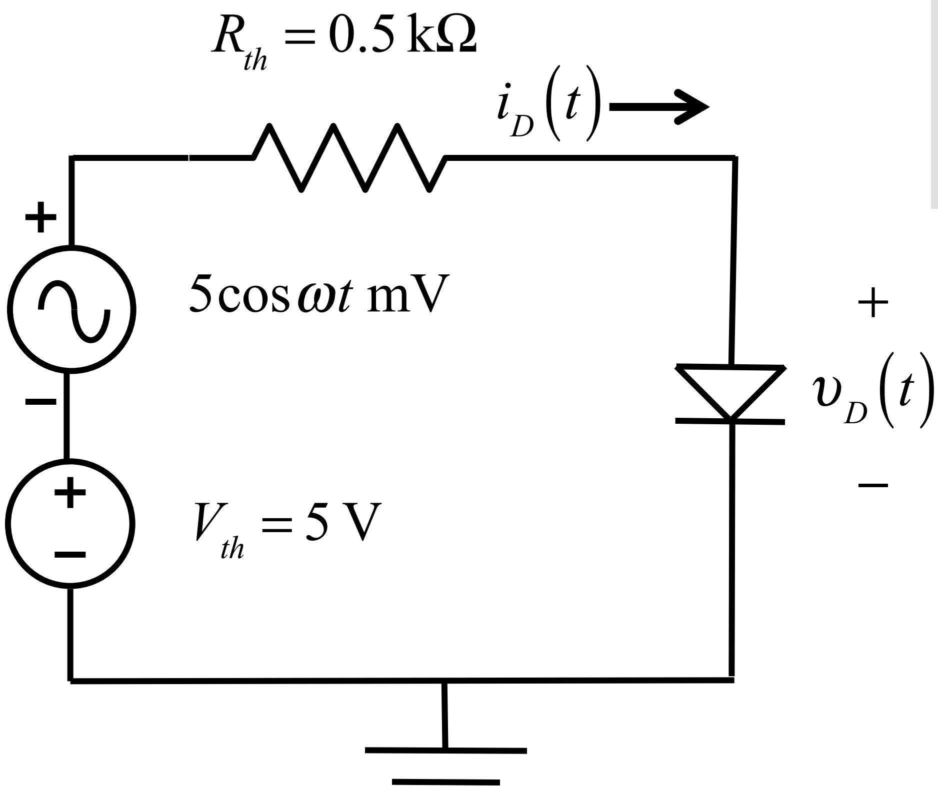
Small letter, capitol subscript means the total time varying quantity.

Large signal:

$$i_D(t) = I_S\left(e^{\upsilon_D(t)/V_T} - 1\right)$$ **+ C's**

Small signal:

Small signal analysis by equations



$$I_D = I_S\, e^{qV_D/k_BT} = I_S e^{V_D/V_T}$$

$$V_T = k_B T/q = 0.026\ \mathrm{V}\ (300\ \mathrm{K})$$

$$i_D(t) = I_S\, e^{\upsilon_D(t)/V_T}$$

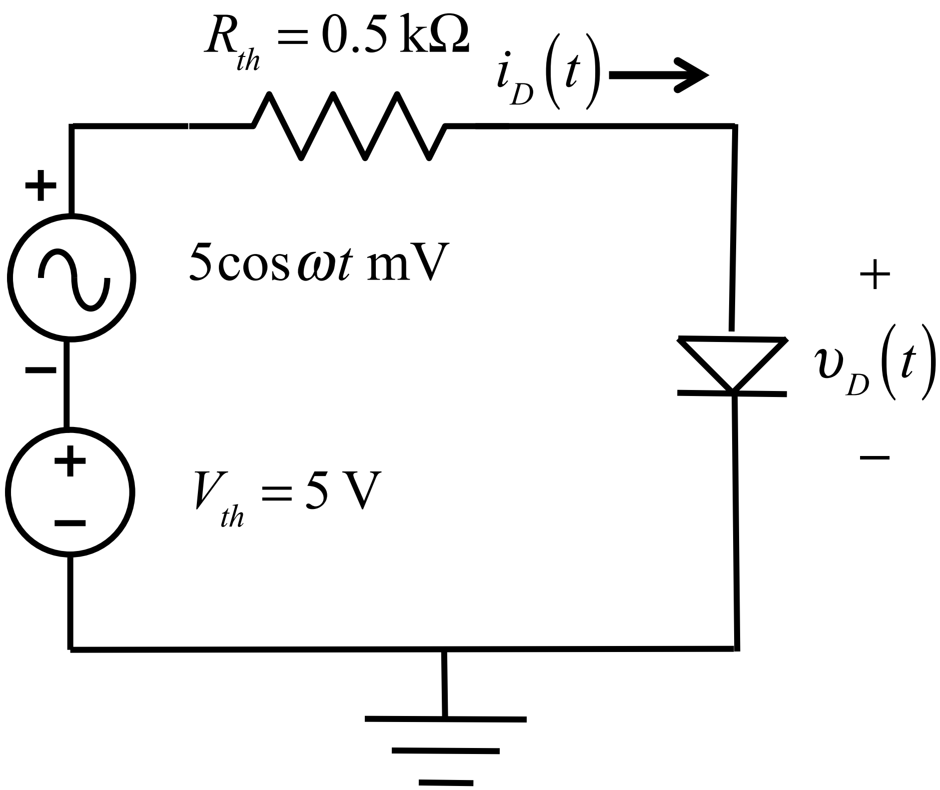
$$\upsilon_D(t) = V_D + \upsilon_d(t)$$

$$\begin{aligned} i_D(t) &= I_S\, e^{(V_D+\upsilon_d)/V_T} \\ &= I_S\, e^{V_D/V_T} e^{\upsilon_d/V_T} \\ &= I_D e^{\upsilon_d/V_T} \\ &\approx I_D\left(1+\upsilon_d/V_T\right) \end{aligned}$$

$$e^x \approx 1 + x \quad x << 1$$

$$\upsilon_d << V_T = 0.026$$

Small signal analysis by equations



$$i_D(t) \approx I_D\left(1 + \upsilon_d / V_T\right)$$

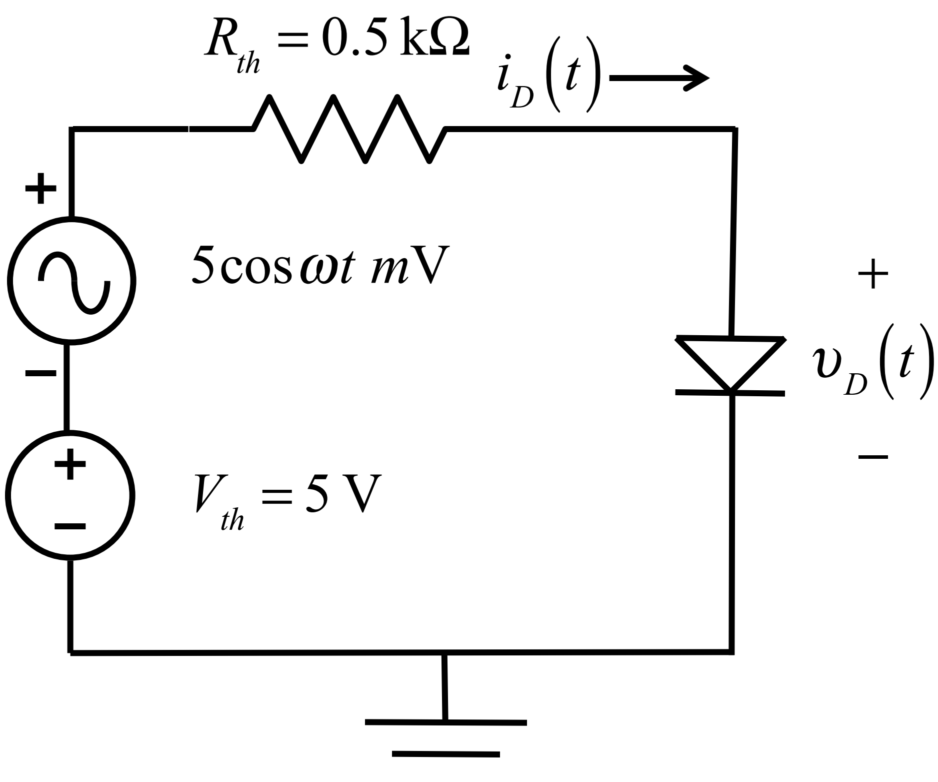
$$i_D(t) \approx I_D + i_d(t)$$

$$i_d = \frac{\upsilon_d}{V_T / I_D} = \frac{\upsilon_d}{r_d}$$

$$r_d = V_T / I_D$$

For small signals, the diode behaves as a resistor. The value of the resistor is determined by the dc current.

