ECE-305: Spring 2018

Carrier Action

Pierret, *Semiconductor Device Fundamentals* (SDF)
Chapter 2 (pp. 22-49)
Chapter 3 (pp. 75-89)

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PURDUE
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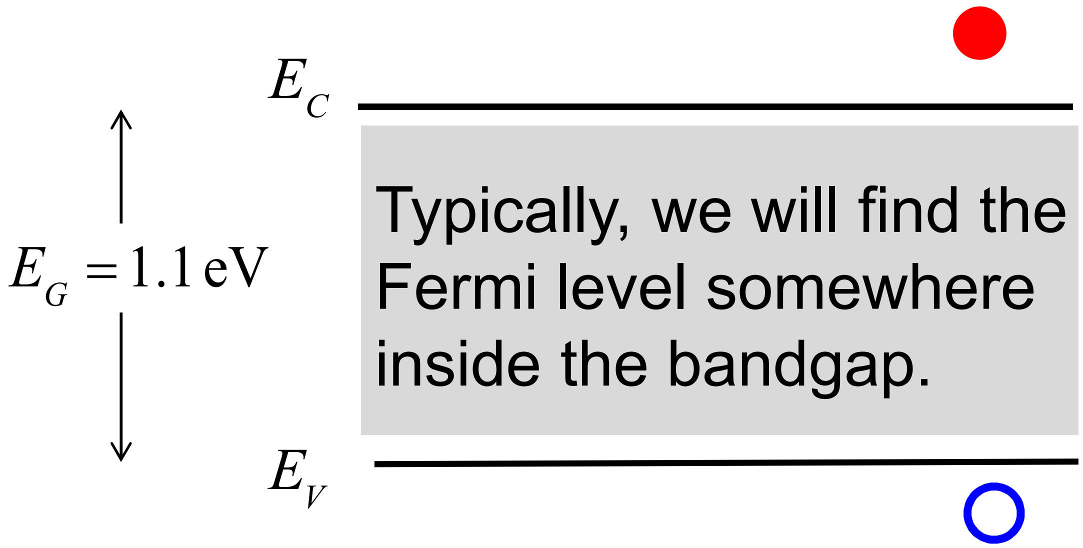
outline

1. Carrier distributions
2. Carrier concentrations
3. Current (drift)
4. Mobility, resistivity, etc.

Carrier Distribution Depends on Fermi Level

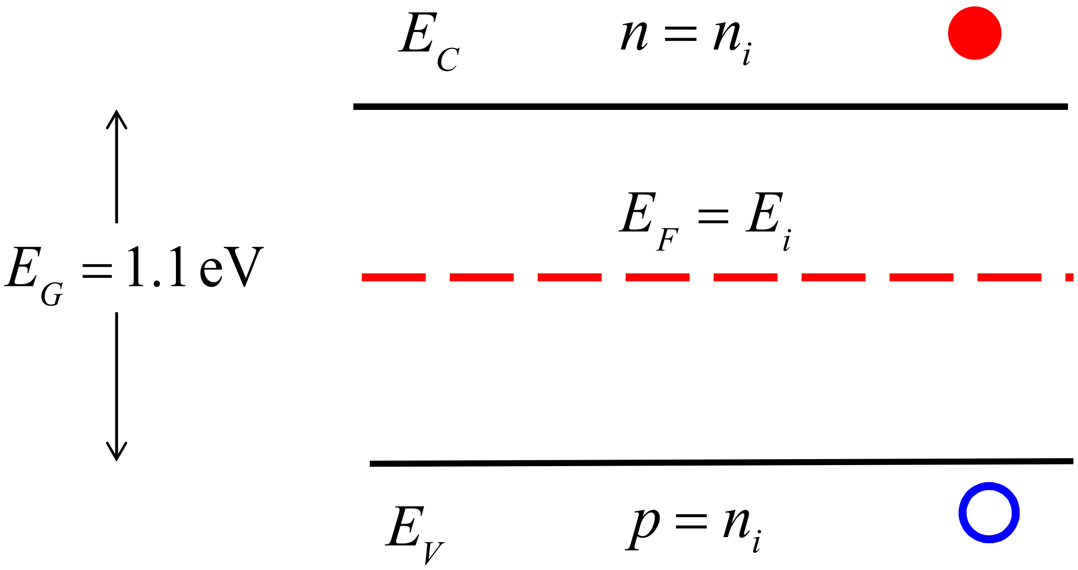
These states are way above the Fermi level.



Typically, we will find the Fermi level somewhere inside the bandgap.

These states are way below the Fermi level.

energy band diagram of an intrinsic semiconductor



E_C $n = n_i$

$E_G = 1.1\,\text{eV}$

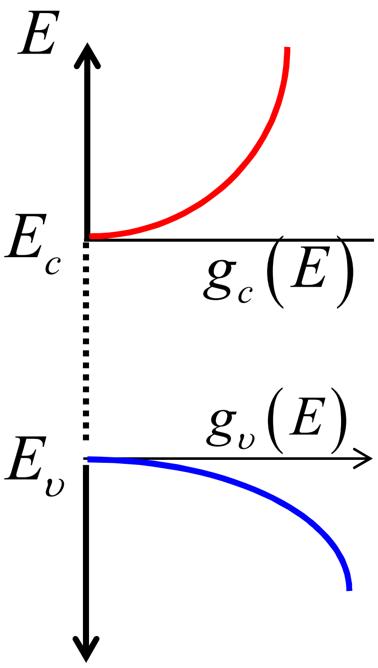
$E_F = E_i$

E_V $p = n_i$

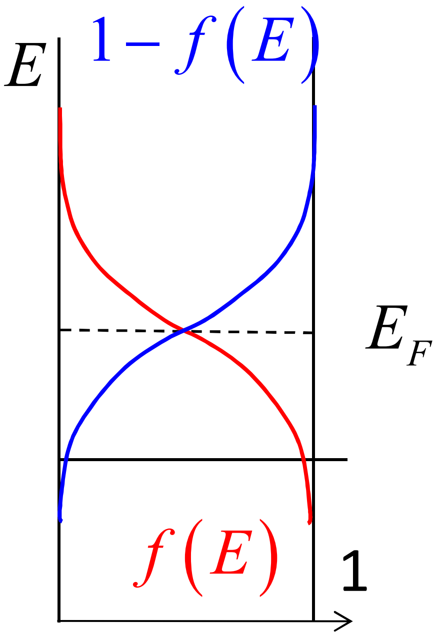
$$f(E) = \frac{1}{1 + e^{(E - E_F)/k_B T}}$$

Carrier Distribution

DOS

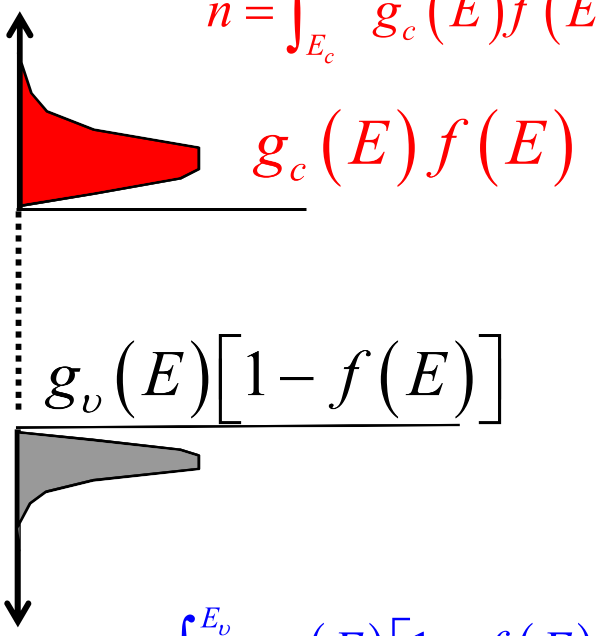


Fermi function



Carrier distribution

$$n=\int_{E_c}^{E_{top}} g_c(E)f(E)\,dE$$

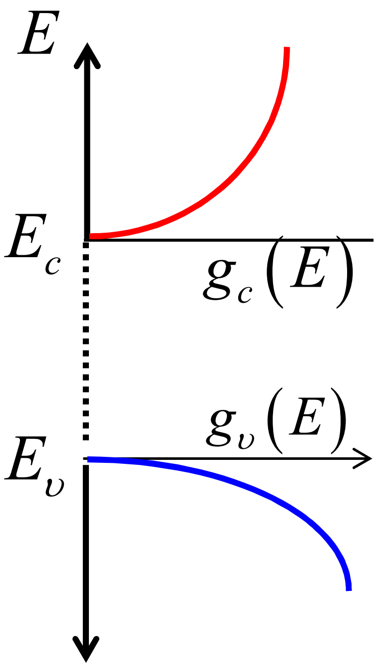


$$p=\int_{E_{bot}}^{E_\upsilon} g_\upsilon(E)\left[1-f(E)\right]dE$$

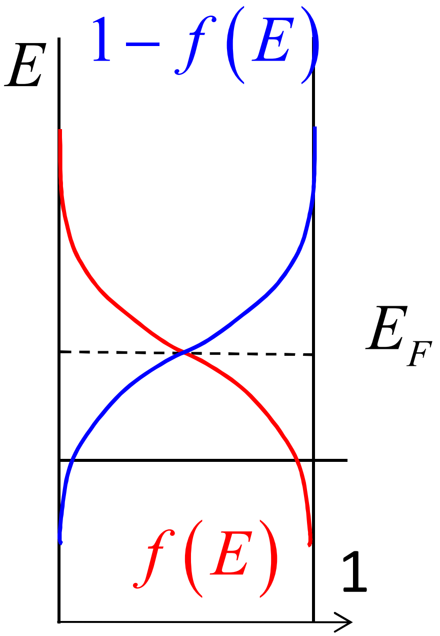
Fermi energy near mid-gap

Carrier Distribution

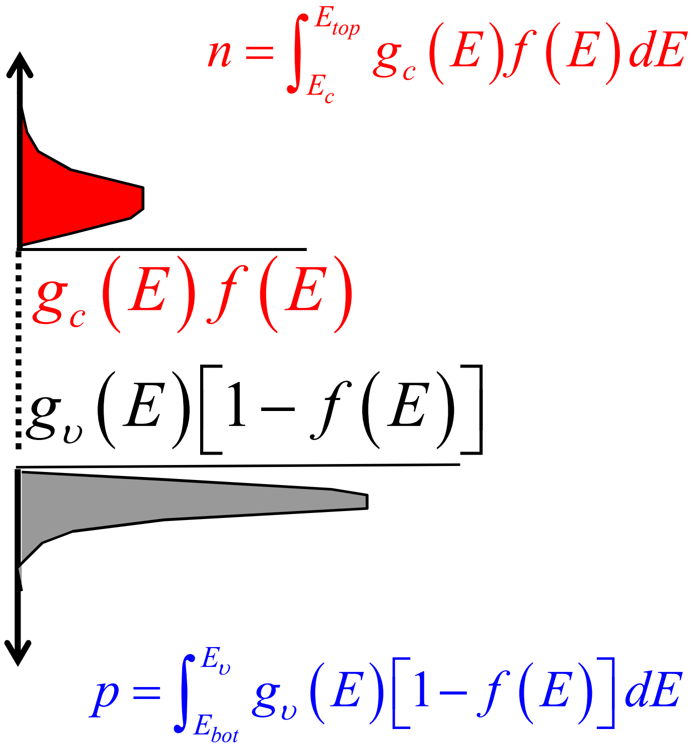
DOS



Fermi function



Carrier distribution



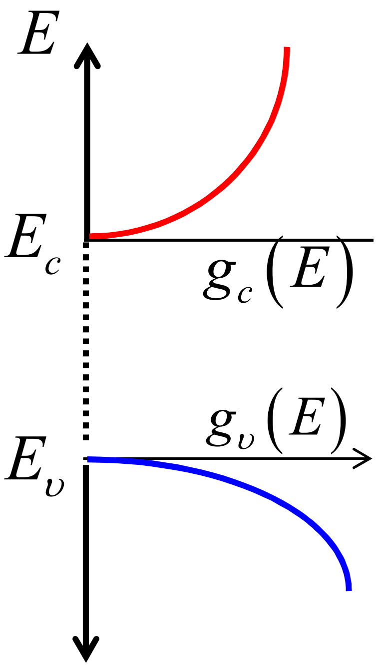
$$n=\int_{E_c}^{E_{top}} g_c(E)f(E)\,dE$$

