

ECE-305: Spring 2018

Material Properties

Pierret, *Semiconductor Device Fundamentals* (SDF)

Chapter 1 (pp. 3-19)

Chapter 2 (pp. 22-32)

Professor Peter Bermel

Electrical and Computer Engineering

Purdue University, West Lafayette, IN USA

pbermel@purdue.edu

outline

1. Graphene
2. Silicon
3. Miller indices
4. Quantization of energy levels
5. Energy bands
6. Electrons and holes
7. Intrinsic carriers
8. Doping

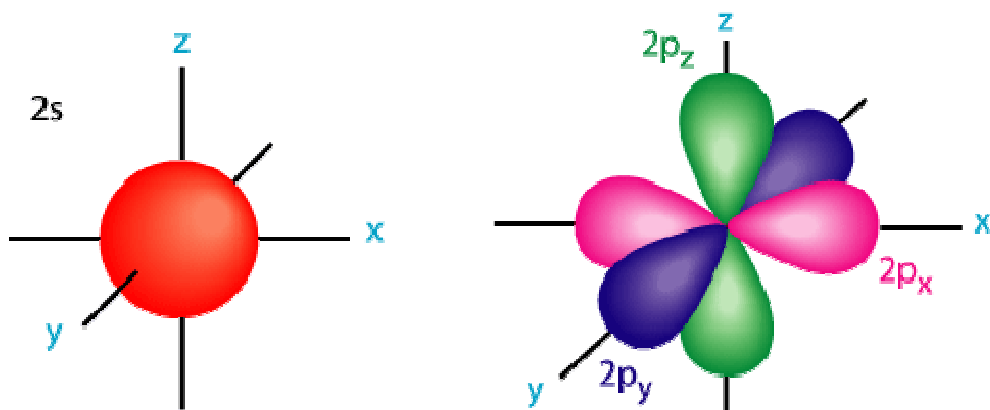
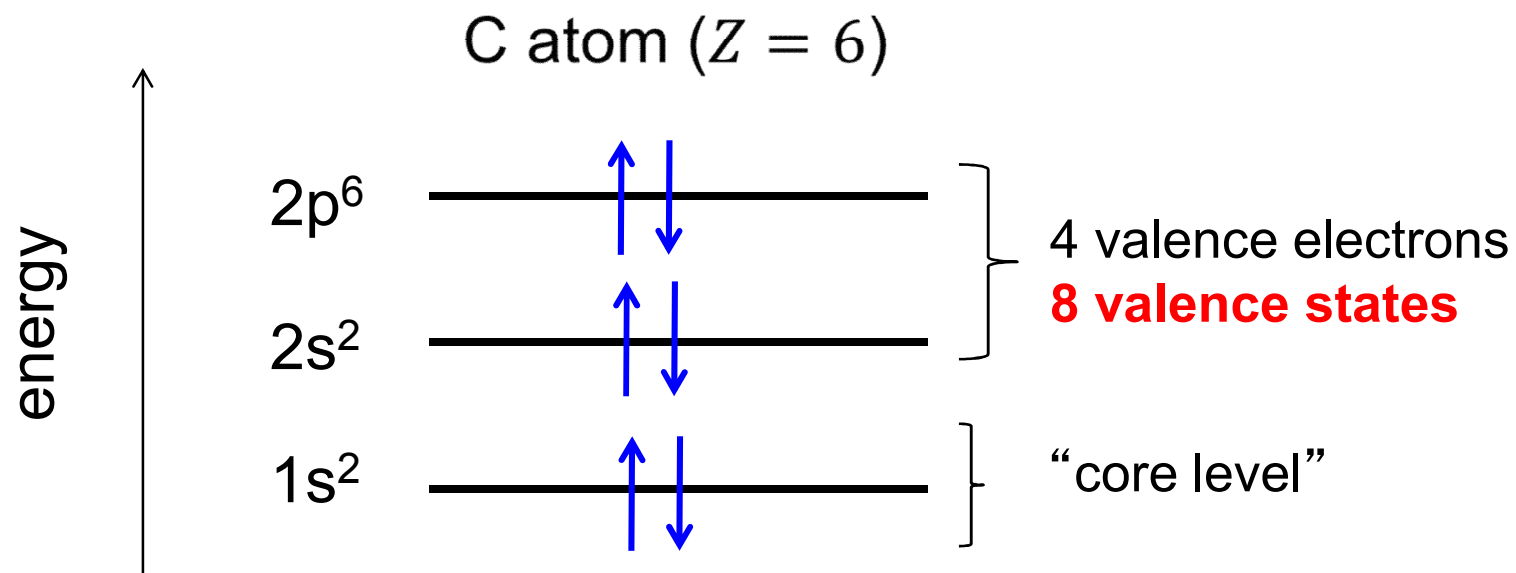
semiconductors

column
4

Period	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	
1	1 H																		2 He
2	3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne	
3	11 Na	12 Mg											13 Al	14 Si	15 P	16 S	17 Cl	18 Ar	
4	19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr	
5	37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe	
6	55 Cs	56 Ba	* Lanthanoids	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn	
7	87 Fr	88 Ra	** Actinoids	104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111 Rg	112 Cn	113 Uut	114 Uuq	115 Uup	116 Uuh	117 Uus	118 Uuo	
			57 La	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu		
			89 Ac	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr		

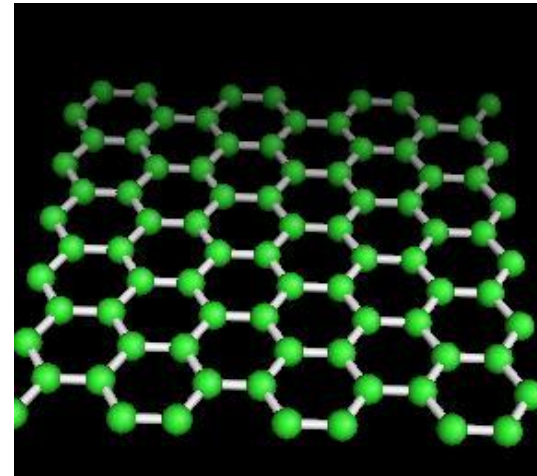
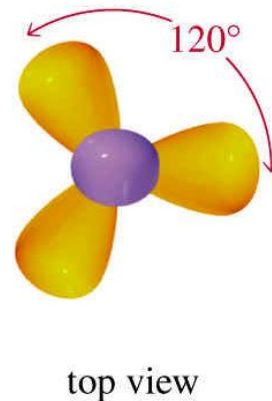
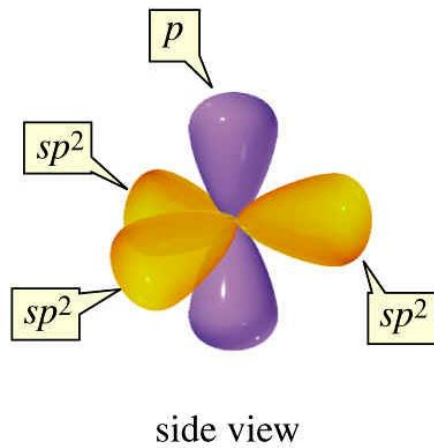
http://en.wikipedia.org/wiki/Periodic_table

carbon energy levels



Graphene: 2011 Nobel Prize in Physics

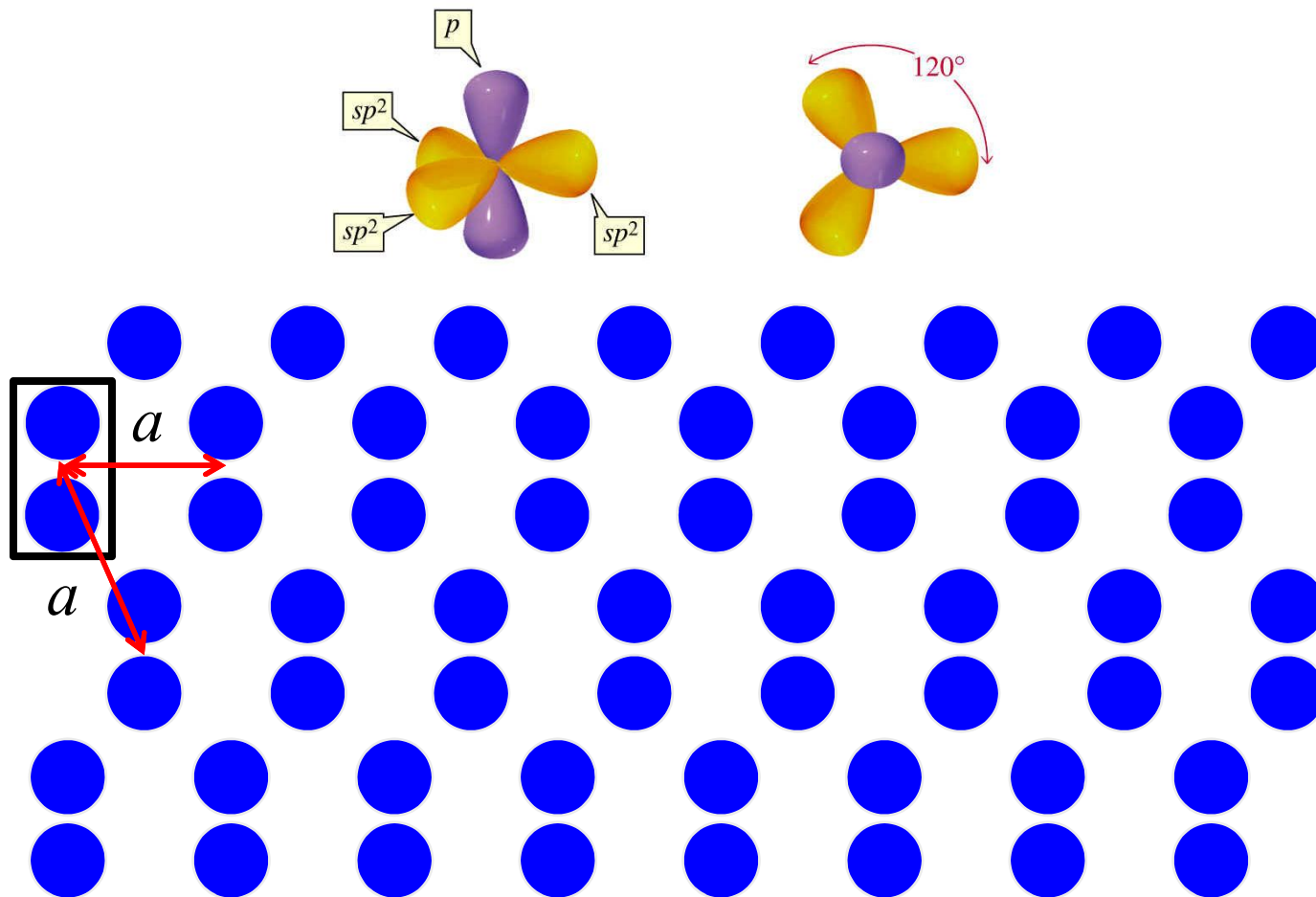
Graphene is a one-atom-thick planar *crystalline* carbon sheet with a triangular lattice with 2 atoms per unit cell.