Analysis procedure



1) Kill the ac source, and analyze the circuit to determine the dc current.

Answer:

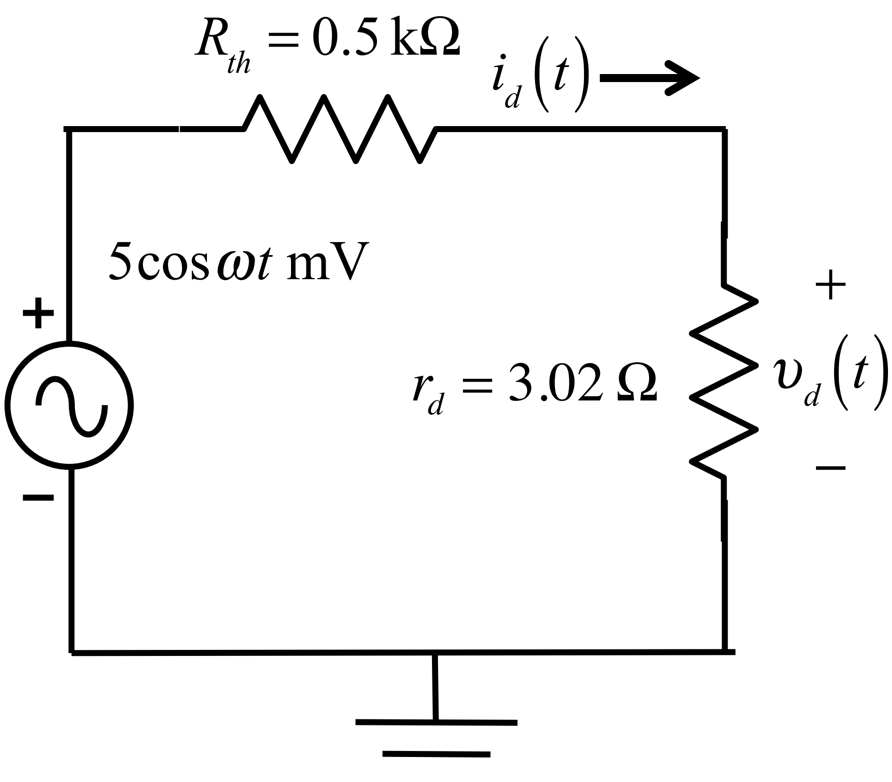
$$I_D = 8.61\,\text{mA} \qquad V_D = 0.7145\ \text{V}$$

2) Determine the value of the small signal resistor.

$$r_d = V_T / I_D = 0.026 / 8.61 \times 10^{-3}$$

$$r_d = 3.02\ \Omega$$

Analysis procedure



3) Kill the **dc** source, and replace the diode by its ac model (resistor).

4) Ac circuit analysis.

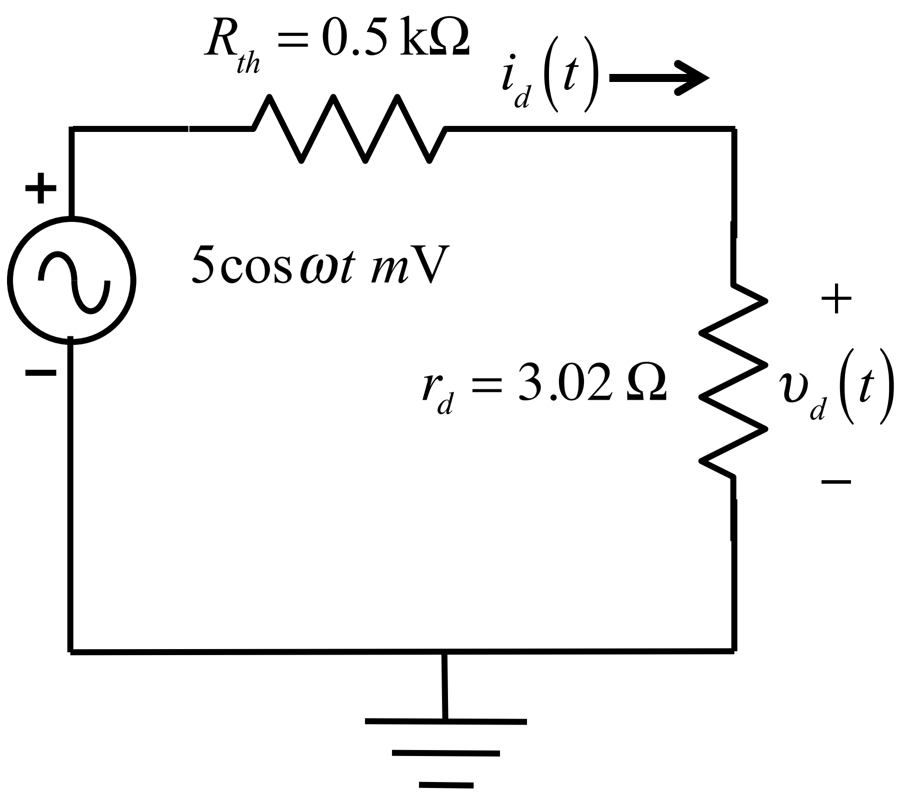
$$\upsilon_d = \left(\frac{r_d}{r_d + R_{th}}\right) 5\cos\omega t \text{ mV}$$

$$\upsilon_d = \left(\frac{3.02}{503}\right) 5\cos\omega t \text{ mV}$$

$$\upsilon_d = 0.03\cos\omega t \text{ mV}$$

$$\upsilon_d = 30\cos\omega t\ \mu\text{V}$$

Analysis procedure



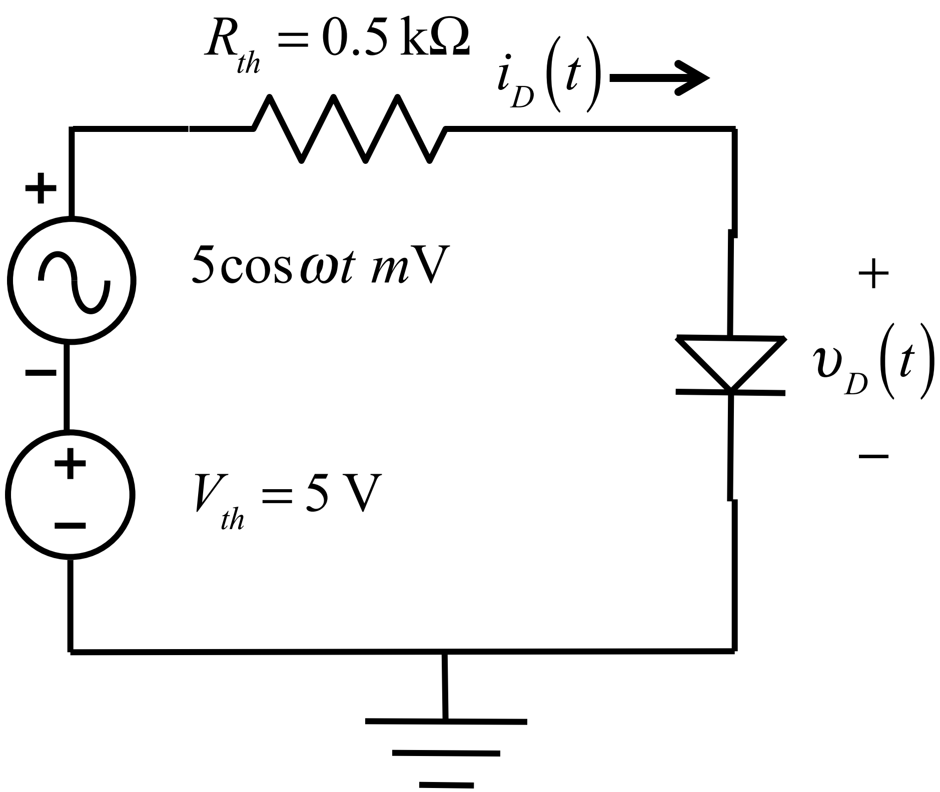
4) ac circuit analysis (cont.)

$$\upsilon_d = 30\cos\omega t\ \mu\text{V}$$

$$i_d = \frac{\upsilon_d}{r_d} = \frac{30\cos\omega t\ \mu\text{V}}{3.02\ \Omega} = 10\cos\omega t\ \mu\text{A}$$

Analysis procedure

5) Add dc and ac.