$$p=\int_{E_{bot}}^{E_\upsilon} g_\upsilon(E)\left[1-f(E)\right]dE$$

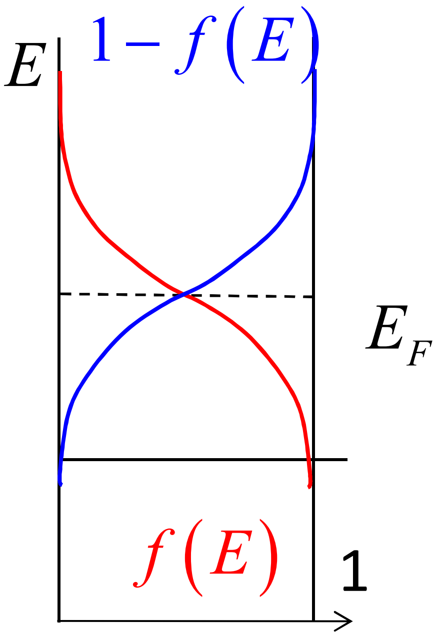
Fermi energy below mid-gap

Carrier Distribution

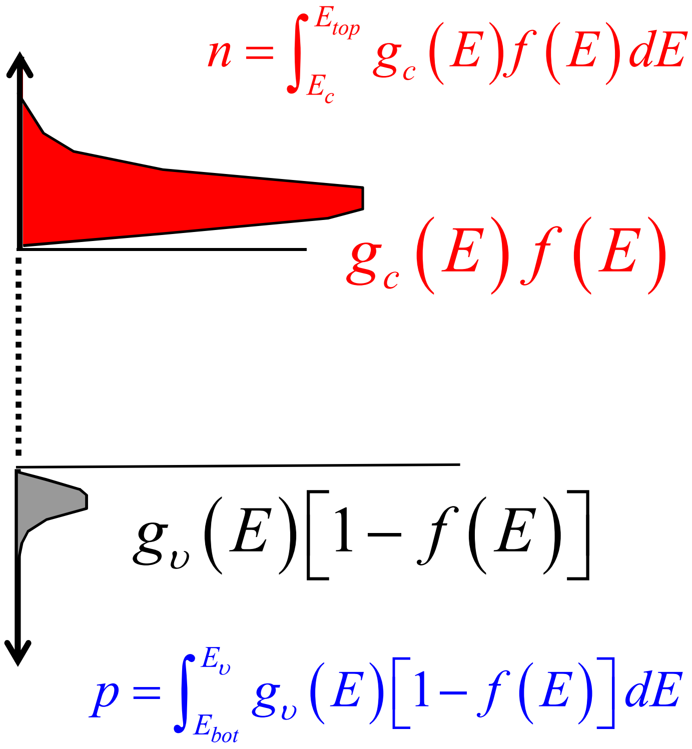
DOS



Fermi function



Carrier distribution



$$n=\int_{E_c}^{E_{top}} g_c(E)f(E)\,dE$$

$$p=\int_{E_{bot}}^{E_v} g_v(E)\left[1-f(E)\right]dE$$

Fermi energy above mid-gap

Carrier Distribution: Boltzmann Approximation

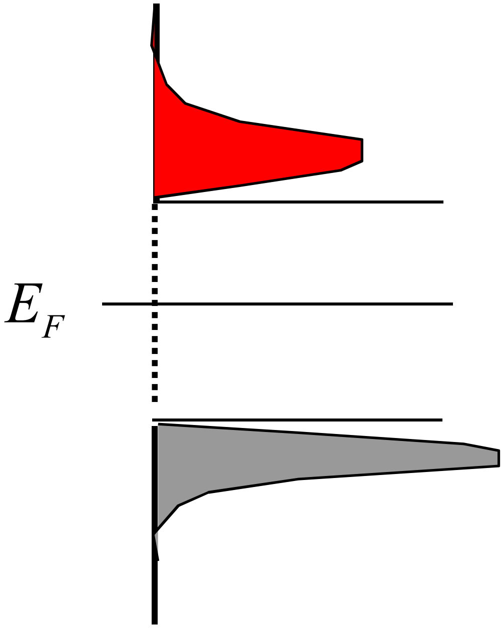
Obtained by integrating carrier distribution using Boltzmann approximation

$$n = \int_{E_c}^{E_{top}} g_c(E) f(E)\, dE$$

$$n = N_C e^{-(E_c - E_F)/k_B T}$$

$$p = \int_{E_{bot}}^{E_\upsilon} g_\upsilon(E)\left[1 - f(E)\right] dE$$

$$p = N_V e^{+(E_\upsilon - E_F)/k_B T}$$



Density of states (electrons)

$$n = N_C e^{(E_F - E_C)/k_B T} \qquad N_C = 2\left[\frac{\left(m_n^* k_B T\right)}{2\pi\hbar^2}\right]^{3/2}$$

"effective density of states"

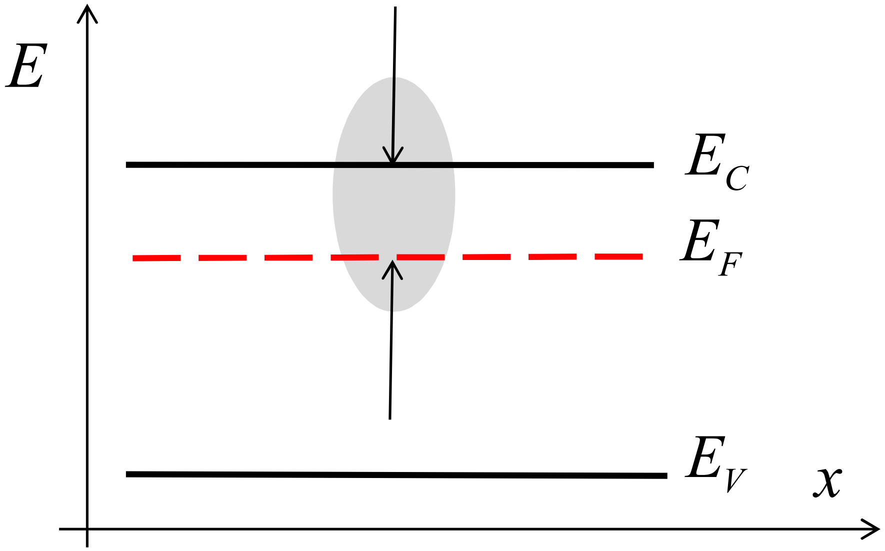
$$E_C - E_F > 3k_B T$$

For Si at *T* = 300K:

$m_n^* = 1.182$ (DOS effective mass)

$N_C = 3.23 \times 10^{19}\ \text{cm}^{-3}$

Fermi level should be at least $3k_BT$ away from a band edge.



Density of states (holes)

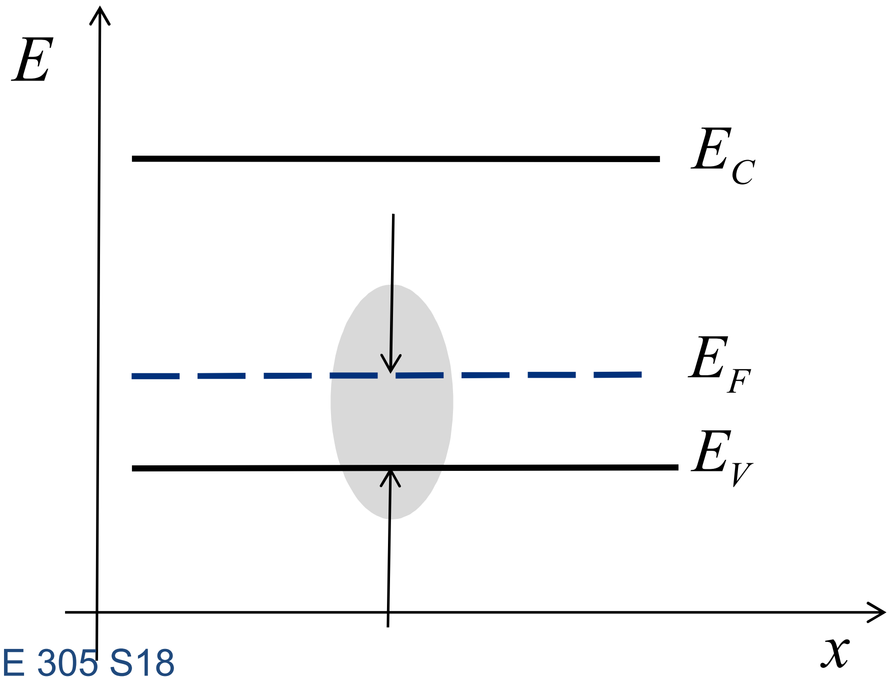
$$p = N_V e^{(E_V - E_F)/k_B T} \qquad N_V = 2\left[\frac{\left(m_p^* k_B T\right)}{2\pi\hbar^2}\right]^{3/2}$$ "effective density of states"

$$E_F - E_V > 3k_B T$$

For Si at *T* = 300K:

$m_n^* = 0.81$ (DOS effective mass)