source: CNTBands 2.7.2

<https://nanohub.org/resources/1838>

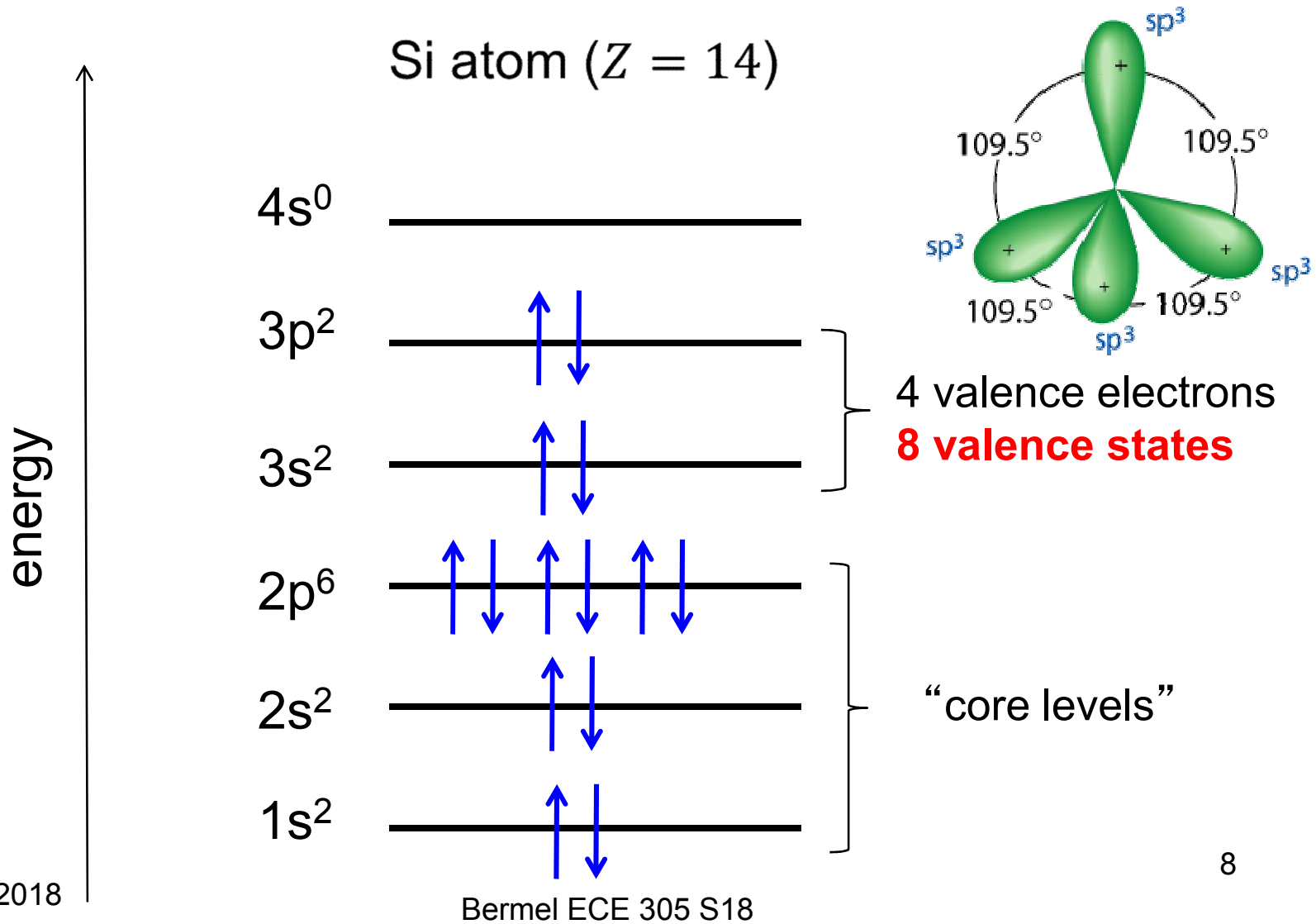
triangular lattice + 2 atom basis



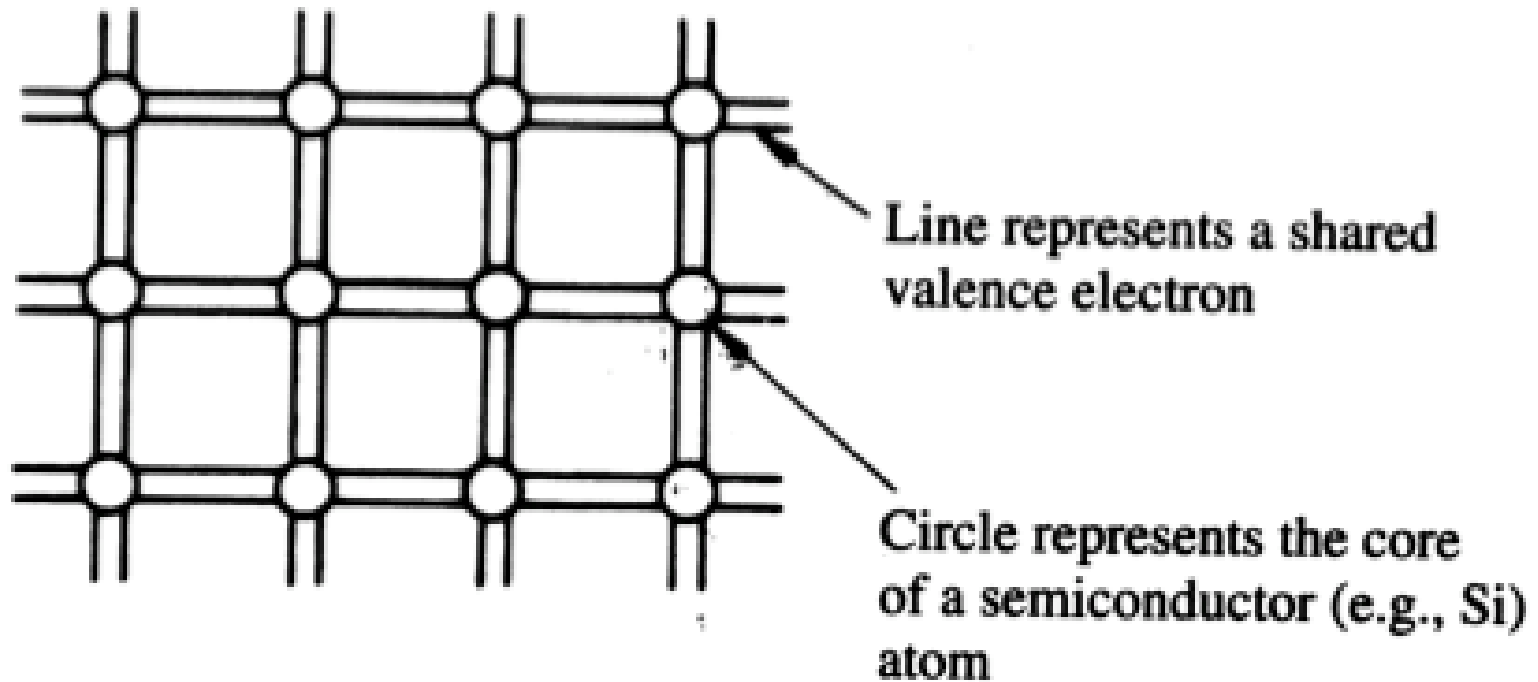
outline

- ✓ 1. Graphene
- 2. Silicon
- 3. Miller indices
- 4. Quantization of energy levels
- 5. Energy bands
- 6. Electrons and holes
- 7. Intrinsic carriers
- 8. Doping

silicon energy levels
































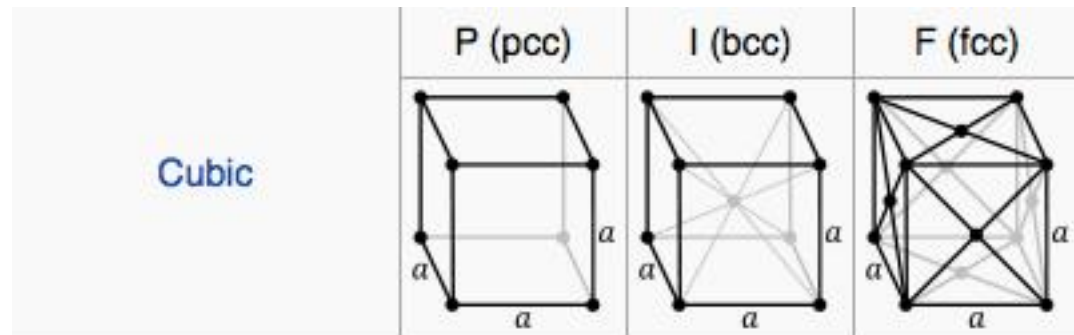
“cartoon” Si crystal



What would you get from combining carbon in the same way?