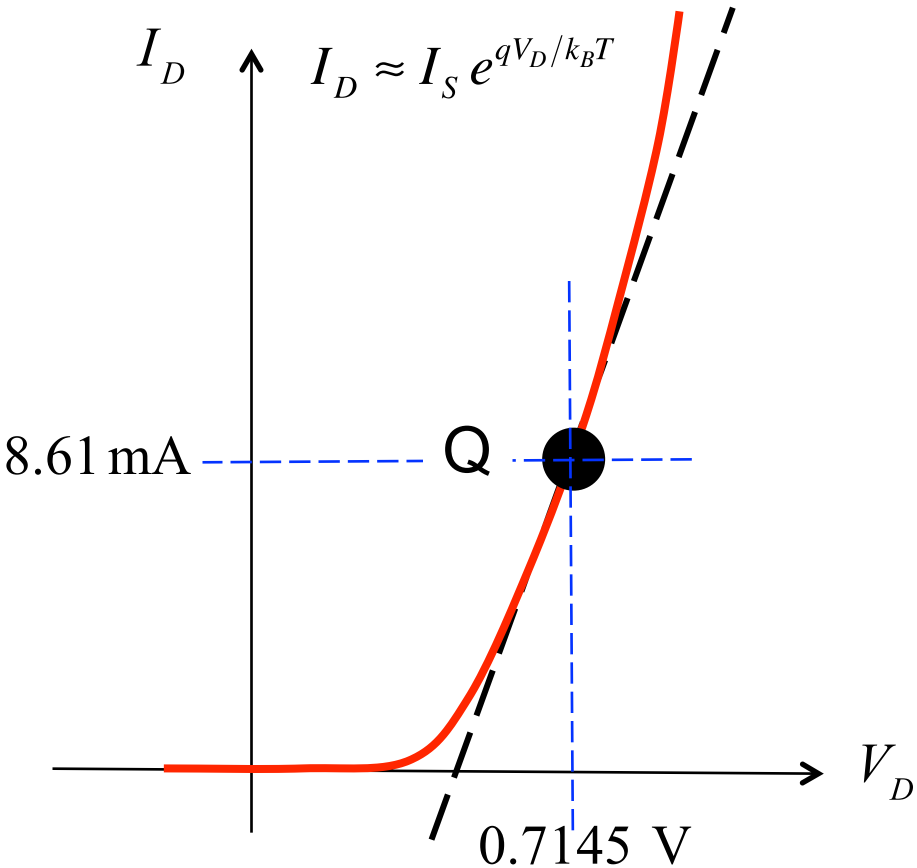
$$i_D(t) = I_D + i_d(t)$$

$$i_D(t) = 8.61\ \text{mA} + 10\cos\omega t\ \mu\text{A}$$

$$\upsilon_D(t) = V_D + \upsilon_d(t)$$

$$\upsilon_D(t) = 0.7145\ \text{V} + 30\cos\omega t\ \mu\text{V}$$

Graphical picture



$$g_d = \left.\frac{dI_D}{dV_D}\right|_Q = \frac{\Delta I_D}{\Delta V_D} = \frac{i_d}{\upsilon_d} = \frac{1}{r_d}$$

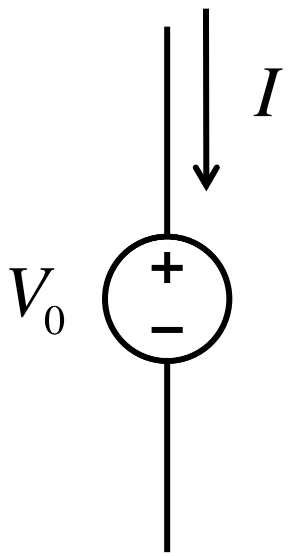
$$r_d = \left(\left.\frac{dI_D}{dV_D}\right|_Q \right)^{-1} = 3.02\ \Omega$$

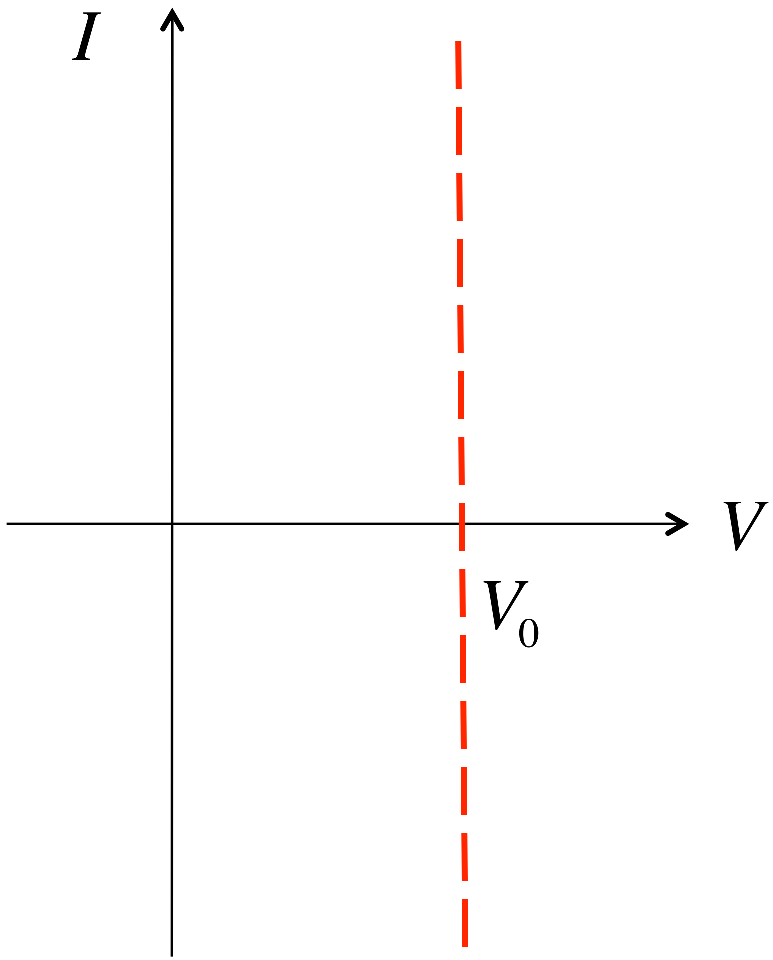
Set DC bias to get the r_d needed.

