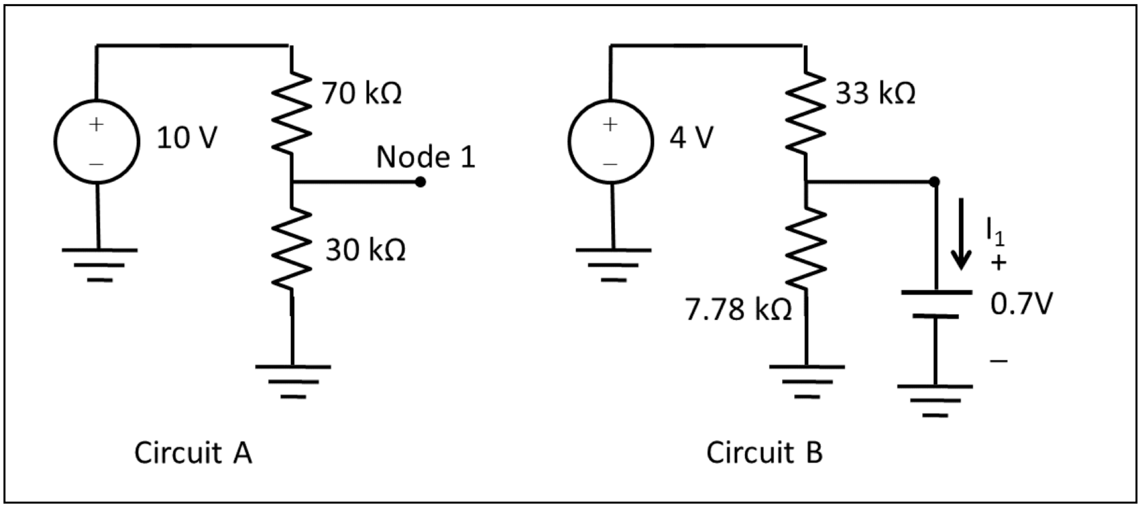
ECE 255 Spring 2019

Homework 1 **SOLUTIONS**

Due 5:00 PM Monday, Jan 14 in MSEE 180 Dropbox

1) (Review Problem) Consider the voltage source and resistor networks shown in Figure A.



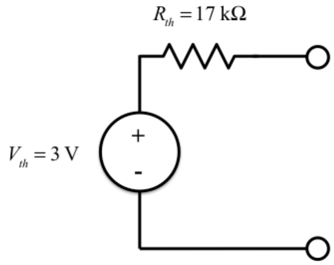
1a) For the circuit shown in Fig A, find the Thevenin equivalent network between node 1 and ground.

Solution:

For Rth, we short the 10 V power supply and see $R_{th} = 70 \,||\, 30 = 21\,\text{k}\Omega$

Vth is the voltage across the 30 k resistor: $V_{th} = \dfrac{30}{30+70}10 = 3\text{ V}$

$$\boxed{\begin{aligned} R_{th} &= 21\,\text{k}\Omega \\ V_{th} &= 3\text{ V} \end{aligned}}$$



HW1 Solutions (cont.)

1b) For the circuit shown in Fig A, what is the open-circuit voltage between node 1 and ground (i.e. voltage on node 1 for circuit shown in figure)?

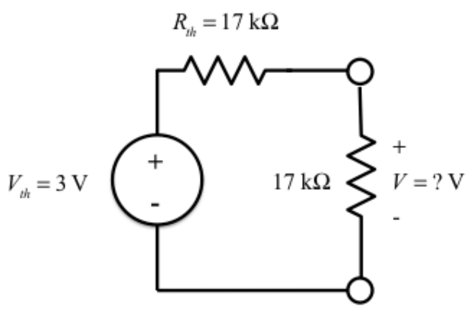
Solution:

$$V_{oc} = V_{th} = \frac{30}{30+70}10 = 3\text{ V} \qquad \boxed{V_{oc} = 3\text{ V}}$$

1c) For the circuit shown in Fig. A, suppose that we connect a load resistor of 21 kΩ between node 1 and ground. What is the voltage across this resistor?

Solution:

We now have the following circuit:



Voltage division gives: $\boxed{V = 1.5\text{ V}}$

1d) For the circuit shown in Fig B, what is the current I1?

Solution:

The 0.7 V battery puts 0.7 V across the 7.78 k resistor. The current through this resistor is:

$I_{7.78\text{k}} = \frac{0.7}{7.78} = 0.09\text{ mA}$. Since we know the voltage on the top and bottom of the 33 k resistor, we can compute the current as $I_{33\text{k}} = \frac{4-0.7}{33} = 0.1\text{ mA}$.