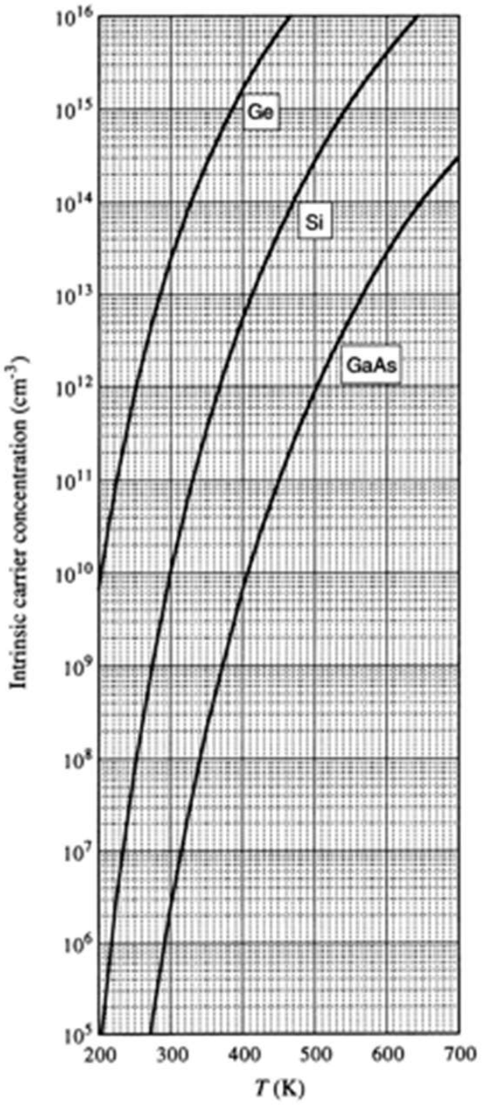
$N_V = 1.83 \times 10^{19}\ \text{cm}^{-3}$



outline

1. Carrier distributions
2. Carrier concentrations
3. Current (drift)
4. Mobility, resistivity, etc.

Carrier Concentration in Intrinsic Semiconductors



$$n = p = n_i$$

$$n \times p = N_C e^{-(E_C - E_F)/k_B T} \times N_V e^{-(E_F - E_V)/k_B T}$$

$$= N_C N_V e^{-(E_C - E_V)/k_B T}$$

$$= N_C N_V e^{-E_g/k_B T}$$

$$= n_i^2$$

$$n_i = \sqrt{N_C N_V}\, e^{-E_g/2k_B T}$$

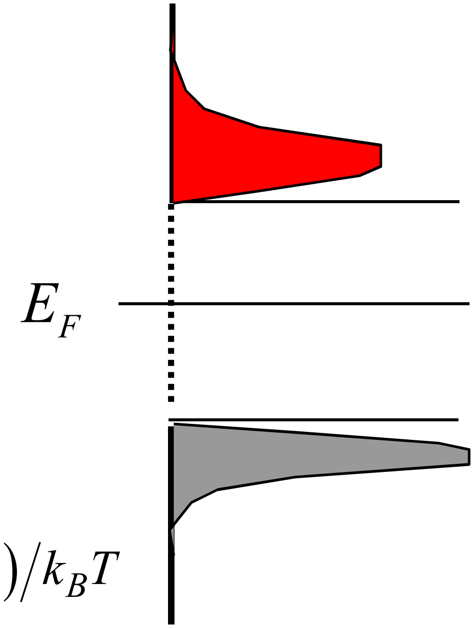
(Eq. from Ex. 2.4
Includes temp.-dependent
changes in effective mass)

$$n_i(Si) = 9.15 \times 10^{19} \left(\frac{T}{300}\right)^2 e^{-1.12/2k_B T}$$

Fermi Level for Intrinsic Semiconductors

$$n_i = \sqrt{N_C N_V}\, e^{-E_g/2k_BT}$$

$$E_F \equiv E_i$$



$$n = p \Rightarrow N_C e^{-(E_c - E_i)/k_BT} = N_V e^{+(E_\upsilon - E_i)/k_BT}$$

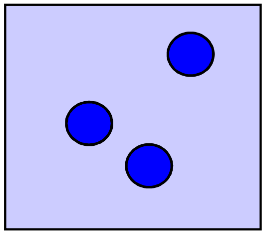
$$E_i = \frac{E_G}{2} + \frac{k_BT}{2} ln \frac{N_V}{N_C}$$

$$N_C \equiv 2\left(\frac{m_n^* k_B T}{2\pi h^2}\right)^{3/2}$$

$$N_V \equiv 2\left(\frac{m_h^* k_B T}{2\pi h^2}\right)^{3/2}$$

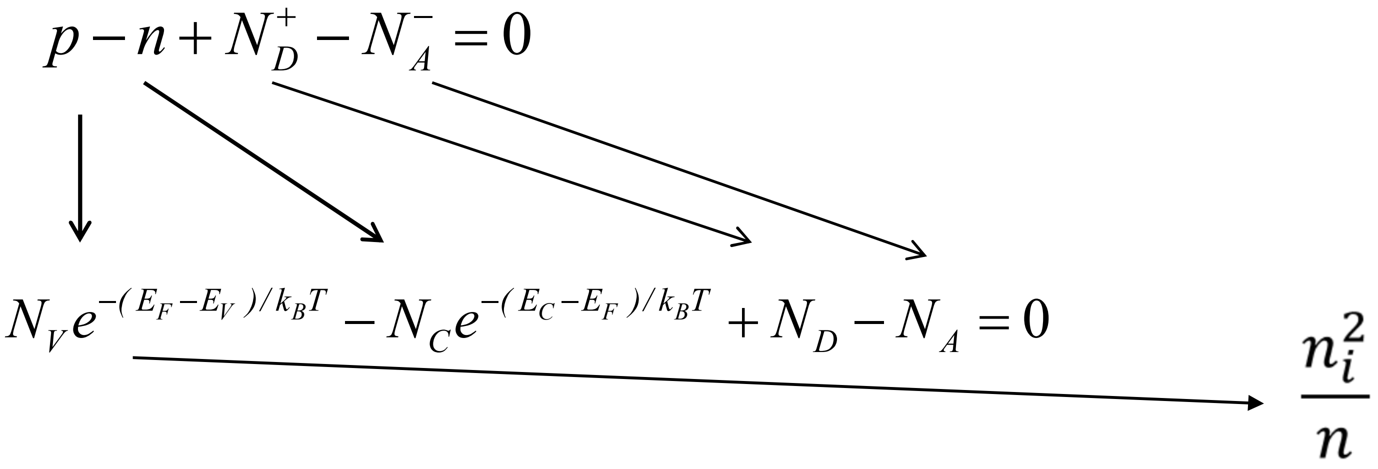
carrier densities (given doping)

A bulk material must be charge neutral overall …



$$\int \left[p - n + N_D^+ - N_A^- \right] dV = 0$$

Further if the material is spatially homogenous and field free:



$$p - n + N_D^+ - N_A^- = 0$$

$$N_V e^{-(E_F - E_V)/k_B T} - N_C e^{-(E_C - E_F)/k_B T} + N_D - N_A = 0$$

$$\frac{n_i^2}{n}$$

Convince yourself that the only unknown is E_F.
Once you know E_F, you can calculate n, p, N_D^+, N_A^-.
Bulk homogenous material has only one Fermi level.

carrier densities (more generally)

(N-type)

$$\frac{n_i^2}{n} - n + N_D - N_A = 0$$

$$n = \frac{N_D - N_A}{2} + \left[\left(\frac{N_D - N_A}{2}\right)^2 + n_i^2\right]^{1/2}$$

$$p = \frac{n_i^2}{n}$$

(P-type)

$$p - \frac{n_i^2}{p} + N_D - N_A = 0$$

$$p = \frac{N_A - N_D}{2} + \left[\left(\frac{N_A - N_D}{2}\right)^2 + n_i^2\right]^{1/2}$$

$$n = \frac{n_i^2}{p}$$

Important Approximate Cases

$$n = \frac{N_D - N_A}{2} + \left[\frac{(N_D - N_A)^2}{4} + n_i^2 \right]^{1/2} \qquad p = \frac{n_i^2}{n} \quad \text{(at equilibrium)}$$

Intrinsic Semiconductor: $N_D \ll n_i$, $N_A \ll n_i$

$n \approx n_i$, $\quad p \approx n_i$

N-doped semiconductor: $N_D \gg n_i \quad N_D > N_A$

$n \approx N_D \quad p \approx n_i^2 / N_D$

P-doped semiconductor: $N_A \gg n_i \quad N_A > N_D$

$n \approx n_i^2 / N_A \quad p \approx N_A$

outline

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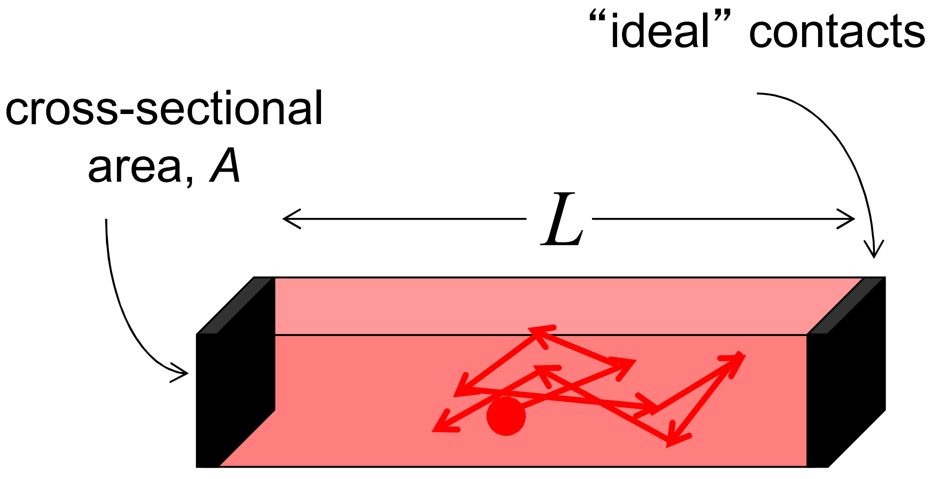
semiconductor in equilibrium



uniform n-type semiconductor

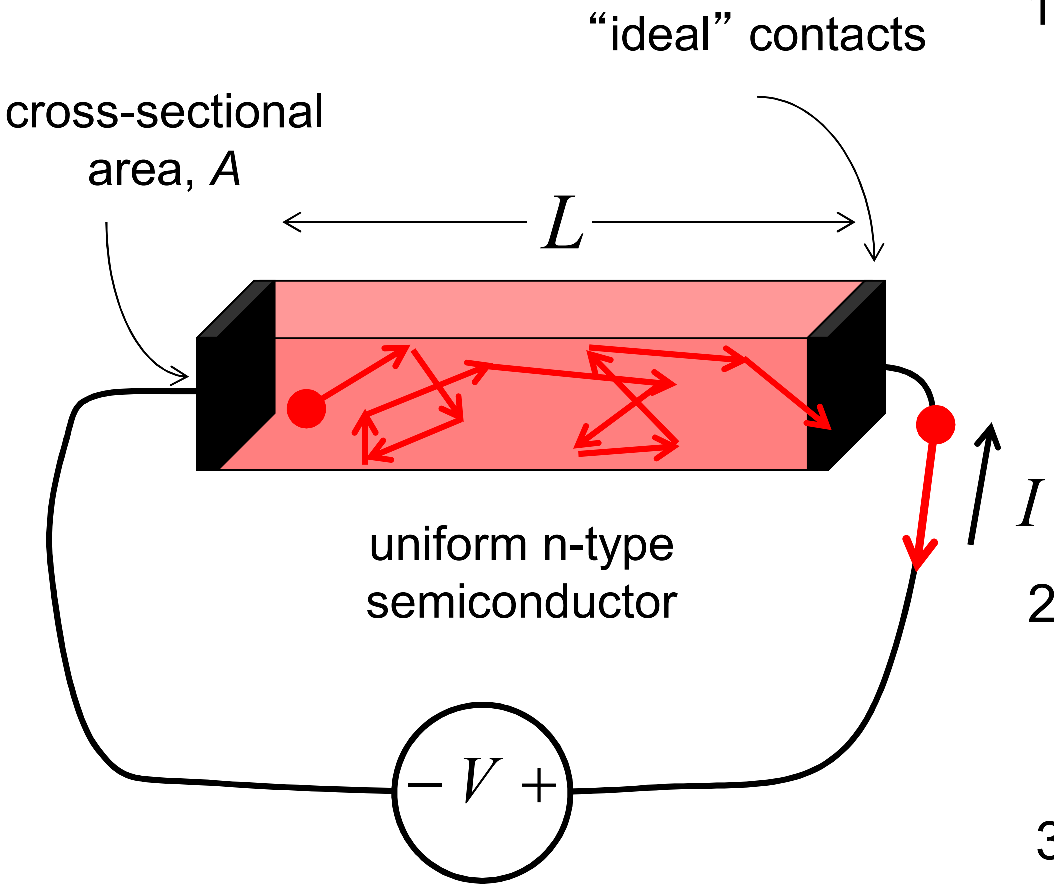
$$\langle KE \rangle = \frac{3}{2} k_B T$$