Modeling Diodes

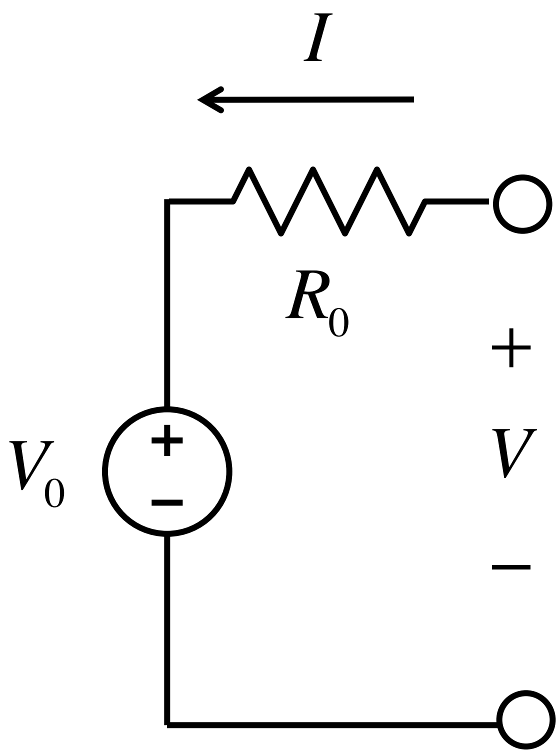
1) Model parameters
2) Exponential model
3) Small signal model
4) **Zener diode model**
5) Zener diode applications