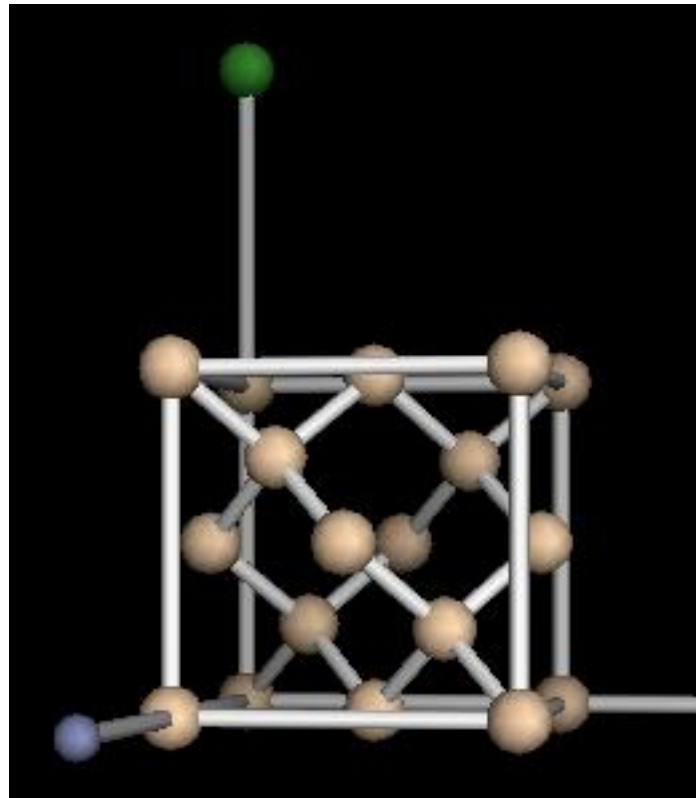
3D crystal structure

The 7 lattice systems	The 14 Bravais lattices			
Triclinic	P $\alpha, \beta, \gamma \neq 90^\circ$ 			
Monoclinic	P $\beta \neq 90^\circ$ $\alpha, \gamma = 90^\circ$ 		C $\beta \neq 90^\circ$ $\alpha, \gamma = 90^\circ$ 	
	 			
Orthorhombic	P $a \neq b \neq c$ 		C $a \neq b \neq c$ 	
	I $a \neq b \neq c$ 		F $a \neq b \neq c$ 	
	   			
	 			
Tetragonal	P $a \neq c$ 		I $a \neq c$ 	
	 			
Rhombohedral	P $a = b = c$ $\alpha = \beta = \gamma \neq 90^\circ$ 			
Hexagonal	P 			
Cubic	P (bcc) 		I (bcc) 	
	F (fcc) 		 	
	  			



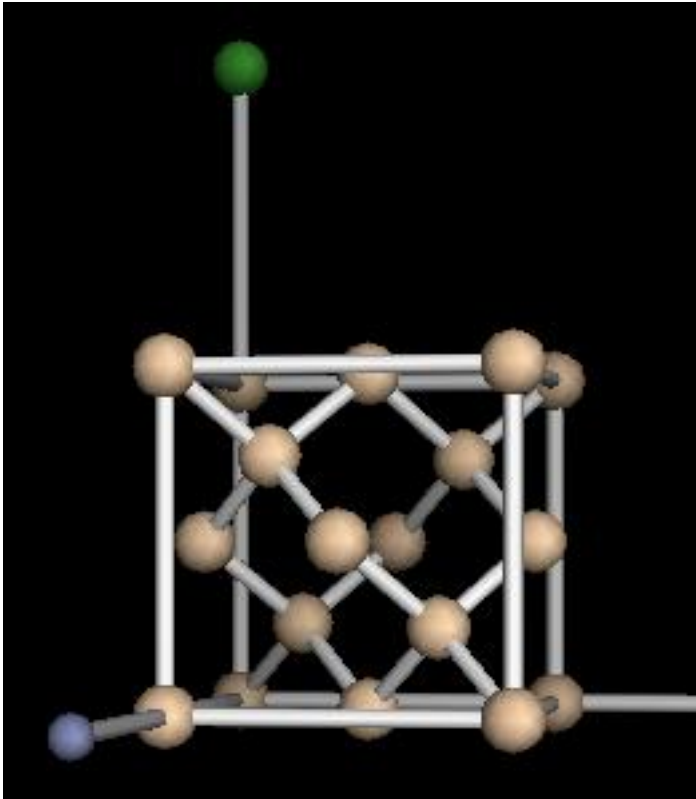
http://en.wikipedia.org/wiki/Bravais_lattice

silicon in diamond lattice



https://nanohub.org/tools/crystal_viewer

The diamond lattice



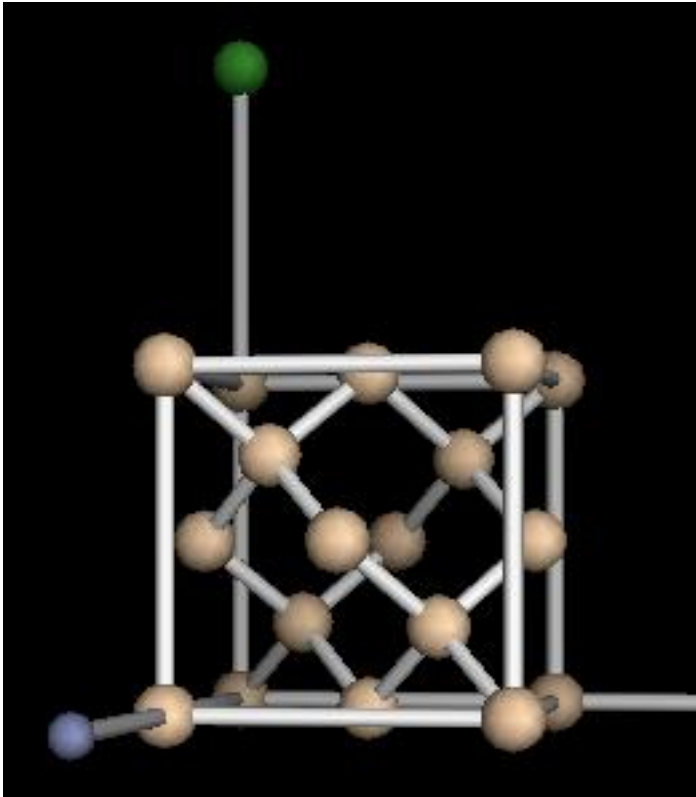
Atoms per unit cell

$$8 \text{ times } 1/8 + 6 \text{ times } 1/2 + 4$$

8 atoms per unit cell

https://nanohub.org/tools/crystal_viewer

Silicon: nearest neighbor (NN) spacing



Lattice constant: $a = 5.4307 \text{ \AA}$

Body diagonal = $a\sqrt{3}$.

NN spacing = $a\sqrt{3}/4$

https://nanohub.org/tools/crystal_viewer

Si atoms in a solid

- 1) In a **Si crystal**, each atom occupies, a specific location in a crystal lattice.
- 2) **Polycrystalline** Si consider of many crystalline “grains” with different orientations.
- 3) In **amorphous** Si, the atoms are more or less randomly distributed throughout the solid.

semiconductors

column
4

Period	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	
1	1 H																		2 He
2	3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne	
3	11 Na	12 Mg											13 Al	14 Si	15 P	16 S	17 Cl	18 Ar	
4	19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr	
5	37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe	
6	55 Cs	56 Ba	* La	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn	
7	87 Fr	88 Ra	** Ac	104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111 Rg	112 Cn	113 Uut	114 Uuq	115 Uup	116 Uuh	117 Uus	118 Uuo	
* Lanthanoids			57 La	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu		
** Actinoids			89 Ac	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr		

http://en.wikipedia.org/wiki/Periodic_table

semiconductors

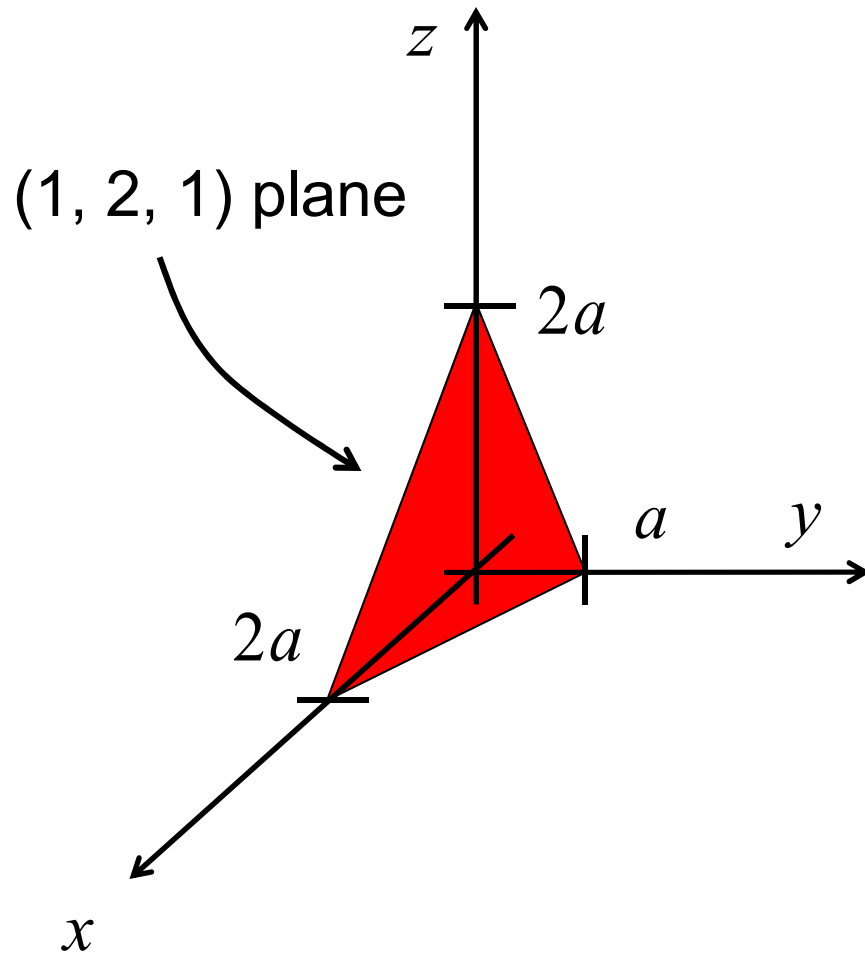
Period											Col. 3	Col. 5					Col. 2	
1	1																	2
	H																	He
2	3	4											5	6	7	8	9	10
	Li	Be											B	C	N	O	F	Ne
3	11	12											13	14	15	16	17	18
	Na	Mg											Al	Si	P	S	Cl	Ar
4	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
	K	Ca	Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn	Ga	Ge	As	Se	Br	Kr
5	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54
	Rb	Sr	Y	Zr	Nb	Mo	Tc	Ru	Rh	Pd	Ag	Cd	In	Sn	Sb	Te	I	Xe
6	55	56	*	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86
	Cs	Ba		Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg	Tl	Pb	Bi	Po	At	Rn
7	87	88	**	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118
	Fr	Ra		Rf	Db	Sg	Bh	Hs	Mt	Ds	Rg	Cn	Uut	Uuq	Uup	Uuh	Uus	Uuo
* Lanthanoids	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71			
	La	Ce	Pr	Nd	Pm	Sm	Eu	Gd	Tb	Dy	Ho	Er	Tm	Yb	Lu			
** Actinoids	89	90	91	92	93	94	95	96	97	98	99	100	101	102	103			
	Ac	Th	Pa	U	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No	Lr			

http://en.wikipedia.org/wiki/Periodic_table

outline

- ✓ 1. Graphene
- ✓ 2. Silicon
3. Miller indices
4. Quantization of energy levels
5. Energy bands
6. Electrons and holes
7. Intrinsic carriers
8. Doping