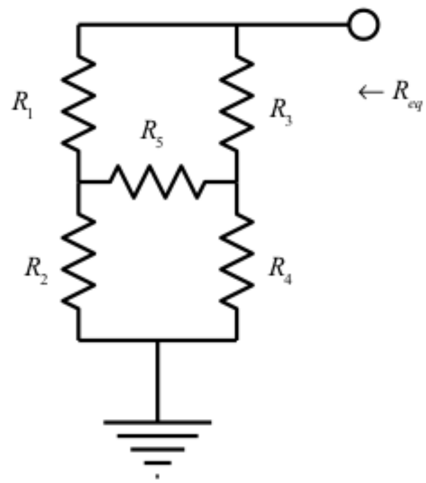
Now apply KCL to the node between the two resistors: $I_{33\text{k}} = I_{7.78\text{k}} + I_1$.

$$I_1 = I_{33\text{k}} - I_{7.78\text{k}} = 0.10 - 0.09 = 0.01\text{ mA}$$

$$\boxed{I_1 = 0.01\text{ mA}}$$

HW1 Solutions (cont.)

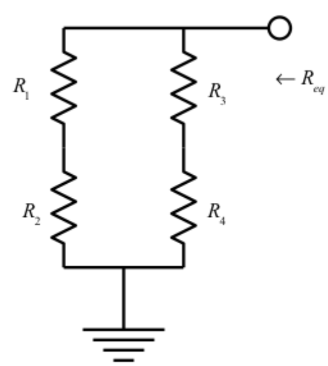
2) (Review Problem) Consider the circuit below



2a) Assume that $R_1 = R_2 = R_3 = R_4 = R_5 = 1\,\text{k}\Omega$ and find the Thevenin equivalent resistance, R_{eq}. (Hint: If you see what's going on, you can solve this problem by inspection.)

Solution:

Because of the symmetry of the circuit, we see that the voltage on each end of R_5 is the same, so no current flows through R_5. We can remove it from the circuit and nothing changes. We now have:



We can now see by inspection that $R_{eq} = \left(R_1 + R_2\right) \| \left(R_3 + R_4\right) = 2 \| 2 = 1\,\text{k}\Omega$

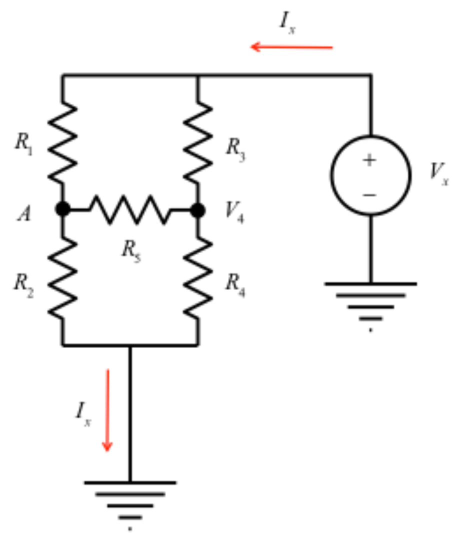
$\boxed{R_{eq} = 1\,\text{k}\Omega}$

HW1 Solutions (cont.)

2b) Assume that $R_1 = 2\ \text{k}\Omega$ and all other resistors are unchanged. Find the Thevenin equivalent resistance, R_{eq}.

Solution:

The problem is more complicated now because we have lost the symmetry that simplified things in part 2a). The circuit to be analyzed is:



KCL at node A gives: $\dfrac{V_A - V_x}{R_1} + \left(\dfrac{V_A - V_B}{R_5}\right) + \dfrac{V_A}{R_2} = \dfrac{V_A - V_x}{2} + \left(V_A - V_B\right) + V_A = 0$

(resistors in k, so currents in mA))

$$5V_A - 2V_B = V_x \quad (1)$$

KCL at node B gives: $\dfrac{V_B - V_x}{R_3} + \left(\dfrac{V_B - V_A}{R_5}\right) + \dfrac{V_B}{R_4} = V_B - V_x + \left(V_B - V_A\right) + V_B = 0$

$$3V_B - V_A = V_x \quad (2)$$

Solve (1) and (2) for:

$$V_A = \frac{2.5}{3.5} V_x \qquad V_B = \frac{6}{13} V_x$$

Solve for $I_x = \dfrac{V_A}{R_2} + \dfrac{V_B}{R_4} = \dfrac{0.38V_x}{1} + \dfrac{0.46V_x}{1} = 0.846V_x$ mA

Finally, $R_{th} = \dfrac{V_x}{I_x} = 1.18\ \text{k}\Omega$ $\qquad \boxed{R_{th} = 1.18\ \text{k}\Omega}$

3) The intrinsic carrier concentration, n_i, is an important parameter for a semiconductor. An approximate expression for n_i is

$$n_i = BT^{3/2}e^{-E_G/2k_BT},$$

where the temperature, T, is in Kelvin, the bandgap, E_G in Joules, and $k_B = 1.38\times10^{-23}$ J/K is Boltzmann's constant.