Recall: ideal voltage source

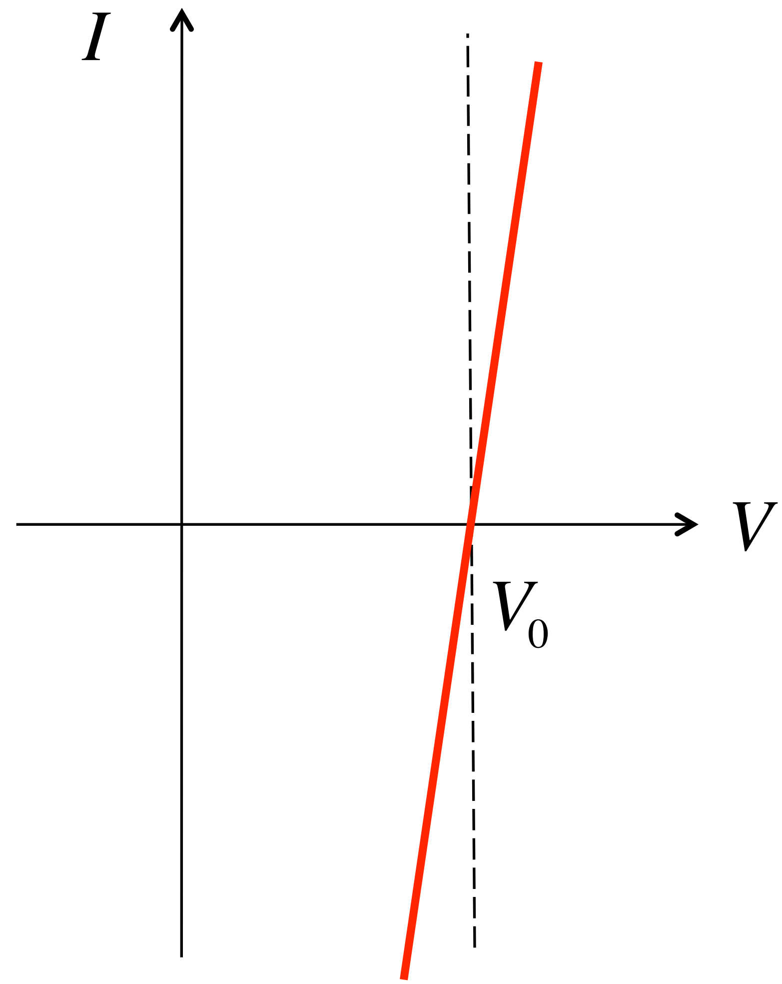




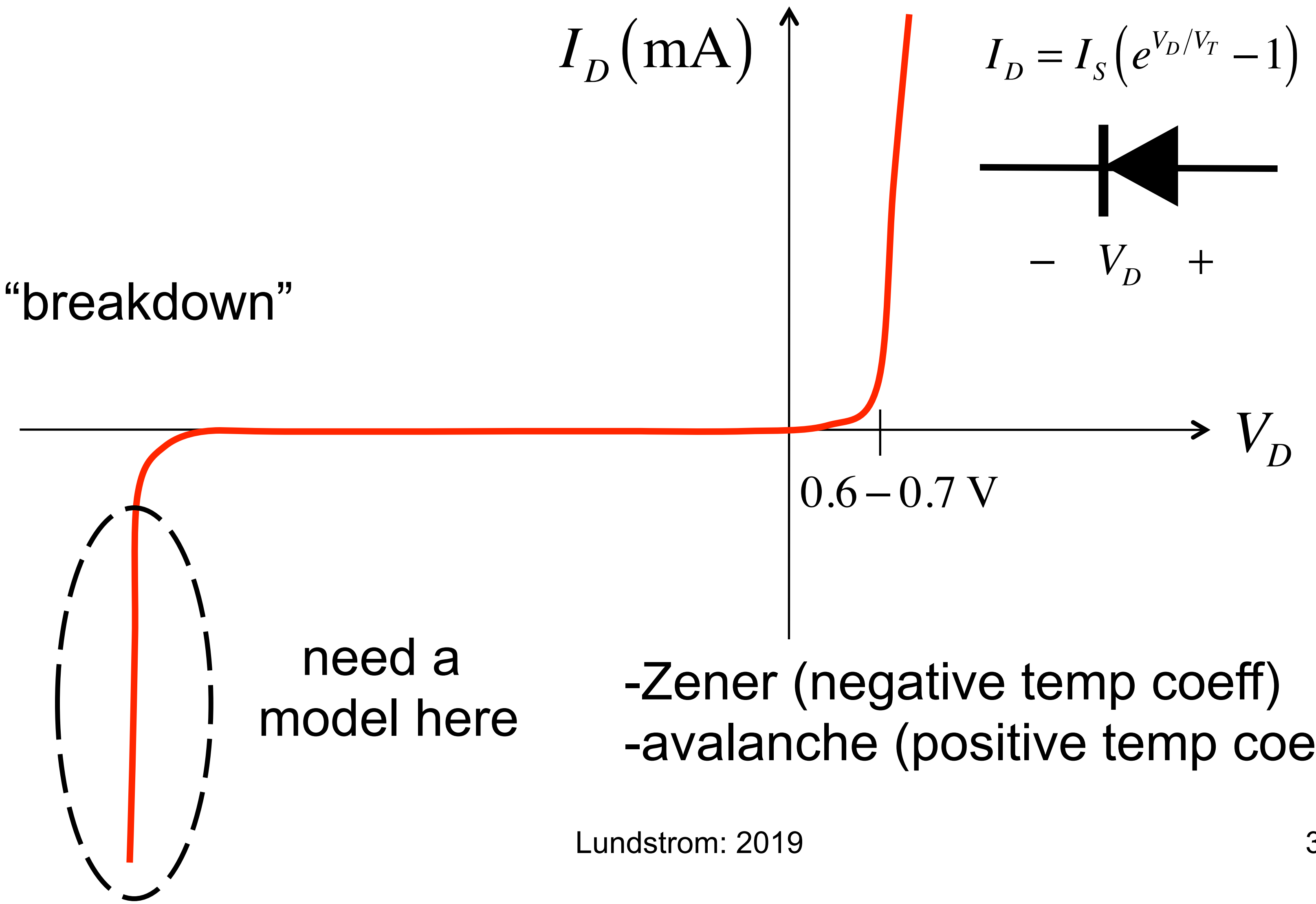
Recall: real voltage source



$$V = V_0 + IR_0$$

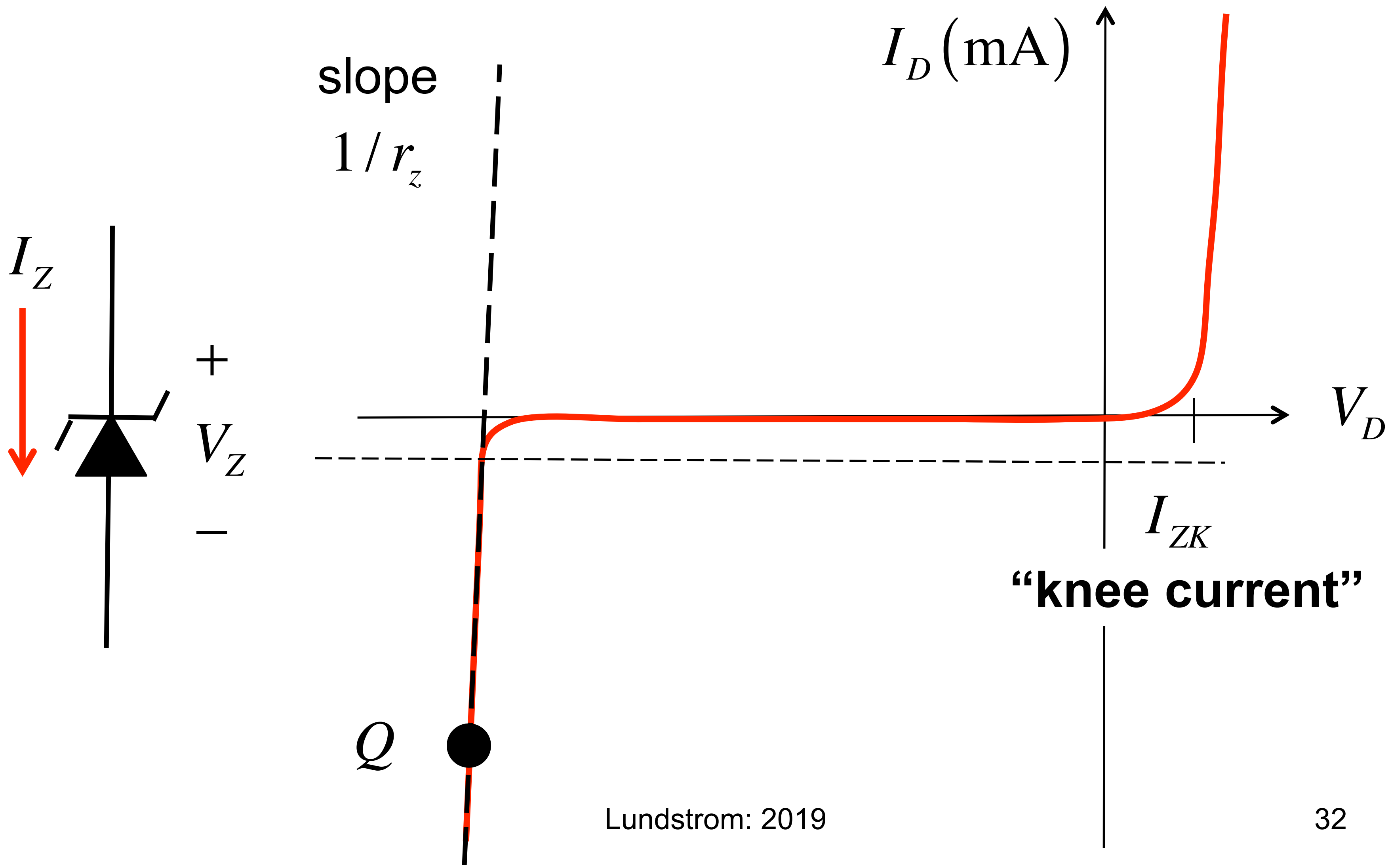


Diode IV: reverse breakdown

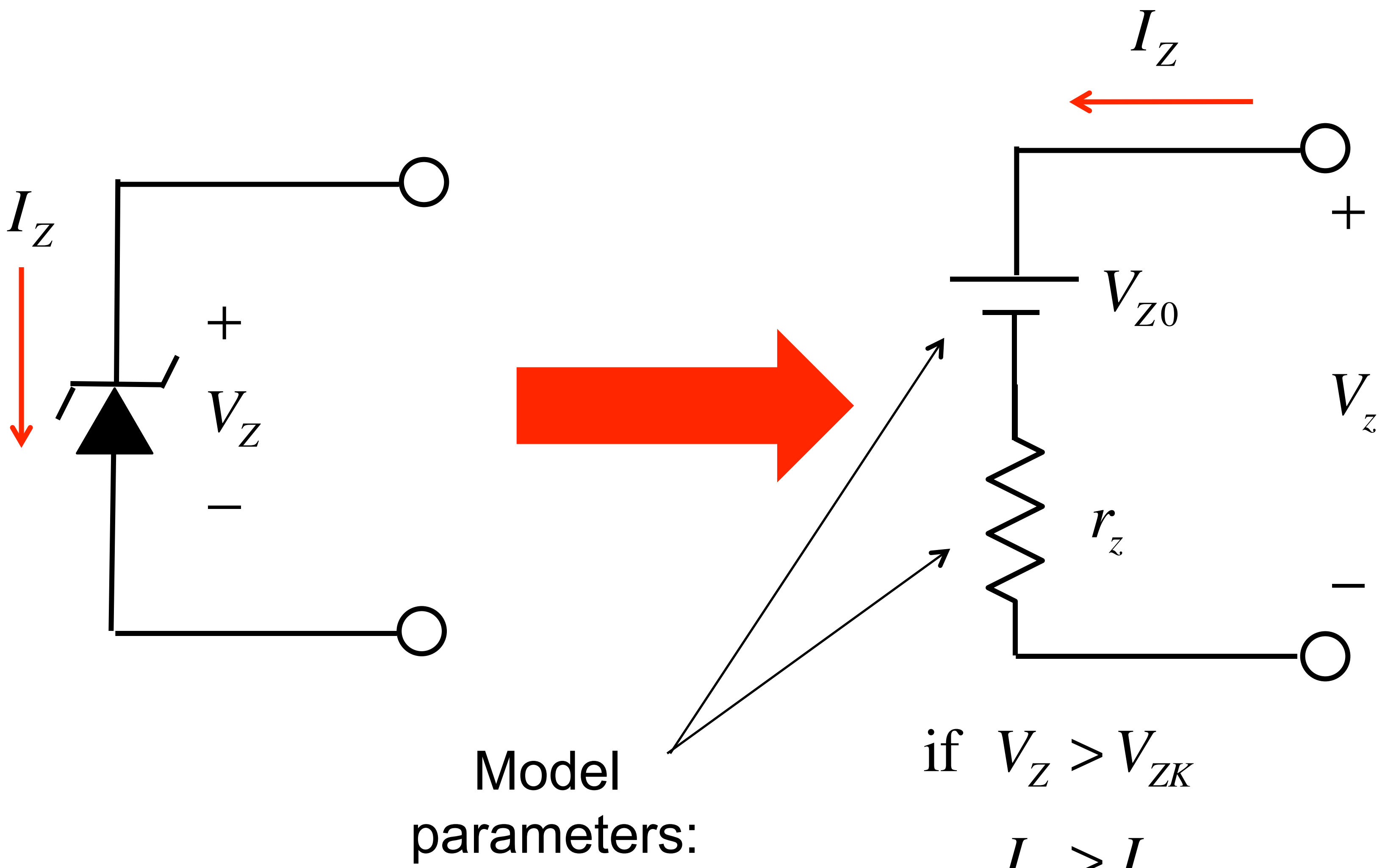


-Zener (negative temp coeff)
-avalanche (positive temp coeff.)