Miller index prescription for describing planes



x, y, and z-axis intercepts:

$2a, 1a, 2a$

$2, 1, 2$

invert:

$\frac{1}{2}, 1, \frac{1}{2}$

Rationalize:

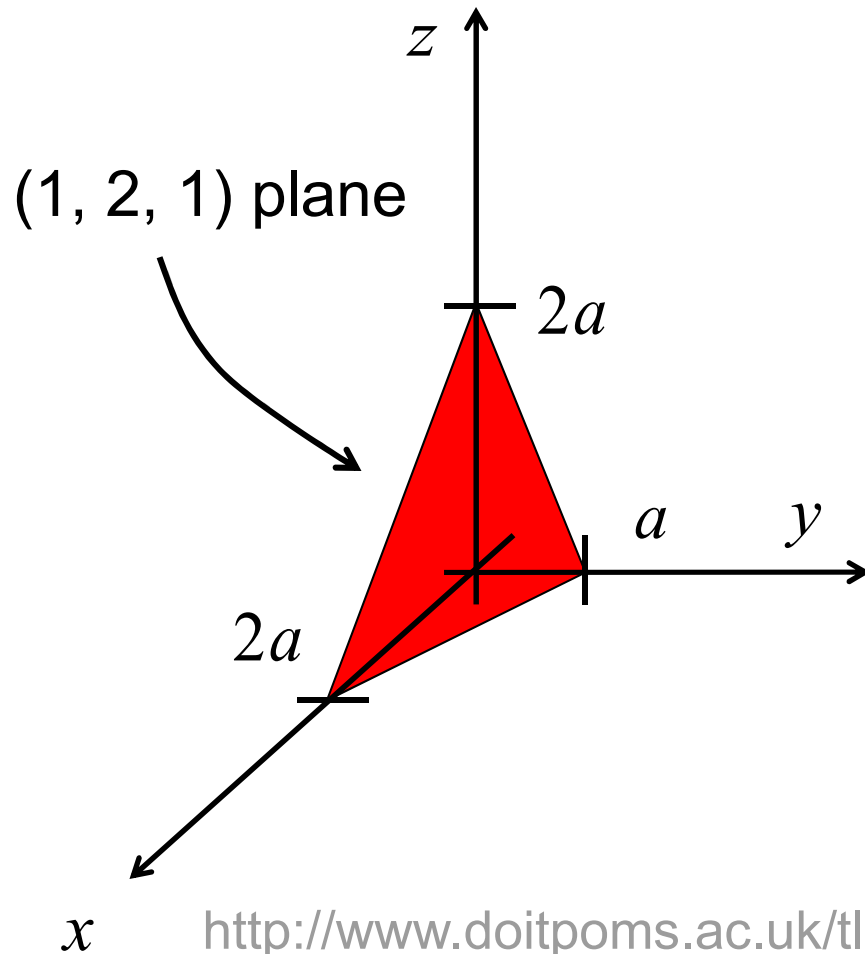
$1, 2, 1$

question

Where does this prescription come from?

Answer: If we remember the equation for a plane, we can figure it out.

where it comes from



equation of a plane:

$$\frac{x}{x_{\text{int}}} + \frac{y}{y_{\text{int}}} + \frac{z}{z_{\text{int}}} = 1$$

describe with the numbers:

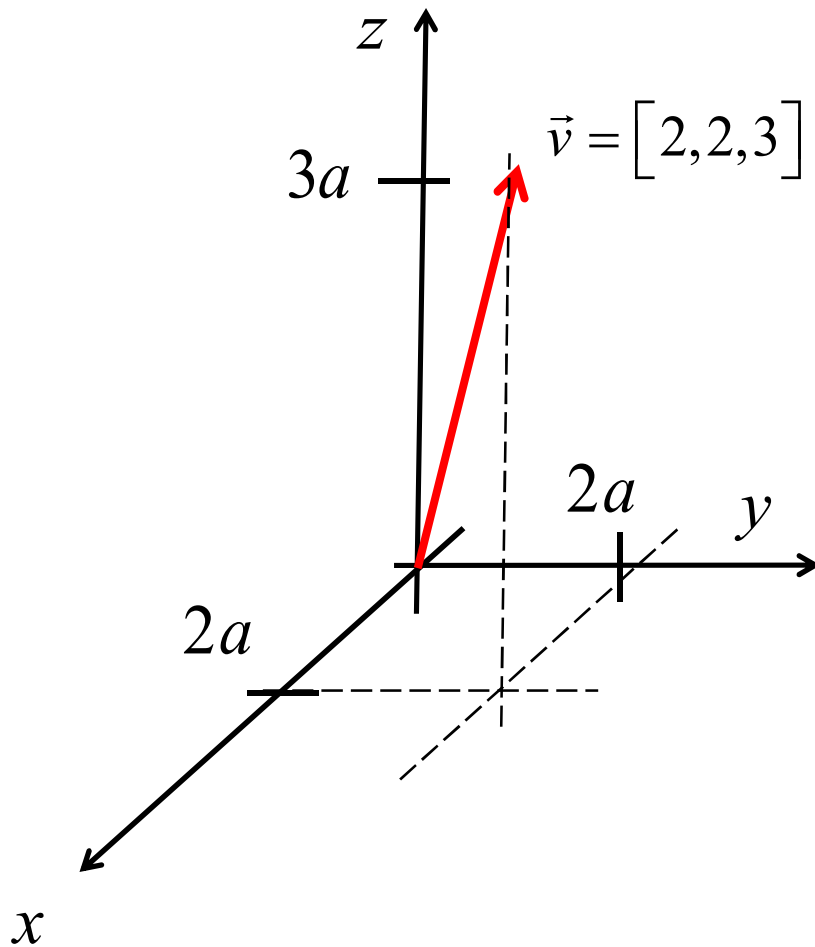
$$\frac{1}{x_{\text{int}}}, \frac{1}{y_{\text{int}}}, \frac{1}{z_{\text{int}}}$$

equivalent to:

$$\frac{1}{x_{\text{int}}/a}, \frac{1}{y_{\text{int}}/a}, \frac{1}{z_{\text{int}}/a}$$

x http://www.doitpoms.ac.uk/tlplib/miller_indices/lattice_draw.php

prescription for describing directions



equation of a vector:

$$\vec{v} = 2a\hat{x} + 2a\hat{y} + 3a\hat{z}$$

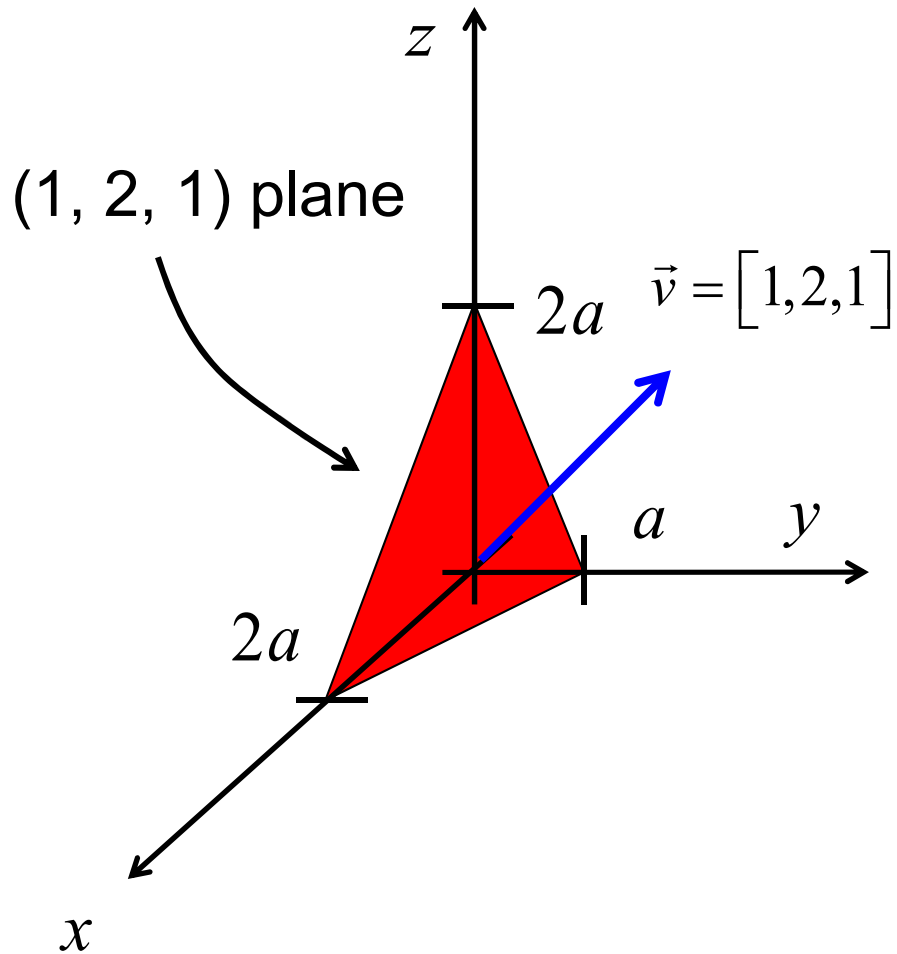
describe with components:

$$2a, 2a, 3a$$

equivalent to:

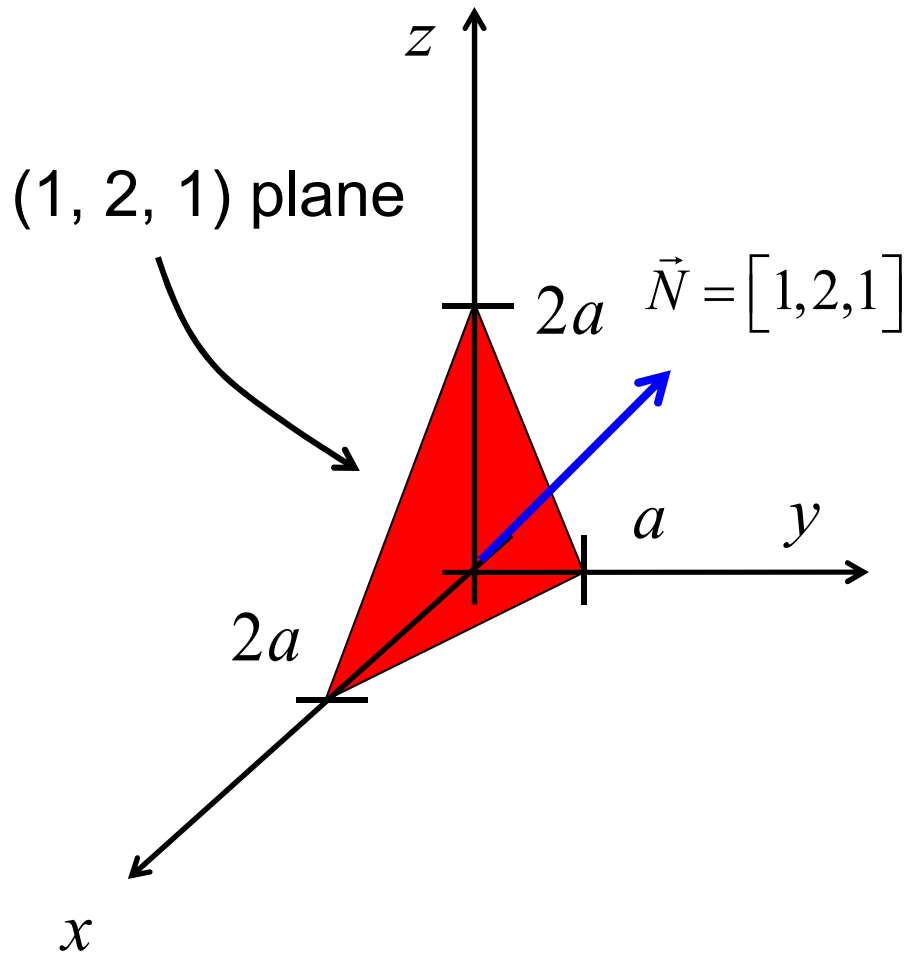
$$2, 2, 3$$

direction normal to a plane



Why is $[1, 2, 1]$ normal to $(1, 2, 1)$?

where it comes from



equation of a plane:

$$f(x, y, z) = \frac{x}{x_{\text{int}}} + \frac{y}{y_{\text{int}}} + \frac{z}{z_{\text{int}}} = 1$$

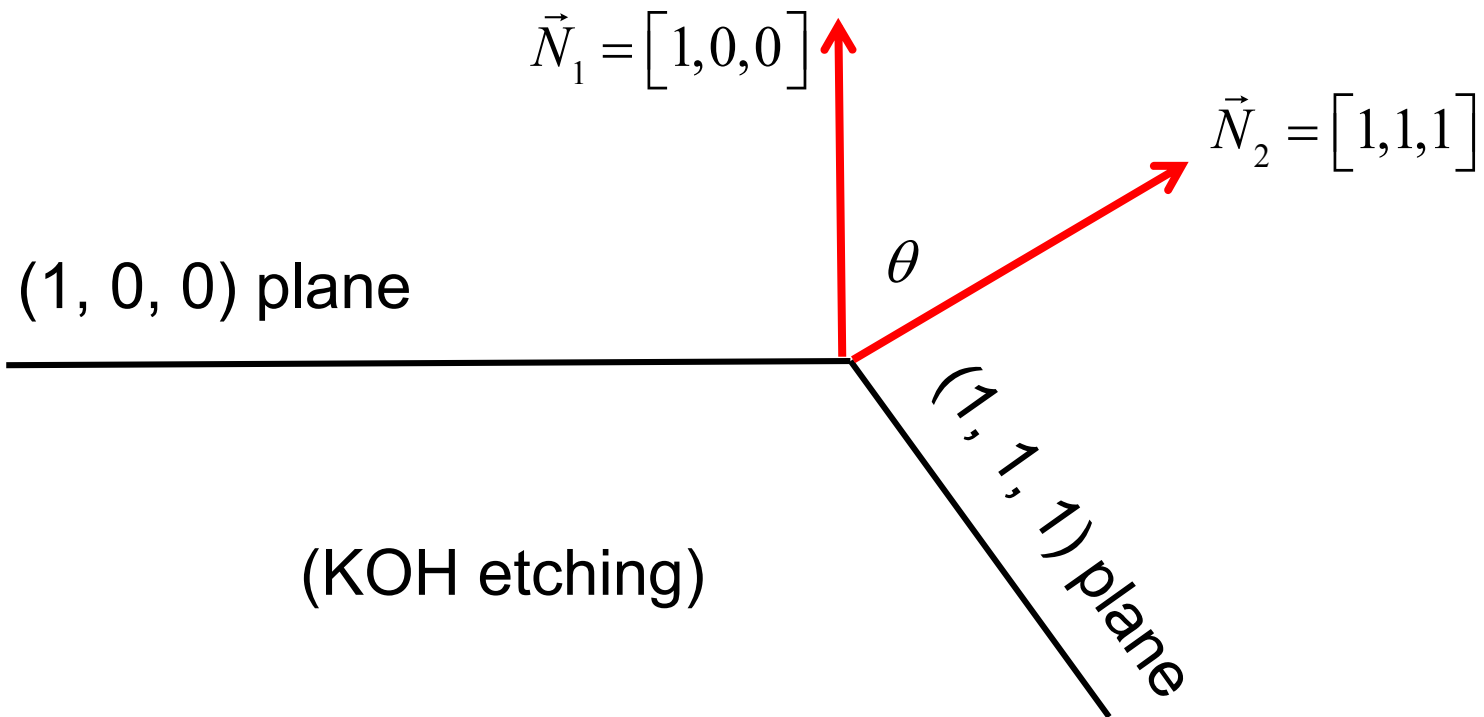
normal to a plane:

$$\vec{N} = \nabla f(x, y, z) = \frac{\partial f}{\partial x} \hat{x} + \frac{\partial f}{\partial y} \hat{y} + \frac{\partial f}{\partial z} \hat{z}$$

(gradient)

$$\vec{N} = \frac{1}{x_{\text{int}}} \hat{x} + \frac{1}{y_{\text{int}}} \hat{y} + \frac{1}{z_{\text{int}}} \hat{z}$$

angle between planes



$$\vec{N}_1 \bullet \vec{N}_2 = N_1 N_2 \cos \theta$$

angle between planes

$$\cos \theta = \frac{\vec{N}_1 \cdot \vec{N}_2}{N_1 N_2}$$

$$\vec{N}_1 = [h_1, k_1, l_1]$$

$$\vec{N}_2 = [h_2, k_2, l_2]$$

$$\cos \theta = \frac{h_1 h_2 + k_1 k_2 + l_1 l_2}{\sqrt{h_1^2 + k_1^2 + l_1^2} \sqrt{h_2^2 + k_2^2 + l_2^2}}$$

$$\vec{N}_1 = [1, 0, 0]$$

$$\vec{N}_2 = [1, 1, 1]$$

$$\cos \theta = \frac{1 + 0 + 0}{\sqrt{1^2 + 0^2 + 0^2} \sqrt{1^2 + 1^2 + 1^2}}$$

$$\cos \theta = \frac{1}{\sqrt{3}}$$

$$\theta = 54.7^\circ$$

summary

$(h\ k\ l)$ A specific plane.

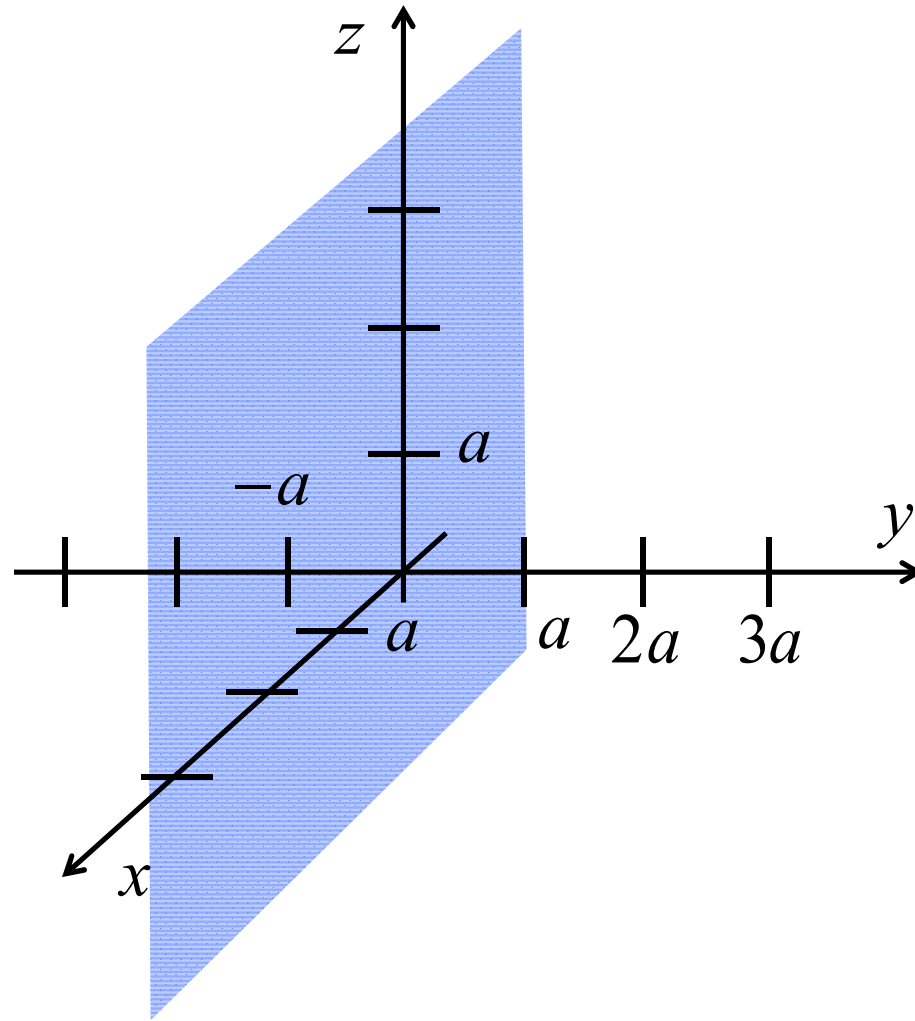
$[h\ k\ l]$ A direction normal to the plane above.