$$\langle KE \rangle = \frac{1}{2} m_n^* \langle \upsilon^2 \rangle$$

$$\sqrt{\langle \upsilon^2 \rangle} = \upsilon_{rms} = \sqrt{\frac{3k_B T}{m_n^*}}$$

$$\upsilon_{rms} \approx 10^7 \text{ cm/s } (T = 300\ K)$$

current flow

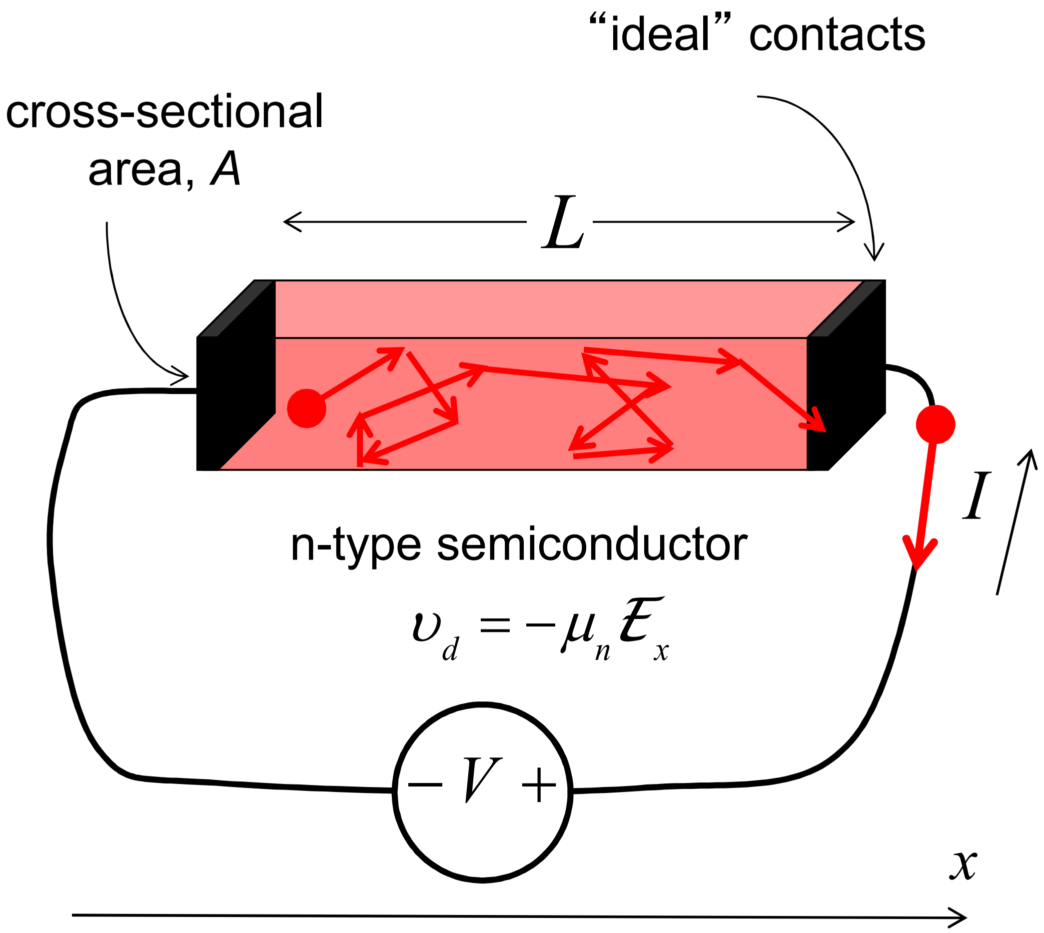


1) random walk with a small bias from left to right

2) assume that electrons "drift" to the right at an average velocity, v_d

3) what is I ?

drift current and velocity



n-type semiconductor

$$\upsilon_d = -\mu_n \mathcal{E}_x$$

$$I = -Q/t_t$$

$$Q = -qnAL$$

$$t_t = L/\upsilon_d$$

$$I = nq\upsilon_d A$$

$$J_{nx} = -nq\upsilon_{dx} \ \text{A/cm}^2$$

$$J_{px} = pq\upsilon_{dx} \ \text{A/cm}^2$$

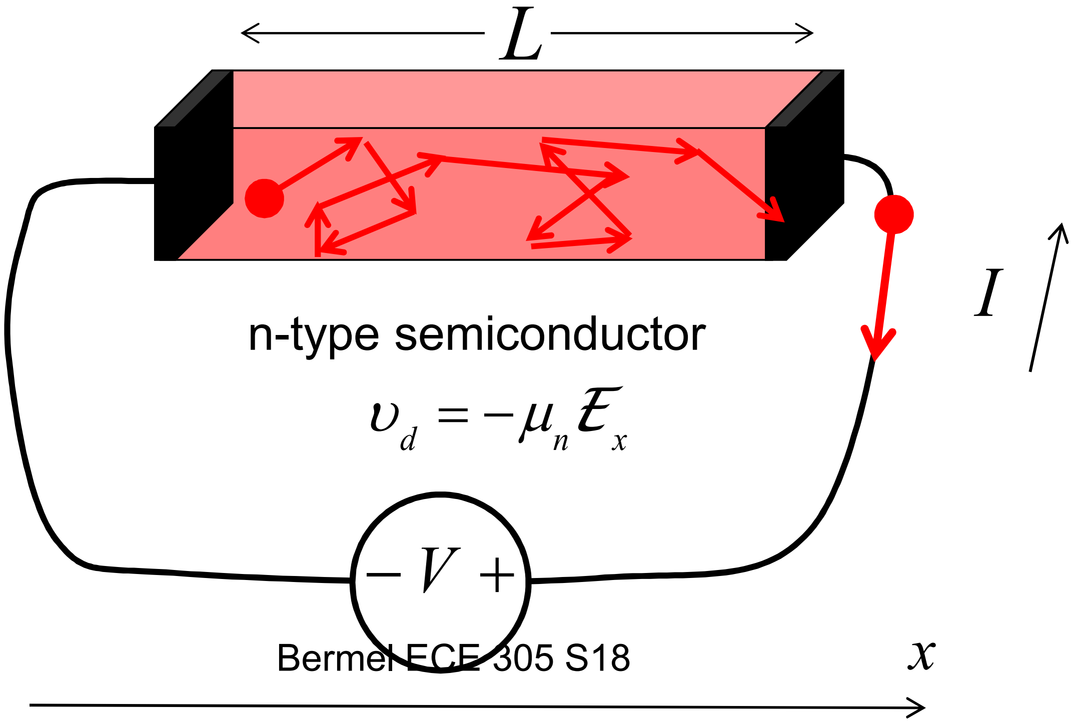
drift current

$$J_{nx} = nq\mu_n \mathcal{E}_x \text{ A/cm}^2 \qquad \upsilon_{dn} = -\mu_n \mathcal{E}$$

$$J_{px} = pq\mu_p \mathcal{E}_x \text{ A/cm}^2 \qquad \upsilon_{dp} = +\mu_p \mathcal{E}$$

$$\mathcal{E} = -\frac{dV}{dx} \text{ V/cm}$$

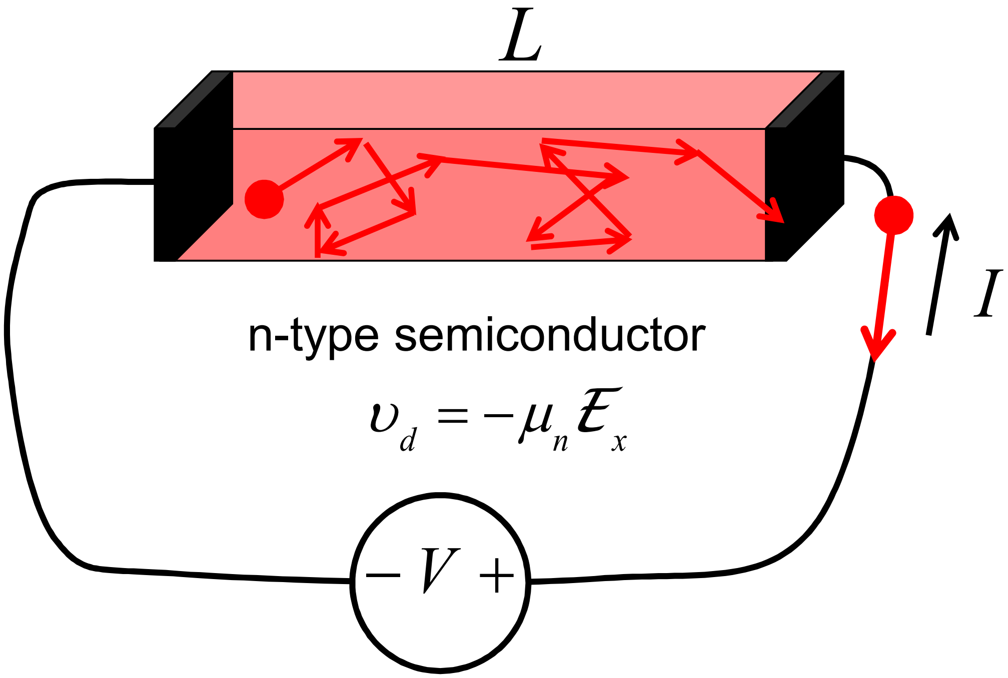
$$\mathcal{E} = -\frac{V}{L} \text{ V/cm}$$



n-type semiconductor

$$\upsilon_d = -\mu_n \mathcal{E}_x$$

resistance



n-type semiconductor

$$\upsilon_d = -\mu_n \mathcal{E}_x$$

$$J_n = \sigma_n \mathcal{E} \text{ A/cm}^2$$

$$I = -AJ_n = \sigma_n A \mathcal{E} \text{ Amps}$$

$$I = \sigma_n A \frac{V}{L}$$

$$I = \left(\sigma_n \frac{A}{L}\right) V = GV = \frac{1}{R} V$$

$$\boxed{R = \rho_n \frac{L}{A}}$$

Definition of Conductivity and Resistivity ….

$$J_n = qn\mu_n\mathcal{E} \equiv \sigma_n\mathcal{E}$$

$$\sigma_n \equiv qn\mu_n$$

$$\mathcal{E} = \left(1/qn\mu_n\right)J_n \equiv \rho_n J_n$$

$$\rho_n \equiv \frac{1}{qn\mu_n}$$

current, conductivity, resistivity

$J_{nx} = nq\mu_n \mathcal{E}_x$ A/cm^2 $\quad J_{nx} = \sigma_n \mathcal{E}_x$ A/cm^2 $\quad \sigma_n = nq\mu_n$ (units?)