Diode IV: reverse breakdown



Modeling the Zener diode



if $V_Z > V_{ZK}$

$I_Z > I_{ZK}$

Modeling diodes

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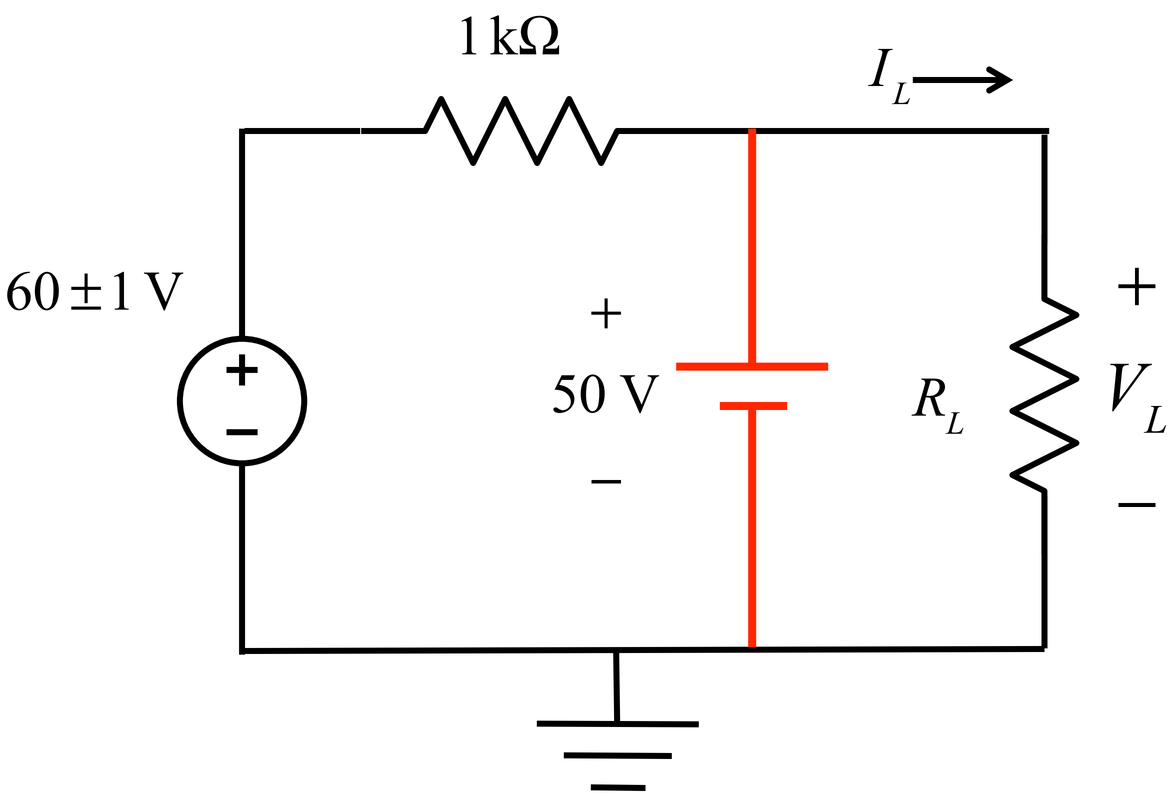
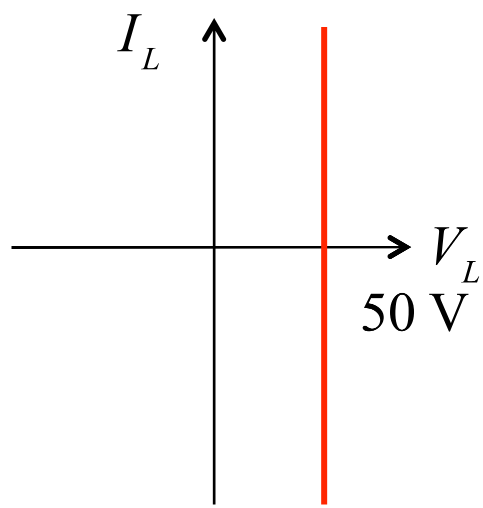
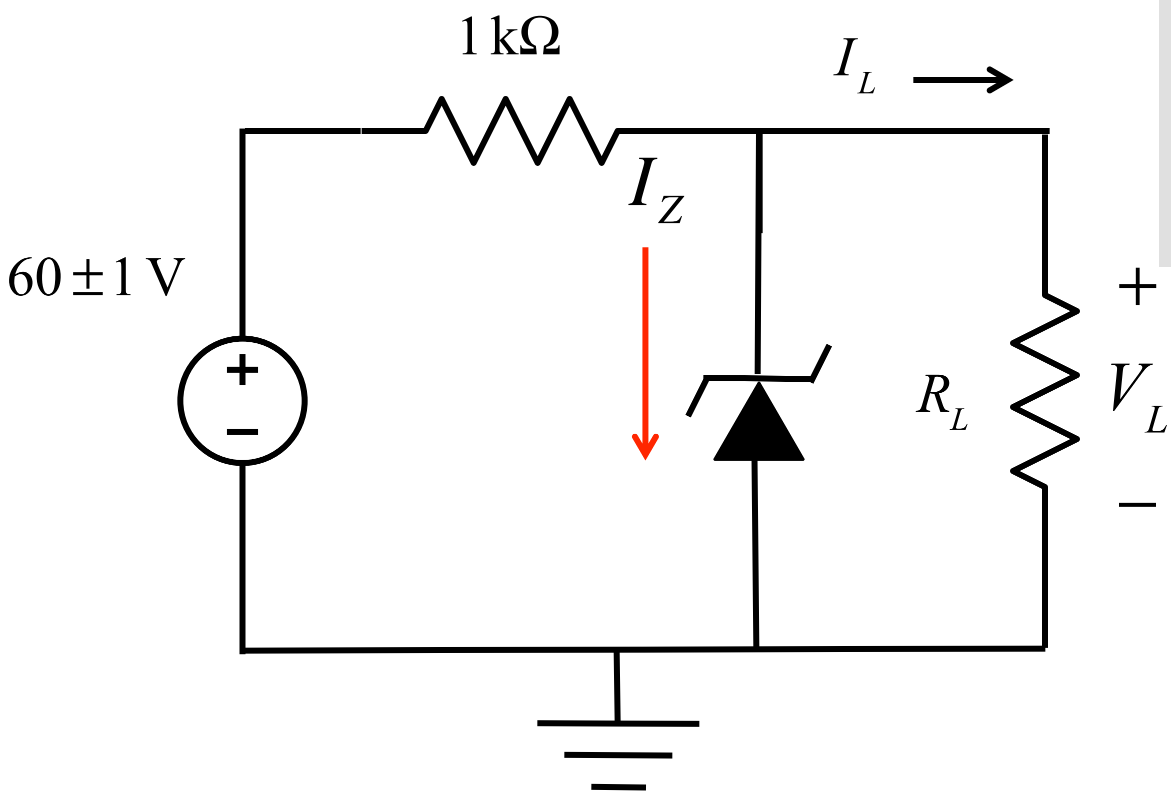
“battery regulator”





"Zener shunt regulator"



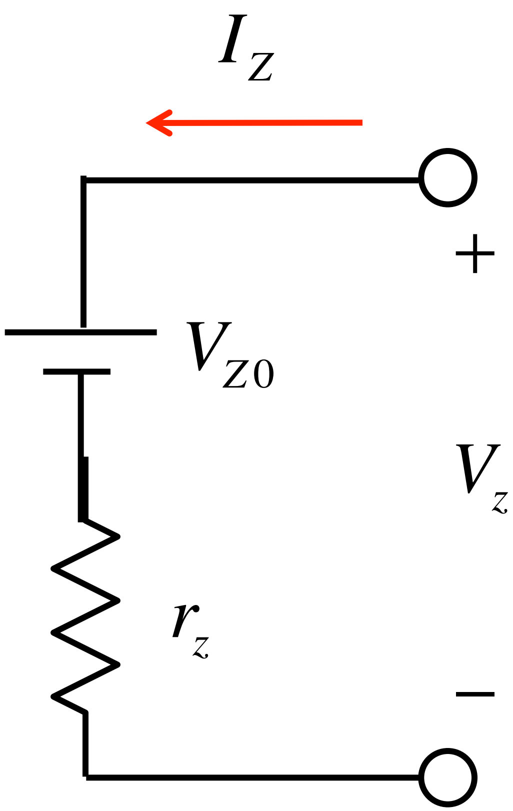
$V_Z = 50$ V at $I_Z = 50$ mA

$r_z = 20\ \Omega$ $V_{Z0} = ?$

$I_{ZK} = 0.3$ mA

At the nominal input voltage, what is V_L when $I_L = 0$?