$$\vec{N} = ha\hat{x} + ka\hat{y} + la\hat{z}$$

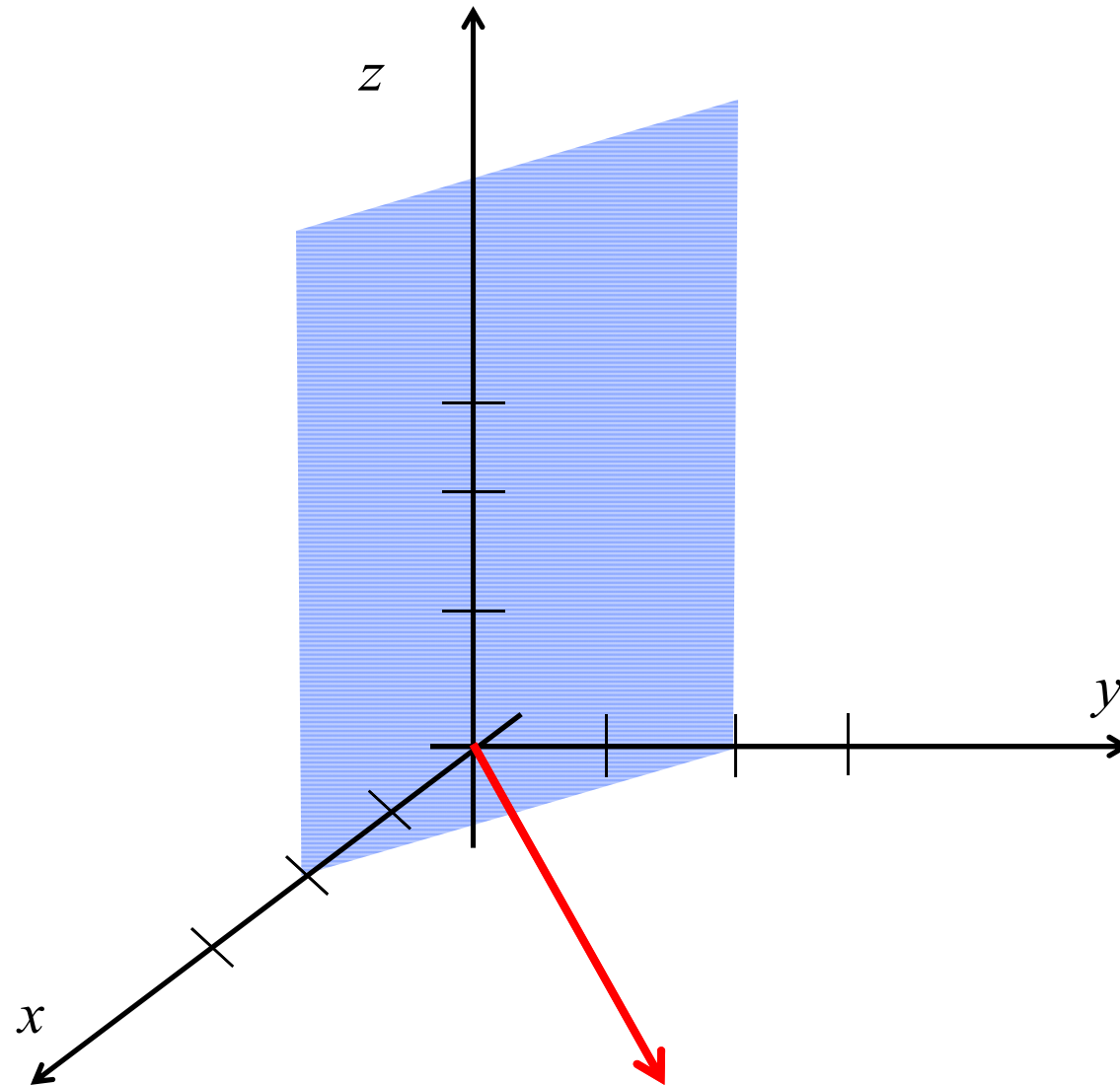
$\{h\ k\ l\}$ A set of equivalent planes.

$\langle h\ k\ l \rangle$ A set of equivalent directions.

what plane is this?



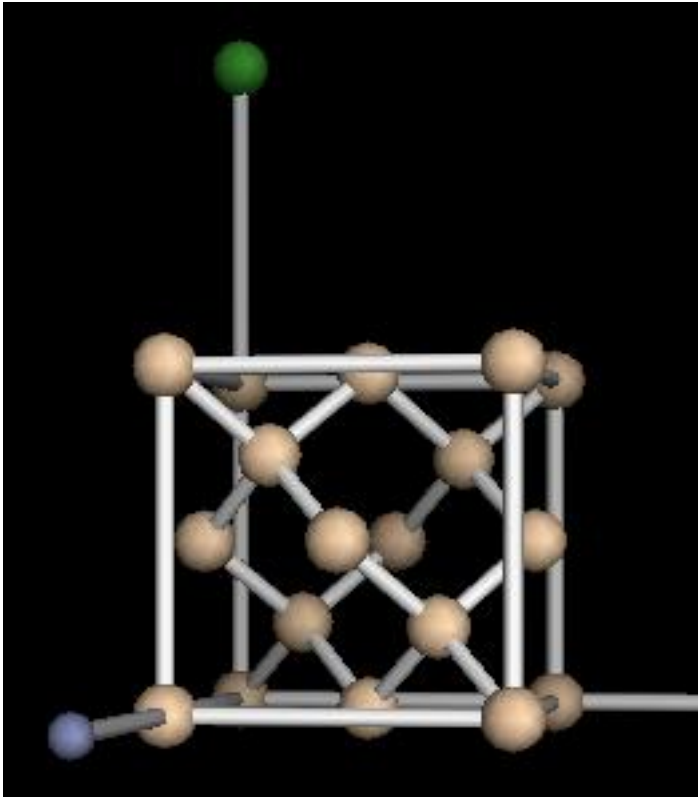
what plane is this?



Bermel ECE 305 S18

1/11/2018

Silicon: atoms / cm² on (100)



Lattice constant: 5.4307 Ang

Atoms on face = 4 times $\frac{1}{4}$ + 1 = 2

$$N_s = 2/a^2$$

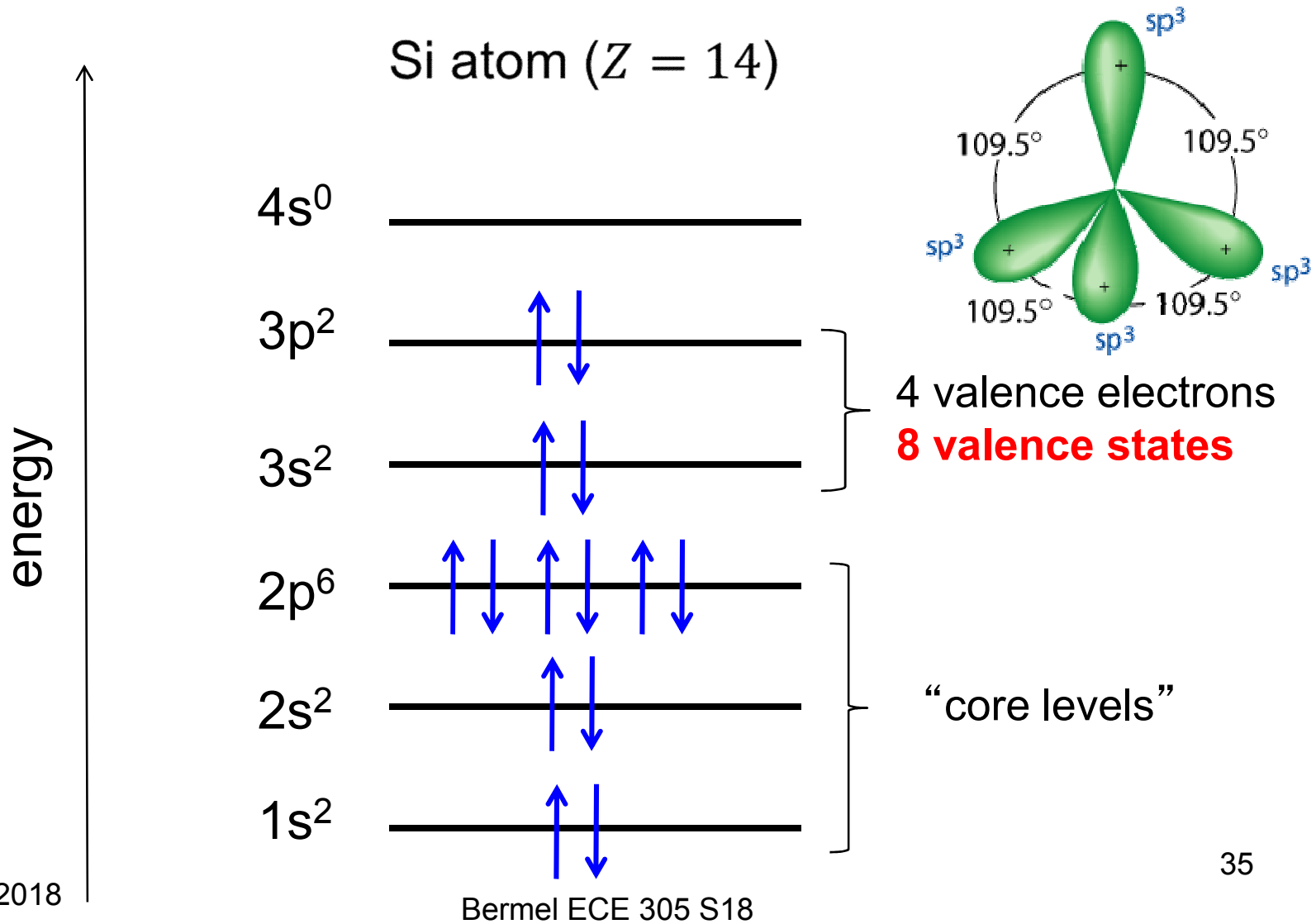
$$N_s = 6.81 \times 10^{14} / \text{cm}^2$$