$J_{px} = pq\mu_p \mathcal{E}_x$ A/cm^2 $\quad J_{px} = \sigma_p \mathcal{E}_x$ A/cm^2 $\quad \sigma_p = pq\mu_p$

$$J_x = J_{nx} + J_{px} = \left(\sigma_n + \sigma_p\right)\mathcal{E}_x = \sigma \mathcal{E}_x \text{ A/cm}^2$$

$$J_x = \sigma \mathcal{E}_x \text{ A/cm}^2 \qquad \mathcal{E}_x = \frac{1}{\sigma} J_x = \rho J_x \text{ V/cm}$$

$$\rho = \frac{1}{\sigma} = \frac{1}{\sigma_n + \sigma_p} = \frac{1}{nq\mu_n + pq\mu_p} \ \Omega\text{-cm}$$

outline

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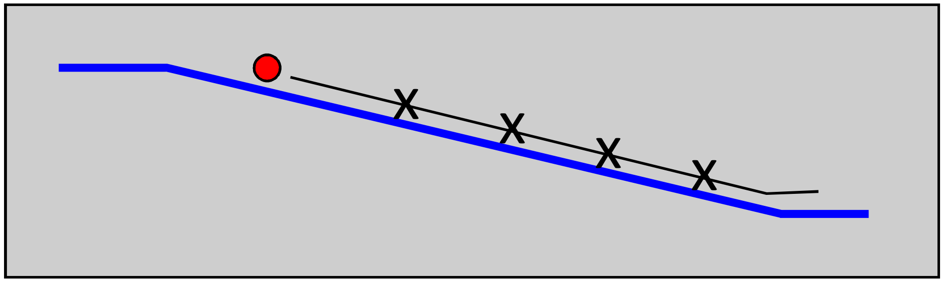
Drift by Electric field ….

$$\upsilon(t) = -\frac{q\tau_n}{m_n^*}\mathcal{E}\left[1 - e^{-\frac{t}{\tau_n}}\right]$$

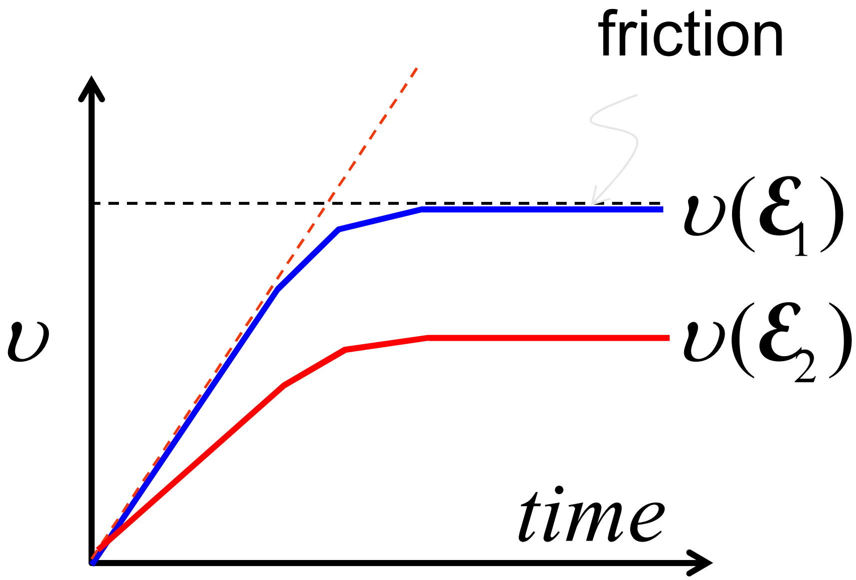


$$= -\frac{q\tau_n}{m_n^*}\mathcal{E} \quad (t \to \infty,\ 1\text{-}2\ \text{ps})$$

$$\equiv -\mu_n\mathcal{E}$$

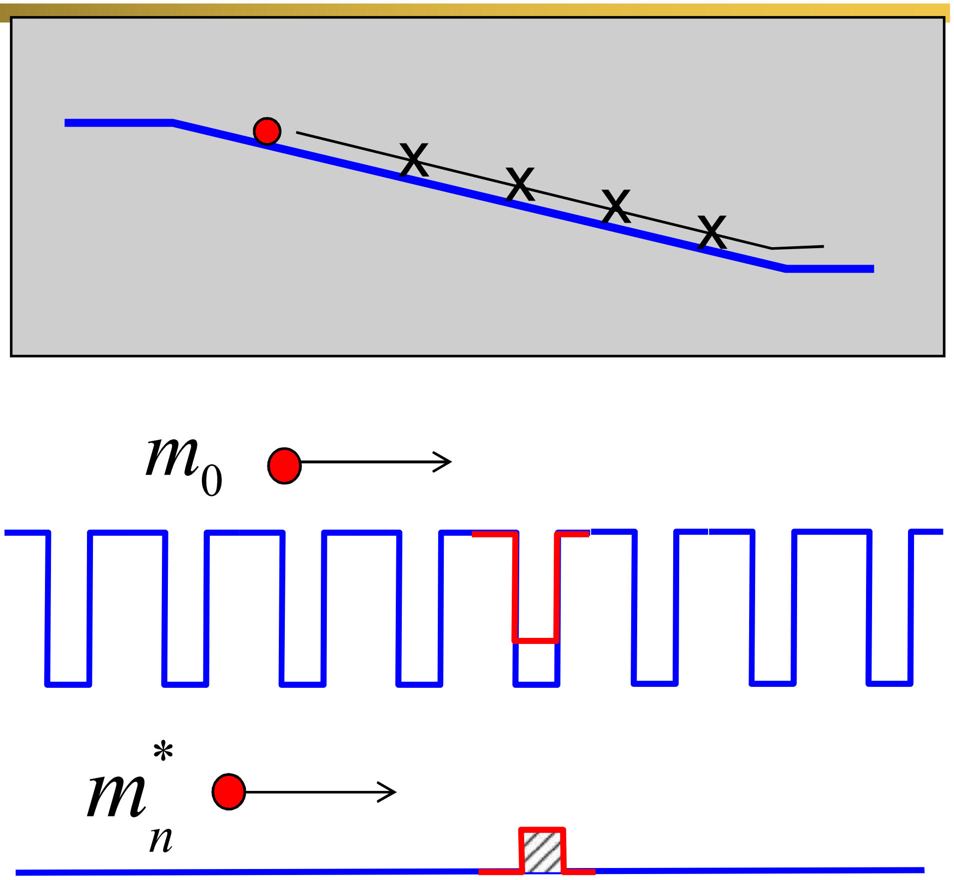
$$J_n = qn\mu_n\mathcal{E}$$

(Theory valid once t > 1-2 ps)



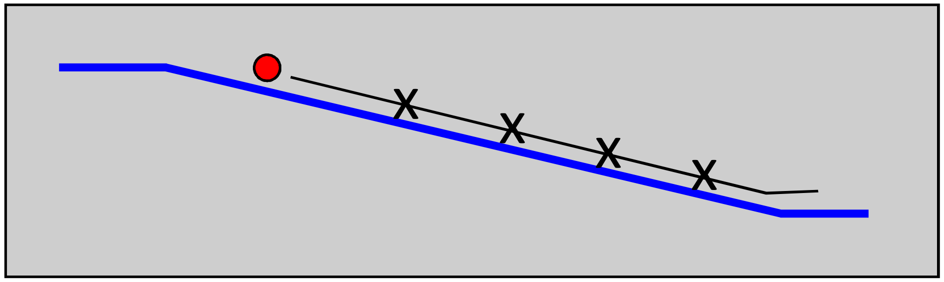
Mobility and Physics of Scattering Time

$$\mu_n = \frac{q\tau_n}{m_n^*}$$



Units of mobility ….

$$\upsilon \equiv \mu_n \mathcal{E}$$



unit of mobility: cm²/V.s

$$\upsilon = \mu_n \mathcal{E} \ldots \frac{\mathrm{cm}^2}{\mathrm{volt} \times \mathrm{sec}} \times \frac{\mathrm{volt}}{\mathrm{cm}} \rightarrow \frac{\mathrm{cm}}{\mathrm{sec}}$$

$$\mu_n = \frac{q\tau_n}{m_n^*} = \frac{(qV)\tau_n \upsilon^2}{V \dfrac{m_n^* \upsilon^2}{2}} = \frac{\mathrm{J} \times \mathrm{sec} \times \mathrm{cm}^2/\mathrm{sec}^2}{\mathrm{volts} \times \mathrm{J}}$$

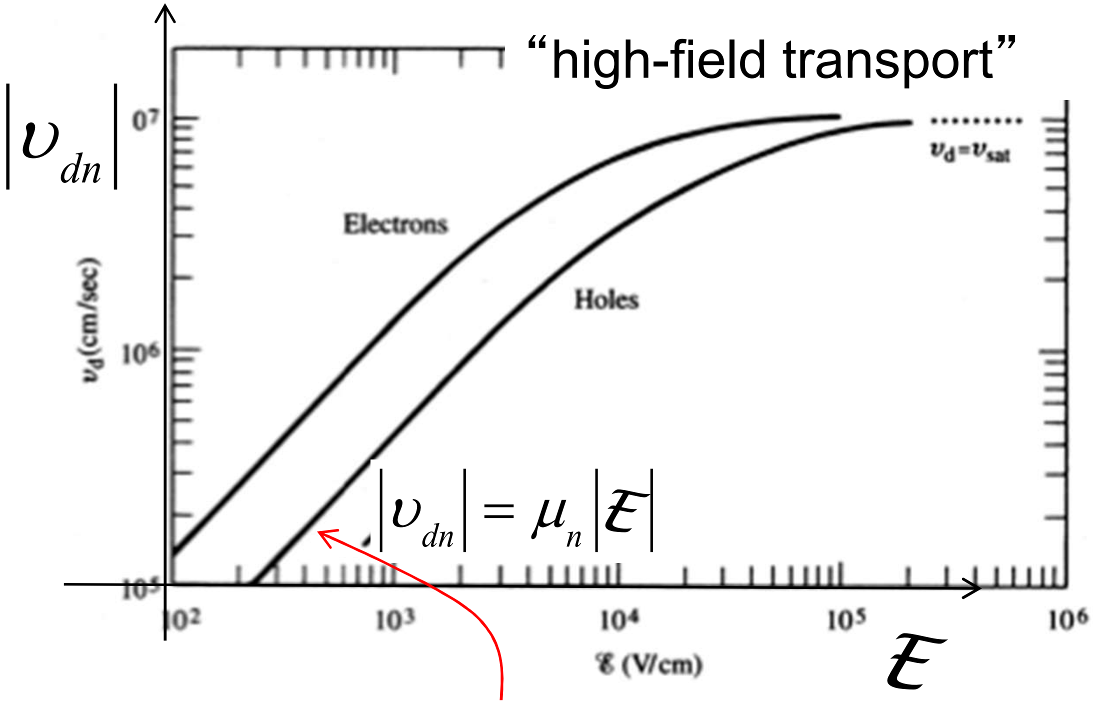
velocity and electric field

$$\upsilon_{dn} = -\mu_n \mathcal{E}$$

$$\mu_n = \left(\frac{q\,\tau}{m_n^*}\right) \mathrm{cm}^2/\mathrm{V\text{-}s}$$

$$\upsilon_{dp} = +\mu_p \mathcal{E}$$

$$\mu_p = \left(\frac{q\,\tau}{m_p^*}\right) \mathrm{cm}^2/\mathrm{V\text{-}s}$$



"high-field transport"