Zener diode model



$V_Z = 50\ \text{V at } I_Z = 50\ \text{mA}$

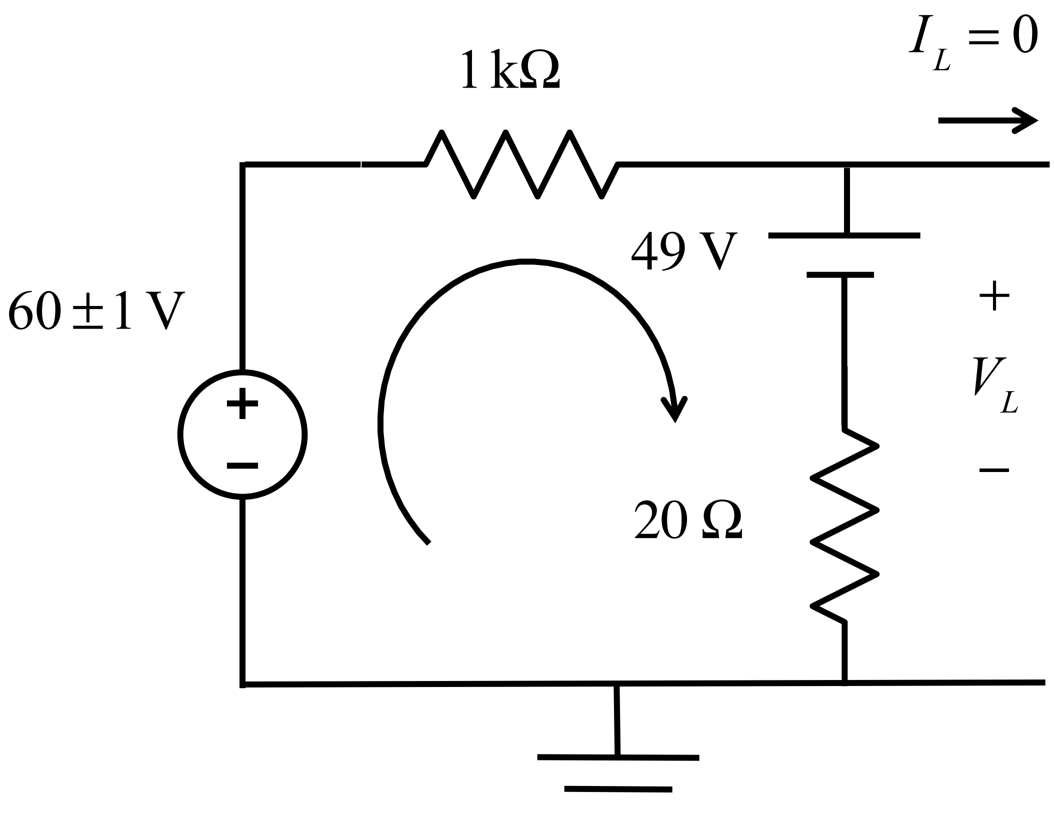
$r_z = 20\ \Omega \qquad I_{ZK} = 0.3\ \text{mA}$

$$V_Z = V_{Z0} + I_Z r_z$$

$$50 = V_{Z0} + 50\,\text{mA} \times 20\ \Omega$$
$$= V_{Z0} + 1$$

$$V_{Z0} = 49\ \text{V}$$ ✓

Zener shunt regulator

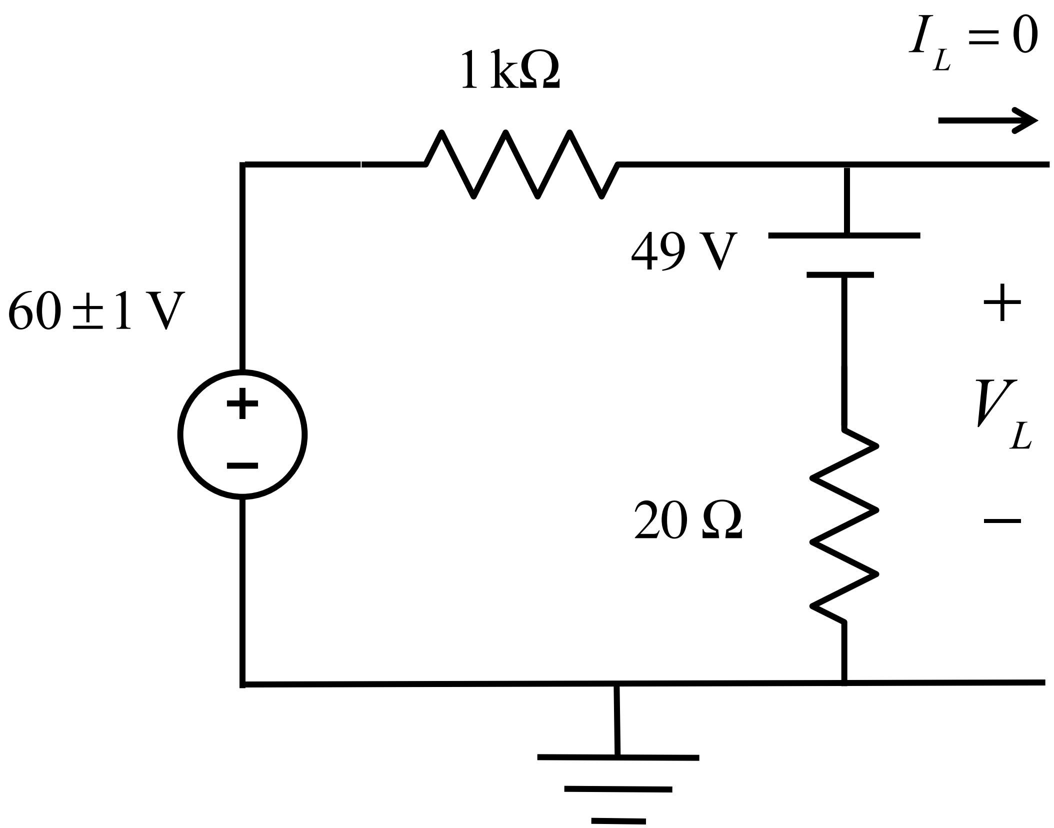


$$I_z = \frac{60-49}{1\text{ k}+0.02\text{ k}}$$
$$= 10.8\text{ mA}$$

$$V_L = 49 + 10.8 \times 0.02$$
$$= 49.2$$

At the nominal input voltage, what is V_L when $I_L = 0$?

Regulation



$$\Delta V_L = \Delta V \frac{r_z}{r_z + R}$$

$$\frac{\Delta V_L}{\Delta V} = \frac{r_z}{r_z + R}$$

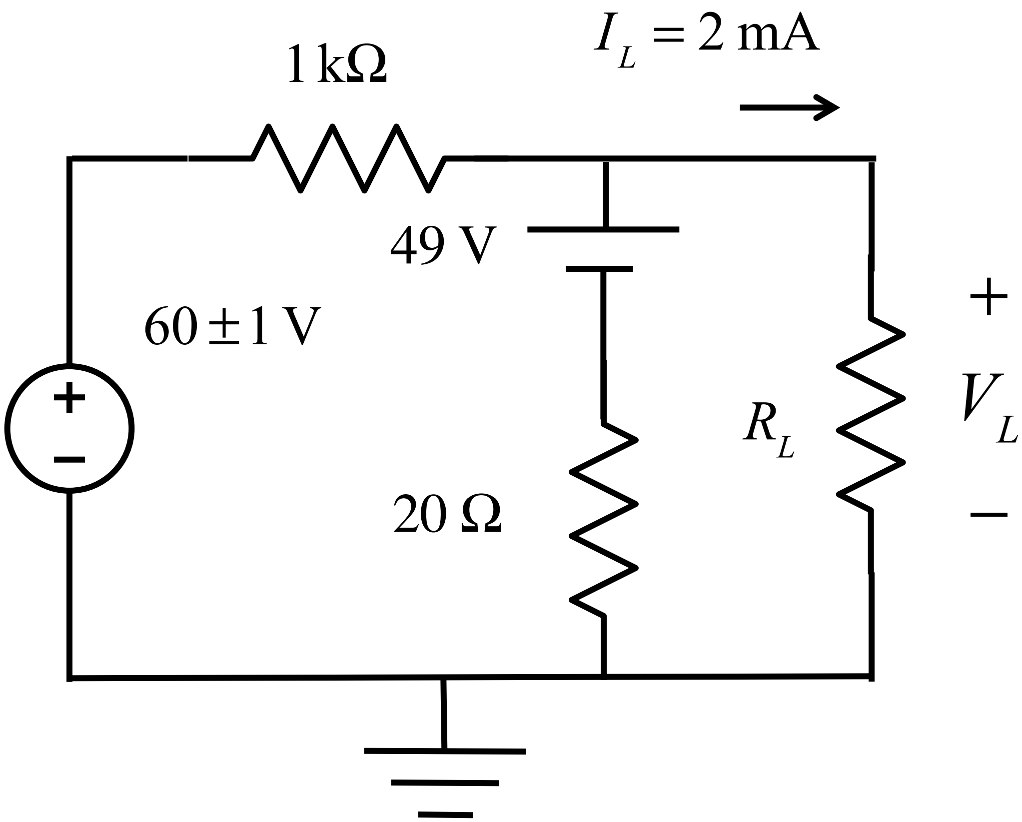
"line regulation"