https://nanohub.org/tools/crystal_viewer

outline

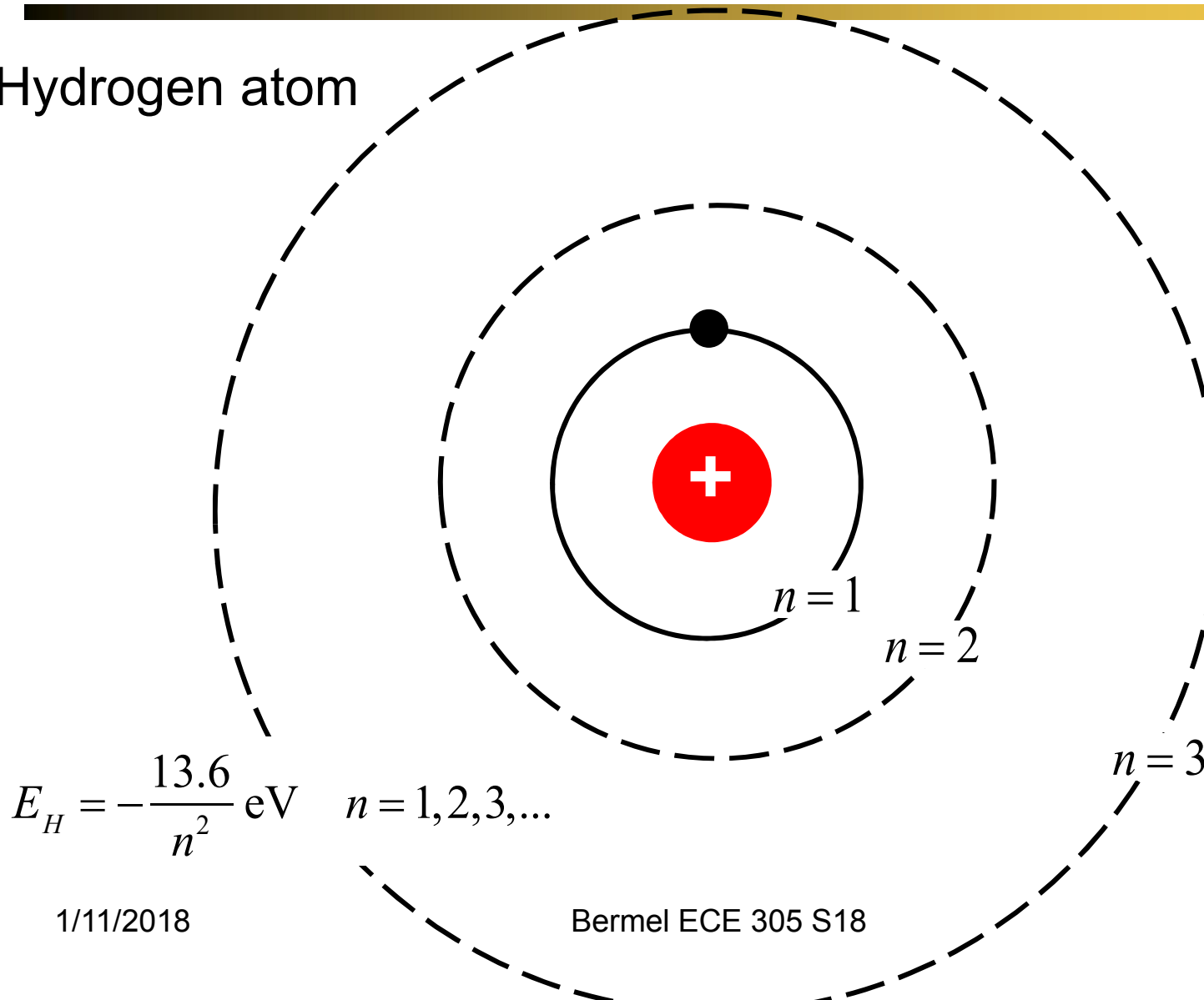
- ✓ 1. Graphene
- ✓ 2. Silicon
- ✓ 3. Miller indices
- 4. Quantization of energy levels
- 5. Energy bands
- 6. Electrons and holes
- 7. Intrinsic carriers
- 8. Doping

silicon energy levels



quantization of energy levels

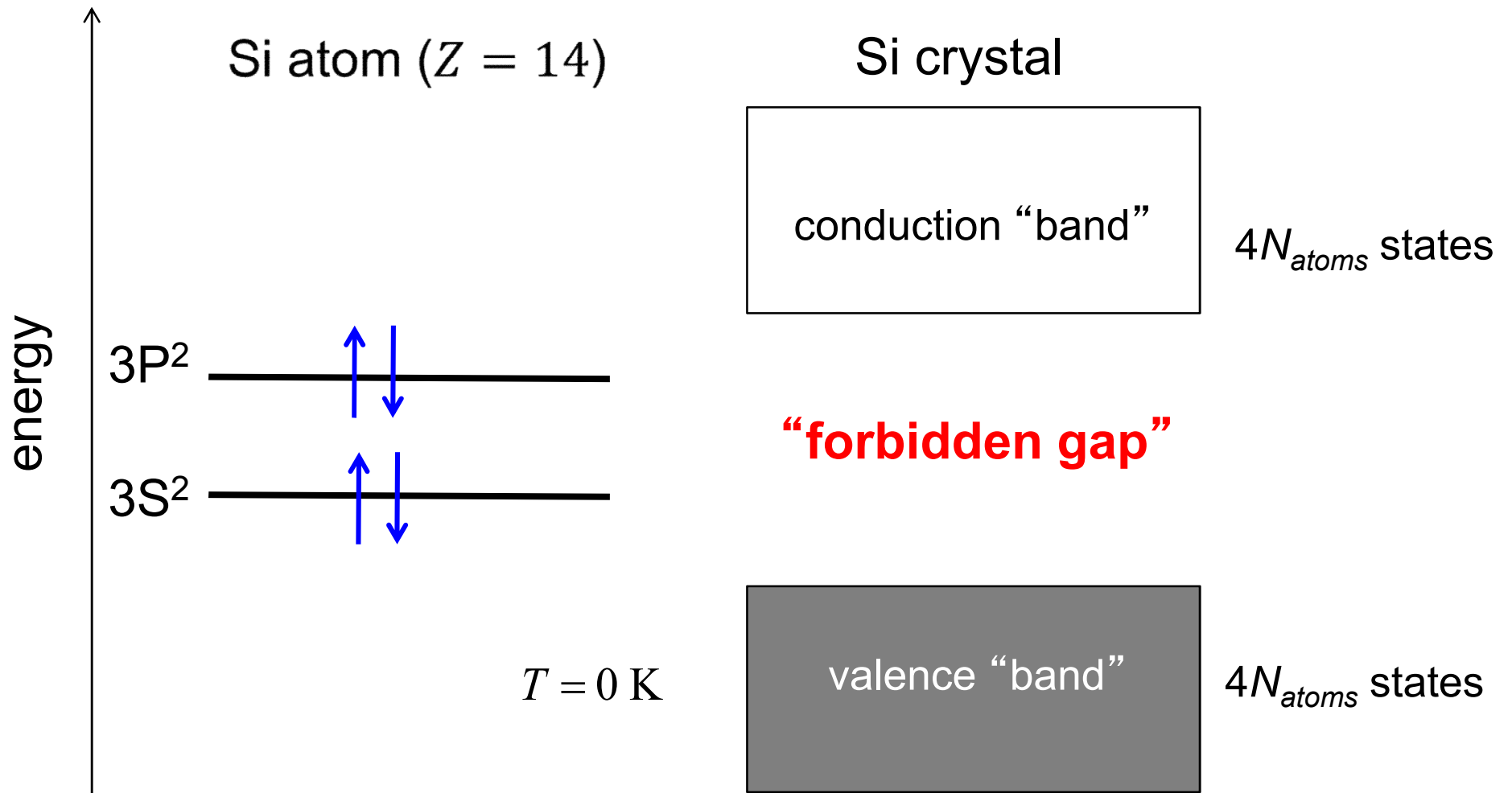
Hydrogen atom



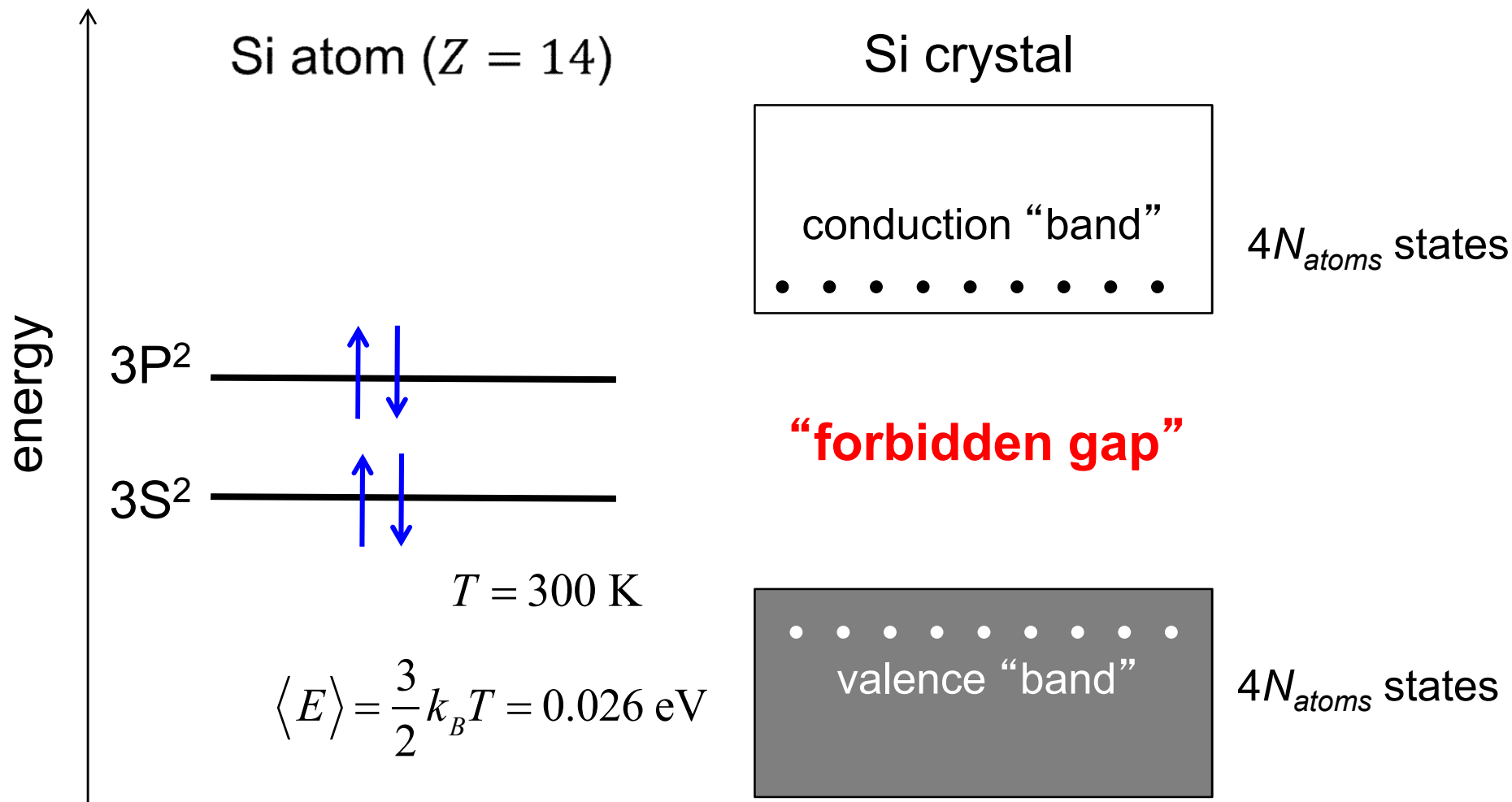
outline

- ✓ 1. Graphene
- ✓ 2. Silicon
- ✓ 3. Miller indices
- ✓ 4. Quantization of energy levels
- 5. Energy bands
- 6. Electrons and holes
- 7. Intrinsic carriers
- 8. Doping