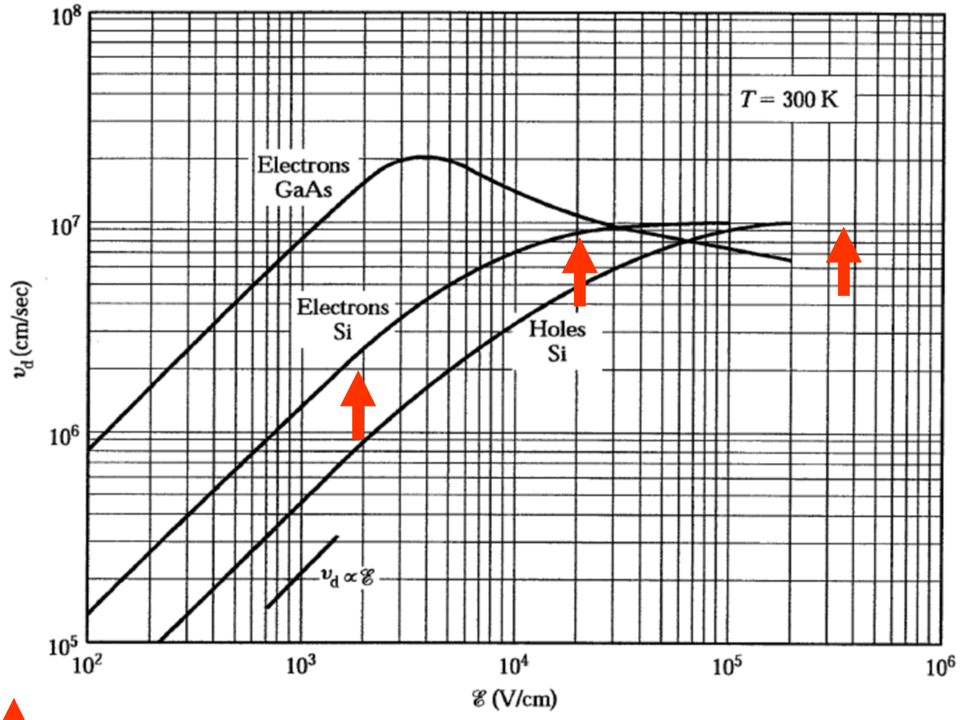
$$|\upsilon_{dn}| = \mu_n |\mathcal{E}|$$

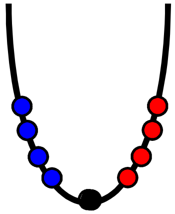
"low-field" or "near-equilibrium" or "linear" transport

from R.F. Pierret, *Semiconductor Device Fundamentals,* Fig. 3.4

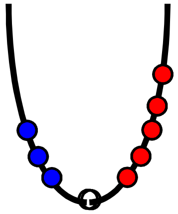
Velocity Saturation in Si/Ge

100 Km/sec !

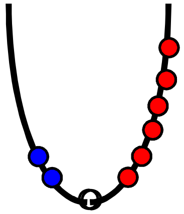




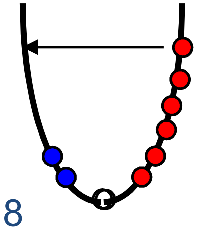
$\mathcal{E} = 0 \quad J_1 = J^+ - J^- = 0$



$\mathcal{E} \ll \mathcal{E}_c \quad J_2 = J^+ - J^- > J_1$

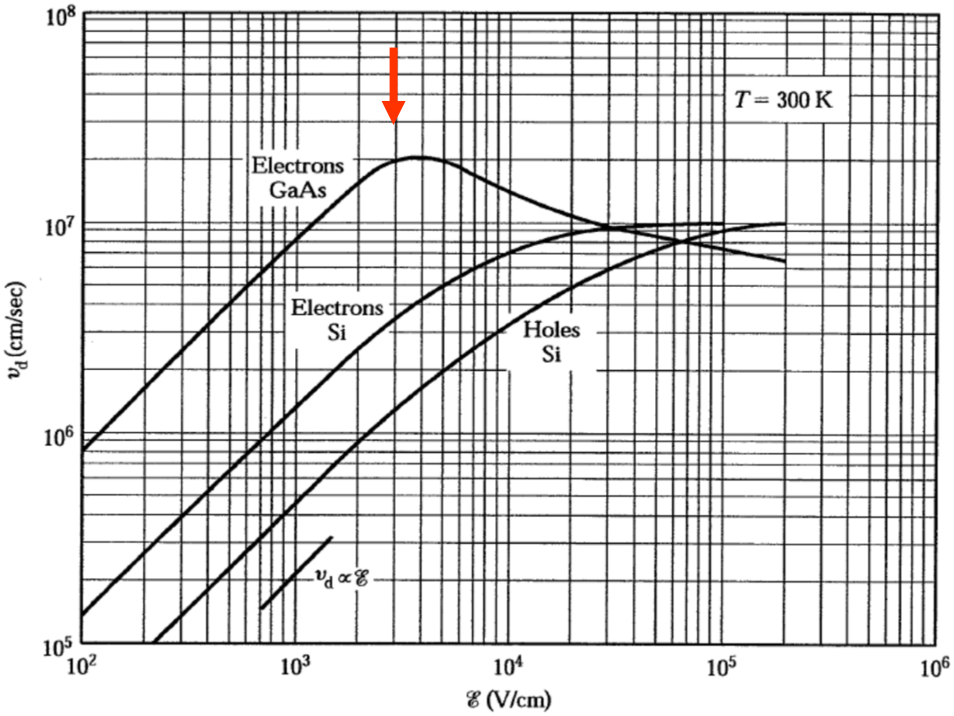


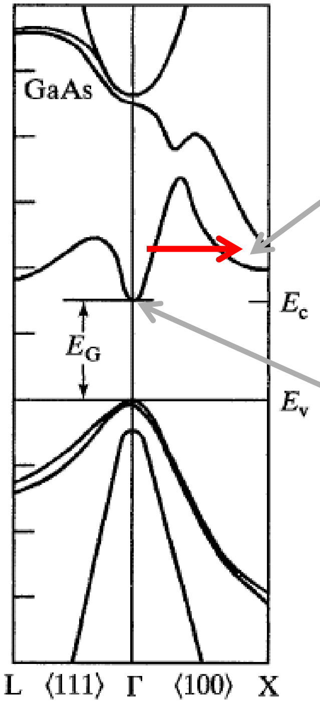
$\mathcal{E} \approx \mathcal{E}_c \quad J_3 = J^+ - J^- > J_2$



$\mathcal{E} \gg \mathcal{E}_c \quad J_4 = J^+ - J^- \approx J_3$

Velocity Overshoot & Inter-valley Transfer



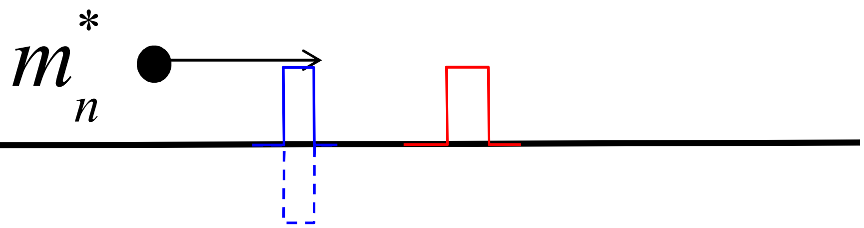


Larger m*

Smaller m*

Multiple Scattering Events

- ☐ Ionized impurity
- ☐ Phonon scattering
- ☐ others ….



$$\frac{1}{\mu_n}=\frac{1}{\mu_{ph}}+\frac{1}{\mu_{II}}$$

$$\Rightarrow \mu_n=\frac{\mu_{ph}\mu_{II}}{\mu_{ph}+\mu_{II}}$$

$$=\mu_{\min}+\left(\frac{\mu_{ph}\mu_{II}}{\mu_{ph}+\mu_{II}}-\mu_{\min}\right)$$

$$=\mu_{\min}+\left(\frac{\mu_0}{1+\left(N_I/N_0\right)^{\alpha}}\right)$$

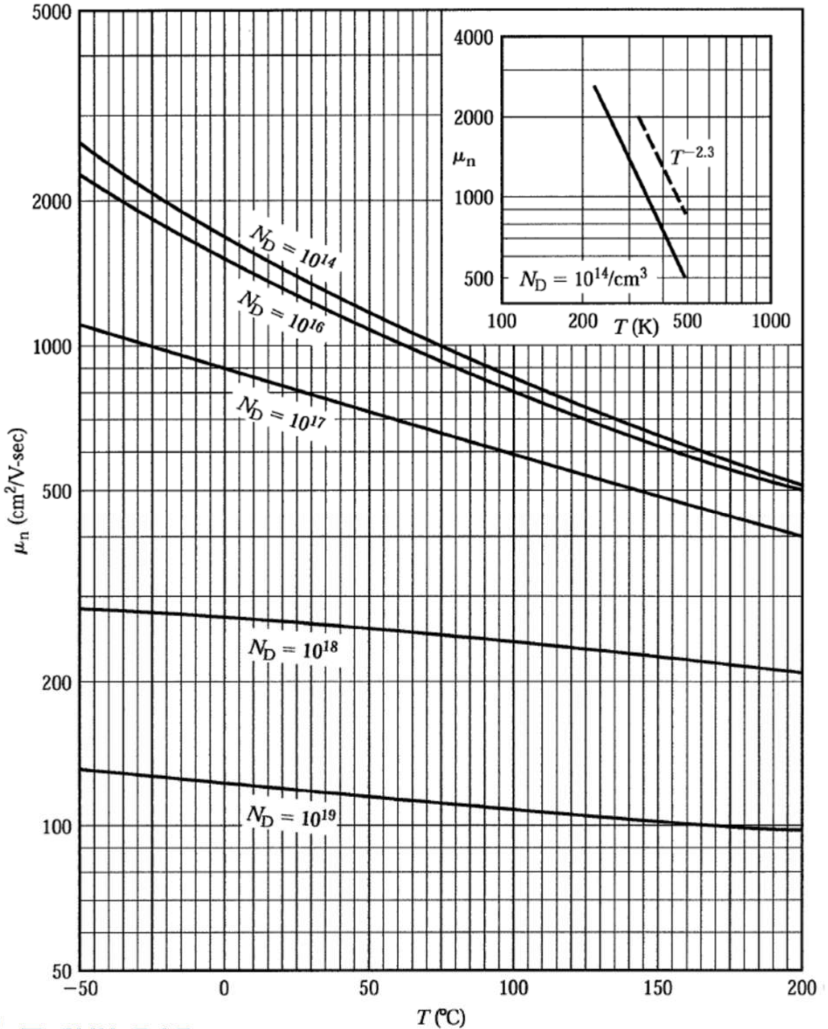
$$\frac{1}{\tau_n}=\frac{1}{\tau_{II}}+\frac{1}{\tau_{ph}}+\frac{1}{\tau_s}+\cdots$$

$$\frac{1}{\mu_n}=\frac{m_n^*}{q\tau_n}$$

Matthession Rule ….