$$\frac{\Delta V_L}{\Delta V} = \frac{0.02}{1.02} = 0.02$$

$$\text{line regulation} = 20\text{ mV/V}$$

What is the change in output voltage for a +/- 1 V change in input voltage?

"Load regulation"



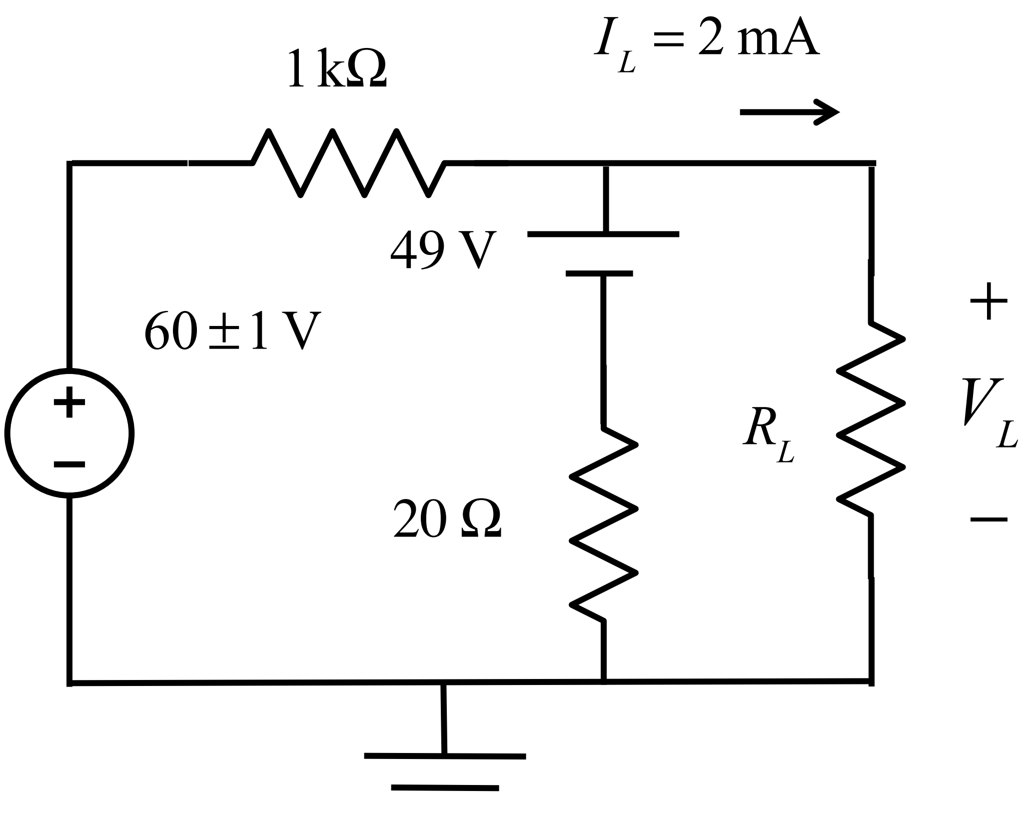
$$\Delta V_L = r_z \Delta I_z = 0.020 \times (-2) = -0.040$$

$$\frac{\Delta V_L}{\Delta I_L} = -0.020\,\frac{\text{V}}{\text{A}} = -20\,\frac{\text{mV}}{\text{mA}}$$

"load regulation"

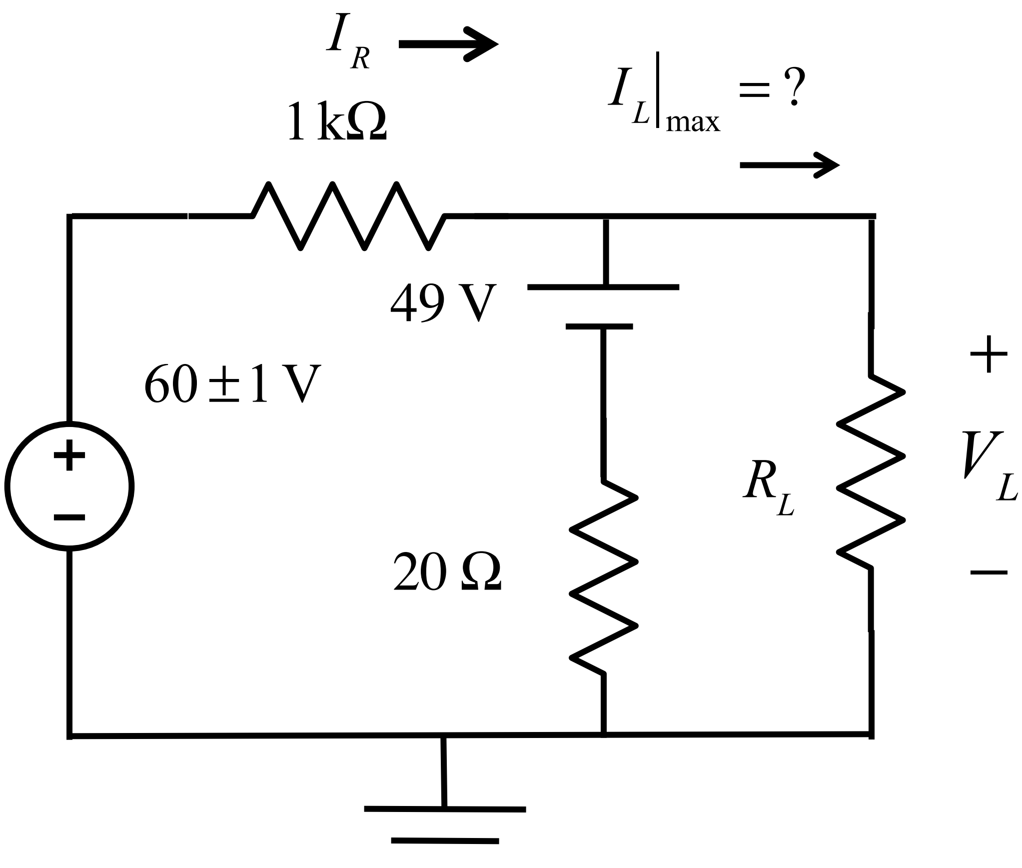
What is the change in output voltage if the load draws 2 mA of current?

Question



Question: What value of load resistor produces 2 mA of load current?

Maximum load current



$$I_{ZK} = 0.3 \text{ mA}$$

$$V_L \approx 49 \text{ V}$$

$$I_R \approx \frac{60-49}{1\text{ k}} = 11 \text{ mA}$$

$$I_L\big|_{\max} = I_R - I_{ZK} = 10.7 \text{ mA}$$

What maximum load current can be drawn if the regulator is to operate properly?

Summary

Model have **model parameters**. To get good results, we need a good model and accurate model parameters.

We discussed three types of diode models: i) ideal, ii) constant-voltage-drop, and iii) mathematical (exponential).

Small signal model parameters depend on the DC bias.

The small signal diode model is a resistor (+ junction and diffusion capacitances).

A good Zener diode in breakdown is like a good battery.

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