3a) Assuming that $B = 4.87\times10^{15}$ cm^{-3}K$^{-3/2}$ and that $E_G = 1.12$ eV for silicon, compute n_i at room temperature (27 °C), -55 °C and +125 °C (the lowest and highest temperatures represent the military specification for the range of temperatures over which electronics must operate). Note that we are ignoring the small but important temperature dependence of the bandgap.

Solution:

Since E_G is in electron volts, we should compute k_BT in electron volts. At 27 °C, $T = 273.15 + 27 = 300.15$

$$\frac{k_BT}{q}(27\ °\mathrm{C}) = \frac{1.38\times10^{-23}(300.15)}{1.6\times10^{-19}} = 0.0259$$

$$\frac{k_BT}{q}(-55\ °\mathrm{C}) = \frac{1.38\times10^{-23}(218.15)}{1.6\times10^{-19}} = 0.0188$$

$$\frac{k_BT}{q}(+125\ °\mathrm{C}) = \frac{1.38\times10^{-23}(398.15)}{1.6\times10^{-19}} = 0.0343$$

$$n_i(27\ °\mathrm{C}) = 4.87\times10^{15}(300.15)^{1.5}e^{-1.12/0.0259} = 1.02\times10^{10}\ \mathrm{cm}^{-3}$$

$$n_i(-55\ °\mathrm{C}) = 1.86\times10^{6}\ \mathrm{cm}^{-3}$$

$$n_i(+125\ °\mathrm{C}) = 3.2\times10^{12}\ \mathrm{cm}^{-3}$$

$$\boxed{\begin{aligned} &n_i(-55\ °\mathrm{C}) = 1.86\times10^{6}\ \mathrm{cm}^{-3}\\ &n_i(27\ °\mathrm{C}) = 1.02\times10^{10}\ \mathrm{cm}^{-3}\\ &n_i(+125\ °\mathrm{C}) = 3.2\times10^{12}\ \mathrm{cm}^{-3}\end{aligned}}$$

Note that the intrinsic concentration is very sensitive to temperature.

HW1 Solutions (cont.)

3b) For operation at high temperatures, wide bandgap semiconductors are needed. Repeat prob. 3a) for gallium nitride (GaN) assuming that $E_G = 3.4$ eV and $B = 1.85\times10^{15}$ cm^{-3}K$^{-3/2}$ and that $T = +125$ °C.

Solution:

$$n_i\left(125\,°\text{C}\right) = 1.85\times10^{15}\left(398.15\right)^{1.5} e^{-3.4/0.0343} = 4.65\times10^{-3}\ \text{cm}^{-3}$$

$$\boxed{n_i\left(125\,°\text{C}\right) = 4.65\times10^{-3}\ \text{cm}^{-3}}$$

Note that the intrinsic concentration is very sensitive to bandgap. Large bandgaps give small intrinsic carrier concentrations.

4) Consider a Si at two different temperatures: 1) room temperature, 300 K and 2) an elevated temperature of 700 K. Assuming that $n_i\left(300\ \text{K}\right) = 1.0\times10^{10}$ cm^{-3} and $n_i\left(700\ \text{K}\right) = 2.9\times10^{16}$ cm^{-3}, calculate the equilibrium electron and hole concentrations (n and p) for each of the following cases. Assume that the dopants are fully ionized.

4a) intrinsic material ($N_D = N_A = 0$)

Solution:

Since there are only intrinsic carriers, $n = p = n_i$

$$\boxed{\begin{aligned} n\left(300\ \text{K}\right) &= p\left(300\ \text{K}\right) = n_i\left(300\ \text{K}\right) = 1.0\times10^{10}\ \text{cm}^{-3} \\ n\left(700\ \text{K}\right) &= p\left(700\ \text{K}\right) = n_i\left(700\ \text{K}\right) = 2.9\times10^{16}\ \text{cm}^{-3} \end{aligned}}$$

4b) $N_D = 1.00\times10^{13}$ cm^{-3} $N_A = 0$

Solution:

At 300 K:

$$n_i << N_D\ ,\ n\left(300\ \text{K}\right) \approx N_D = 1.00\times10^{13}\ \text{cm}^{-3}$$

$$p\left(300\ \text{K}\right) \approx n_i^2\left(300\ \text{K}\right)/n\left(300\ \text{K}\right) = 10^{20}/10^{13} = 1.0\times10^{7}\ \text{cm}^{-3}$$