Temperature-dependent Mobility

$$\mu_n = \frac{q\tau_n}{m_n^*} \sim T^{-3/2}$$



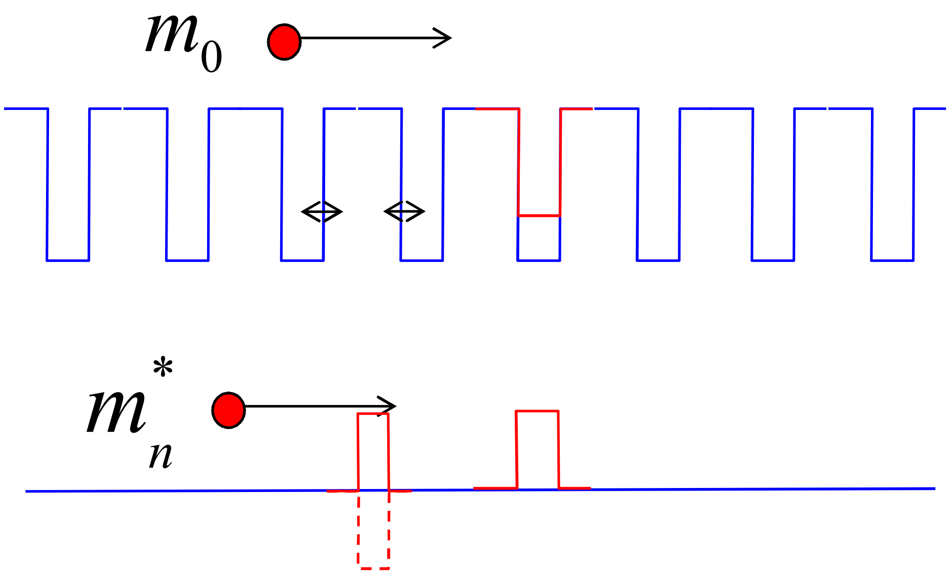
Phonon and Ionized Impurity Scattering

Ionized impurity

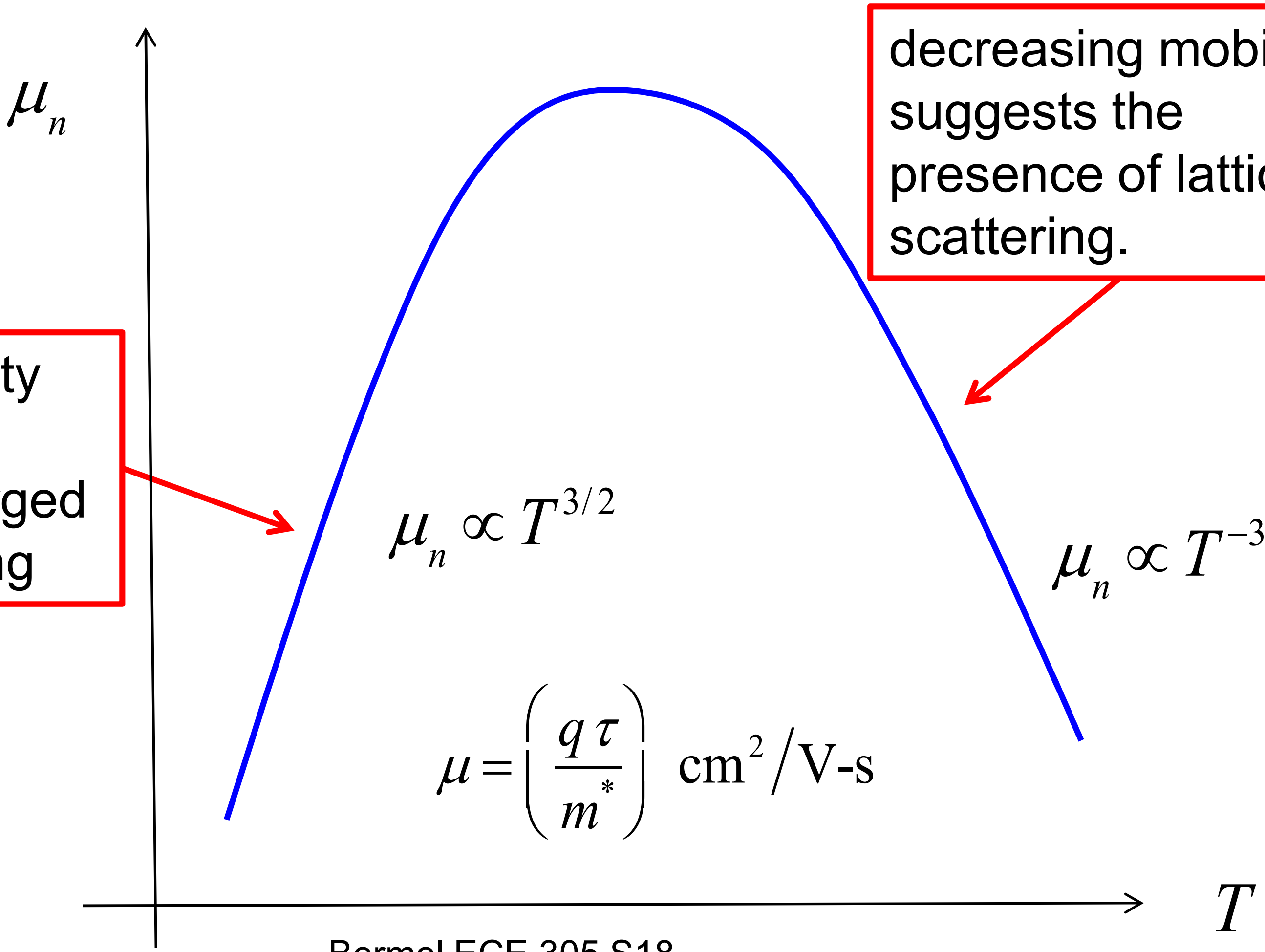
$$\tau_n \sim \frac{T^{3/2}}{N_D}$$

Higher temperature, more phonon scattering

$$\tau_n \sim T^{-3/2}$$



mobility vs. temperature



increasing mobility suggests the presence of charged impurity scattering

decreasing mobility suggests the presence of lattice scattering.

$$\mu_n \propto T^{3/2}$$

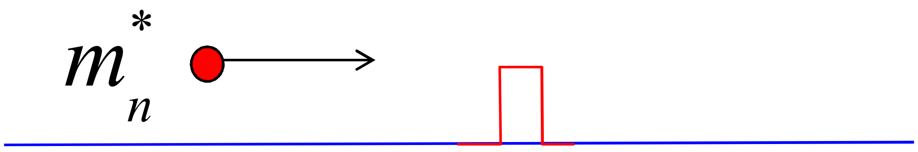
$$\mu_n \propto T^{-3/2}$$

$$\mu = \left(\frac{q\,\tau}{m^*} \right) \ \mathrm{cm}^2/\mathrm{V\text{-}s}$$

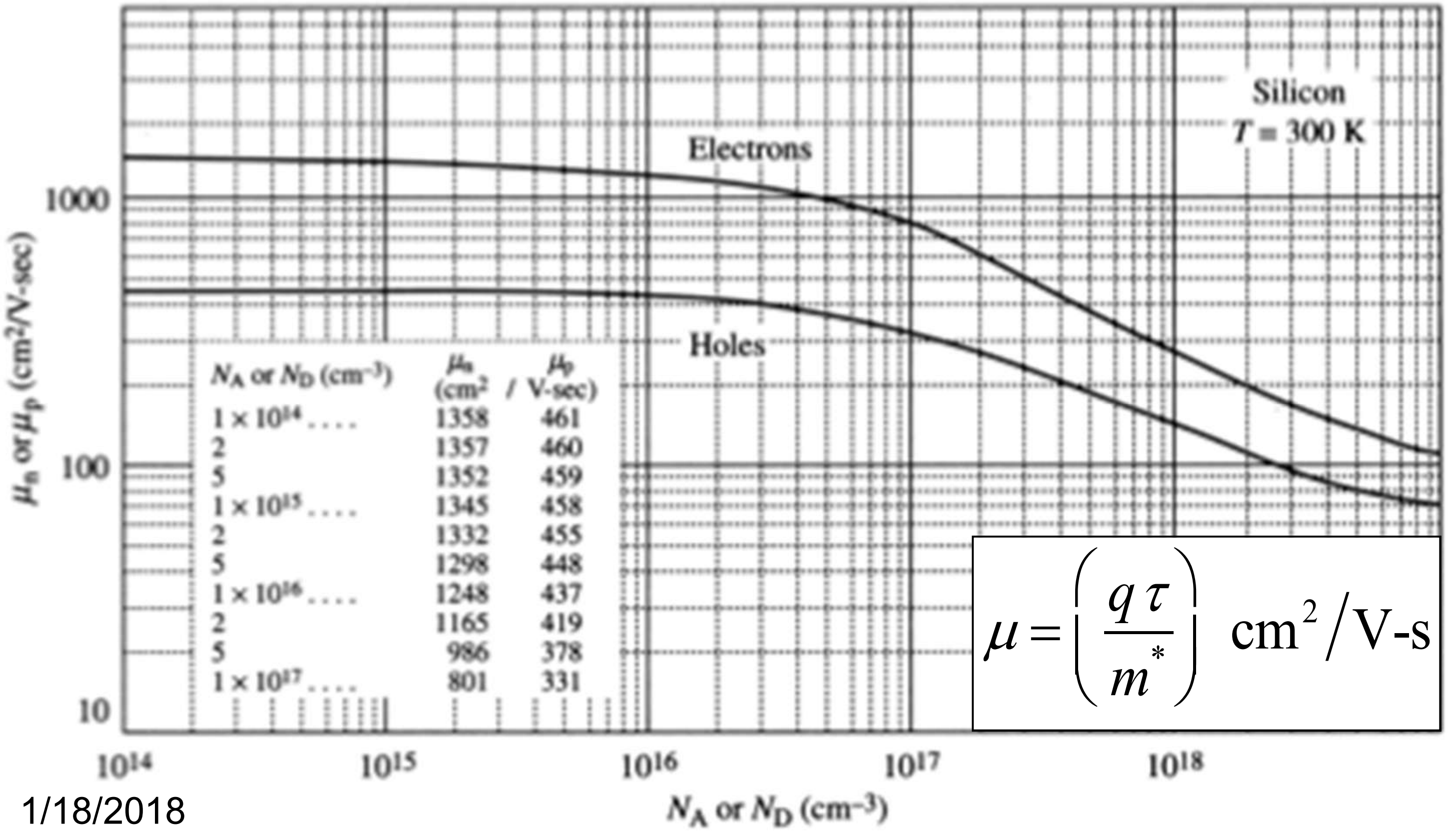
Model for Ionized impurity Scattering

from R.F. Pierret, *Semiconductor Device Fundamentals,* Fig. 3.5 (a)