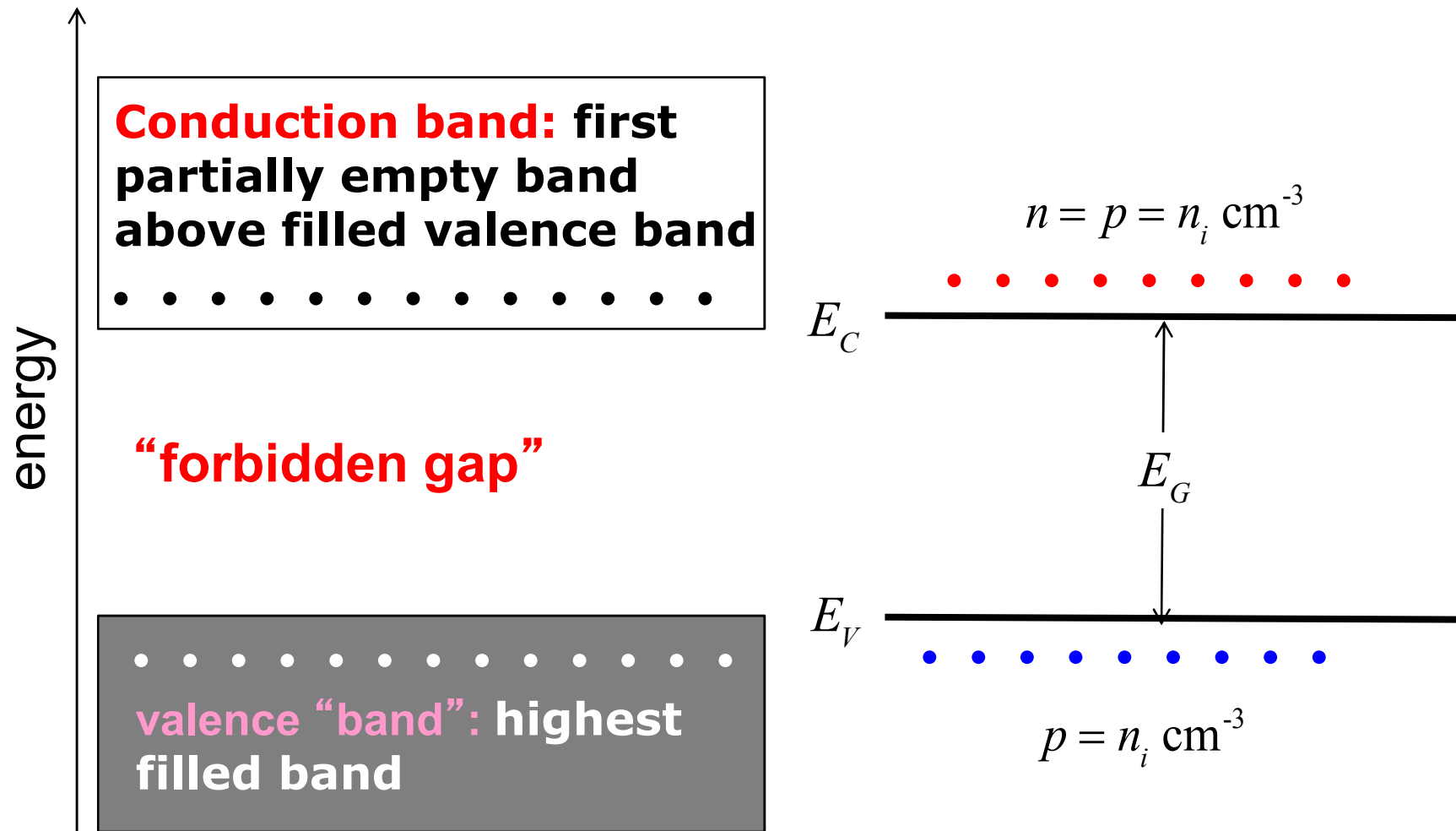
silicon energy levels → energy bands



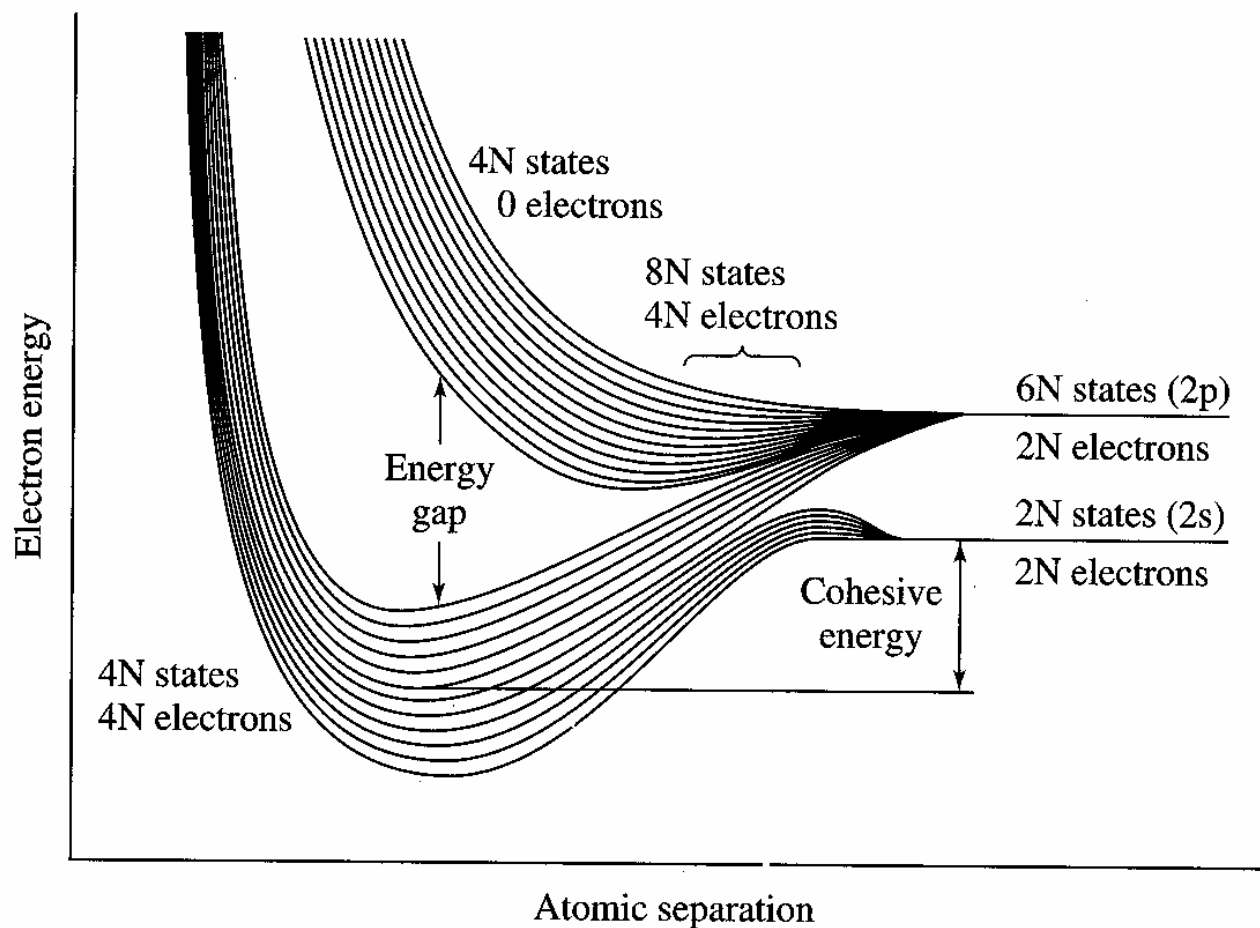
silicon energy levels → energy bands



energy band diagrams



energy bands versus atomic separation



Si atoms
 $1s^2 2s^2 2p^6 3s^2 2p^2$
 C atoms
 $1s^2 2s^2 2p^2$

2s – 2 states
 2p – 6 states

For N atoms:

2s line –
 2N-fold degenerate

2p line –
 6N-fold degenerate

insulators

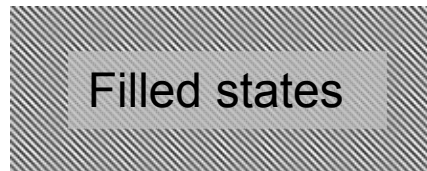
metals

semiconductors

Energy



$$E_G \approx 9 \text{ eV (SiO}_2\text{)}$$



don't conduct electricity well

1/11/2018

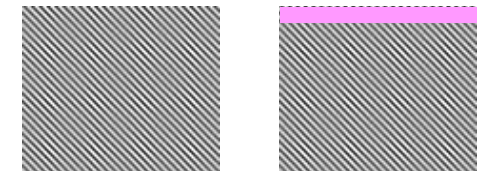


do conduct electricity well

Bermel ECE 305 S18



$$E_G \approx 1.1 \text{ eV (Si)}$$



in-between, **but** can be controlled

42

outline

- ✓ 1. Graphene
- ✓ 2. Silicon
- ✓ 3. Miller indices
- ✓ 4. Quantization of energy levels
- ✓ 5. Energy bands
- 6. Electrons and holes
- 7. Intrinsic carriers
- 8. Doping

summary

1. Most solid materials are crystals, which fill space with periodically repeated elements
2. Showed how atomic energy level quantization leads to energy band formation in materials
3. Three types of materials: insulators, metals, and semiconductors