At 700 K:

$$n_i >> N_D,\ n\left(700\ \text{K}\right) = p\left(700\ \text{K}\right) = n_i\left(700\ \text{K}\right) = 2.9\times10^{16}\ \text{cm}^{-3}$$

$$\boxed{\begin{array}{l} n(300\text{ K}) \approx 1.0\times10^{17}\text{ cm}^{-3} \\ p(300\text{ K}) \approx 1.0\times10^{7}\text{ cm}^{-3} \\ n(700\text{ K}) = p(700\text{ K}) = n_i(700\text{ K}) = 2.9\times10^{16}\text{ cm}^{-3} \end{array}}$$

4c) $N_D = 5.00\times10^{16}\text{ cm}^{-3}\quad N_A = 0$

Solution:

At 300 K:

$n_i \ll N_D$, $n(300\text{ K}) \approx N_D = 5.00\times10^{16}\text{ cm}^{-3}$

$p(300\text{ K}) \approx n_i^2(300\text{ K})/n(300\text{ K}) = 10^{20}/(5\times10^{16}) = 2.0\times10^{3}\text{ cm}^{-3}$

At 700 K:

$n_i < N_D$, but the intrinsic concentration is not negligible, so we should be more careful. As discussed in the notes, assuming space charge neutrality and equilibrium, we find (for fully ionized dopants):

$$n = \frac{N_D - N_A}{2} + \left[\left(\frac{N_D - N_A}{2}\right)^2 + n_i^2\right]^{1/2}$$

Plugging in numbers:

$$n = \frac{5\times10^{16} - 0}{2} + \left[\left(\frac{5\times10^{16} - 0}{2}\right)^2 + \left(2.9\times10^{16}\right)^2\right]^{1/2} = 6.33\times10^{16}$$

$$p(700\text{ K}) = n_i^2(700\text{ K})/n(700\text{ K}) = \left(2.9\times10^{16}\right)^2/\left(6.33\times10^{16}\right) = 1.33\times10^{16}\text{ cm}^{-3}$$

$$\boxed{\begin{array}{l} n(300\text{ K}) \approx 5.00\times10^{16}\text{ cm}^{-3} \\ p(300\text{ K}) \approx 2.00\times10^{3}\text{ cm}^{-3} \\ n(700\text{ K}) = 6.33\times10^{16}\text{ cm}^{-3} \\ p(700\text{ K}) = 1.33\times10^{16}\text{ cm}^{-3} \end{array}}$$

4d) $N_D = 0\quad N_A = 5.00\times10^{16}\text{ cm}^{-3}$

Solution:

This is just the P-type version of the N-type problem, 4c). No work is necessary, the answers are:

$$\boxed{\begin{aligned} &p(300\text{ K}) \approx 5.00\times10^{16}\text{ cm}^{-3}\\ &n(300\text{ K}) \approx 2.00\times10^{3}\text{ cm}^{-3}\\ &p(700\text{ K}) = 6.33\times10^{16}\text{ cm}^{-3}\\ &n(700\text{ K}) = 1.33\times10^{16}\text{ cm}^{-3} \end{aligned}}$$

4e) $N_D = 1.00\times10^{18}\text{ cm}^{-3}$ $N_A = 3.00\times10^{18}\text{ cm}^{-3}$

Solution:

Only the net doping, $N_A - N_D = 2.00\times10^{18}\text{ cm}^{-3}$, matters. We see that at both temperatures, $n_i << N_A - N_D$, so the semiconductor is always **extrinsic**.

$$p(300\text{ K}) \approx p(700\text{ K}) \approx 2.0\times10^{18}\text{ cm}^{-3}$$

At 300 K:

$$n(300\text{ K}) \approx n_i^2(300\text{ K})/p(300\text{ K}) = 10^{20}/(2.0\times10^{18}) = 5.0\times10^{1}\text{ cm}^{-3}$$

At 700 K

$$n(700\text{ K}) \approx n_i^2(700\text{ K})/p(700\text{ K}) = (2.9\times10^{16})^2/(2.0\times10^{18}) = 4.21\times10^{14}\text{ cm}^{-3}$$

$$\boxed{\begin{aligned} &p(300\text{ K}) \approx 2.00\times10^{18}\text{ cm}^{-3}\\ &n(300\text{ K}) \approx 5.00\times10^{1}\text{ cm}^{-3}\\ &p(700\text{ K}) = 2.00\times10^{18}\text{ cm}^{-3}\\ &n(700\text{ K}) = 4.21\times10^{14}\text{ cm}^{-3} \end{aligned}}$$