$$\mu_n = \mu_{n,\min} + \left(\frac{\mu_{0,n}}{1 + \left(N_I / N_{0,n} \right)^{\alpha_n}} \right)$$



m_n^*

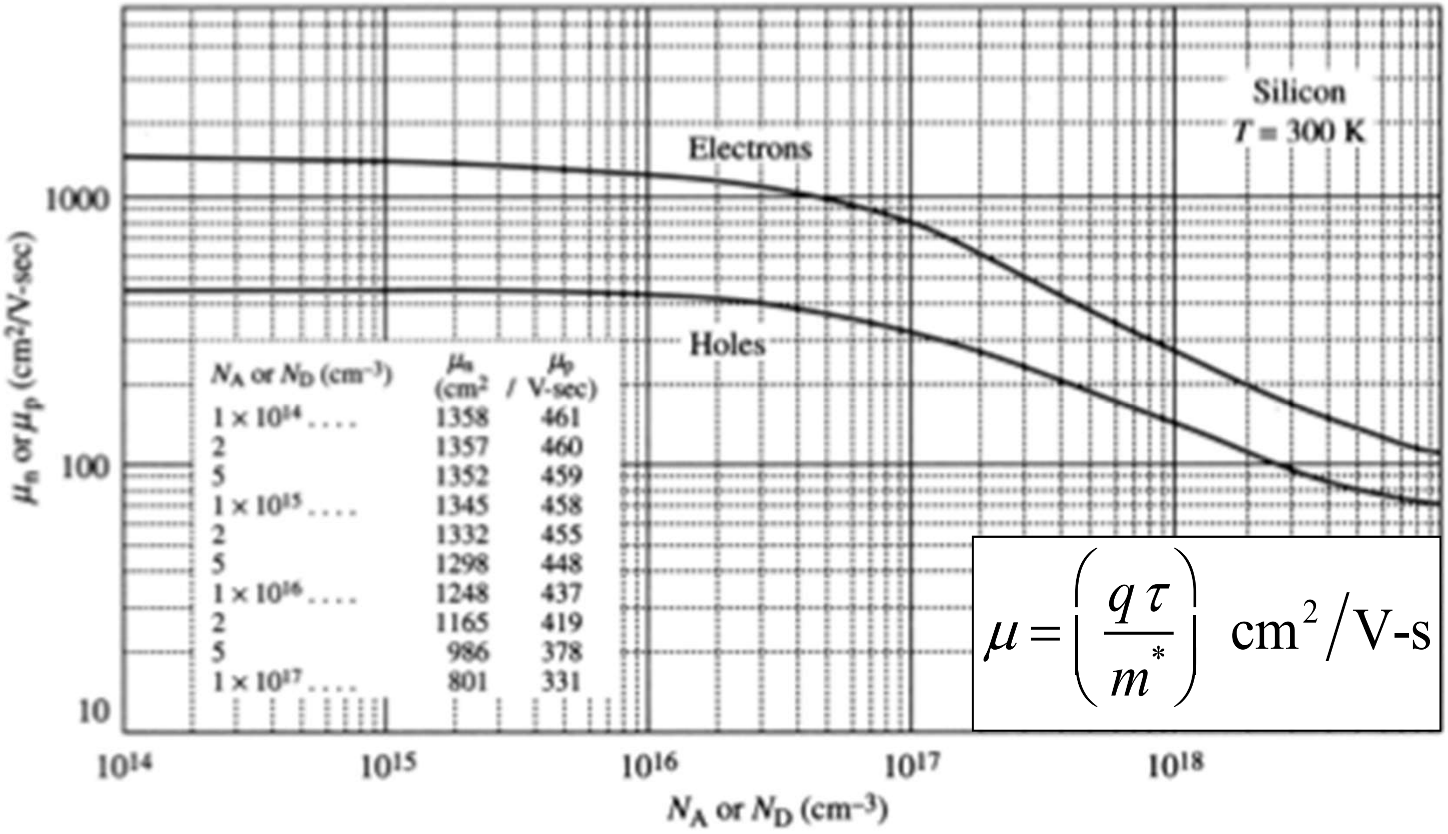


N_A or N_D (cm⁻³)	μ_n (cm²	μ_p / V-sec)
1×10^{14}....	1358	461
2	1357	460
5	1352	459
1×10^{15}....	1345	458
2	1332	455
5	1298	448
1×10^{16}....	1248	437
2	1165	419
5	986	378
1×10^{17}....	801	331

$\mu_{n,\min}$

$$\boxed{\mu = \left(\frac{q\,\tau}{m^*} \right) \ \mathrm{cm}^2/\mathrm{V\text{-}s}}$$

Si carrier mobility vs. doping

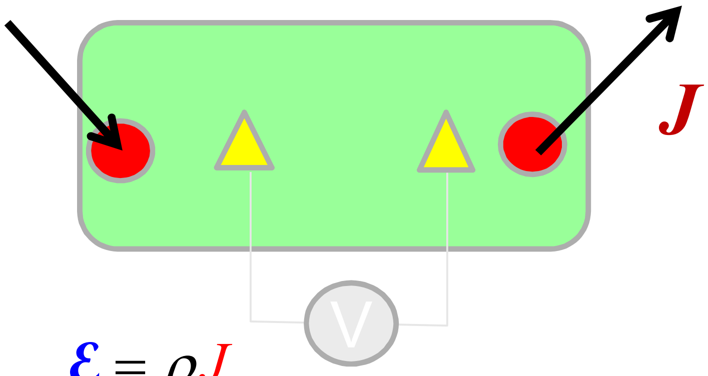


N_A or N_D (cm^{-3})	μ_n (cm^2	μ_p / V-sec)
1×10^{14}	1358	461
2	1357	460
5	1352	459
1×10^{15}	1345	458
2	1332	455
5	1298	448
1×10^{16}	1248	437
2	1165	419
5	986	378
1×10^{17}	801	331

$$\mu = \left(\frac{q\,\tau}{m^*} \right) \ \mathrm{cm}^2/\mathrm{V\text{-}s}$$

from R.F. Pierret, *Semiconductor Device Fundamentals,* Fig. 3.5 (a)

Doping dependent Resistivity



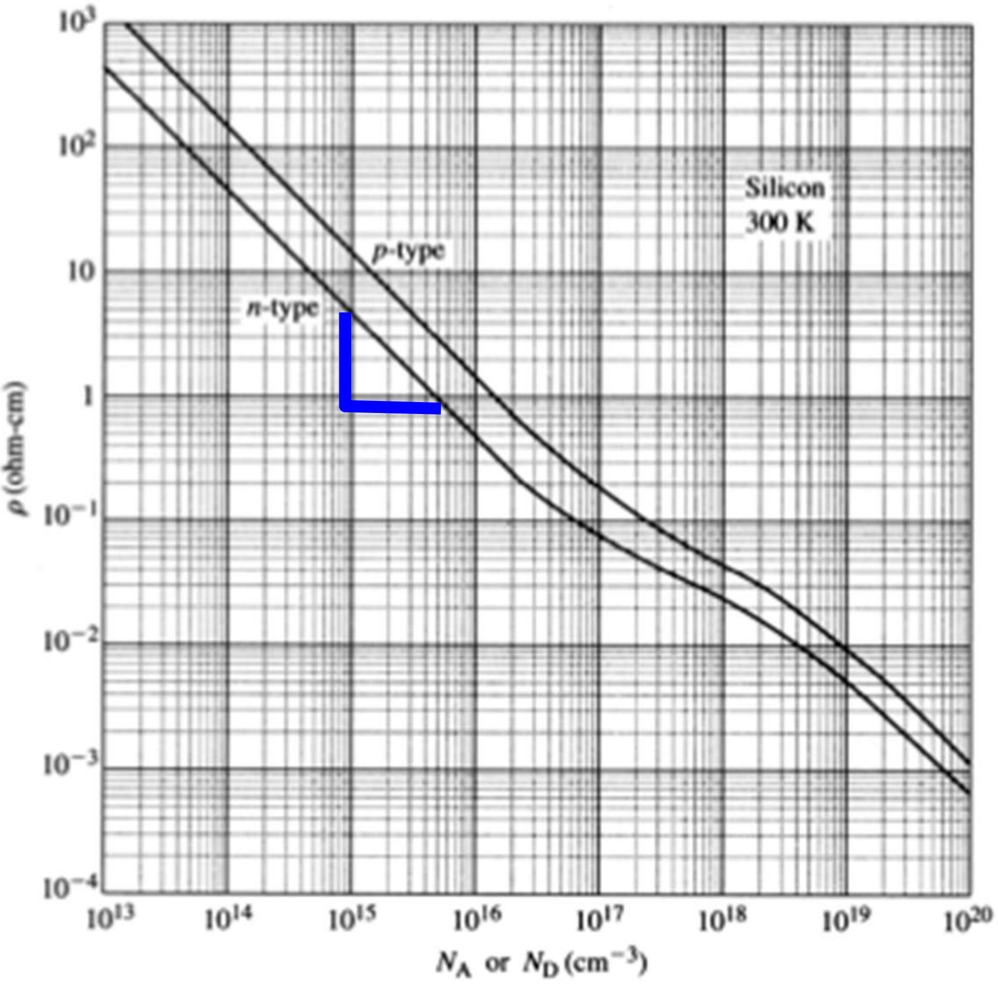
$$\mathcal{E} = \rho J$$

$$J = q(\mu_n n + \mu_p p)\mathcal{E}$$

$$\rho = \frac{1}{nq\mu_n + pq\mu_p}\ \Omega\text{-cm}$$

$$= \frac{1}{q\mu_n N_D} \quad \text{(for n-type)}$$

$$= \frac{1}{q\mu_p N_A}. \quad \text{(for p-type)}$$



summary

1) In <u>equilibrium</u>, the Fermi level and temperature gives the probability that a state at energy, *E*, is occupied by an electron or hole
2) Multiplying with the density of states yields the carrier densities
3) One can also work backwards from doping to calculate carrier densities and thus Fermi energy
4) Can calculate conductivity from carrier densities; can then use Ohm’s law to calculate current
5) Conductivity can depend on electric field, doping, and temperature, which is all captured as carrier *mobility*