ECE-305: Spring 2018

Carrier Currents + Band Structures

Pierret, *Semiconductor Device Fundamentals* (SDF)
Chapter 3 (pp. 75-104)

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1/30/18

PURDUE
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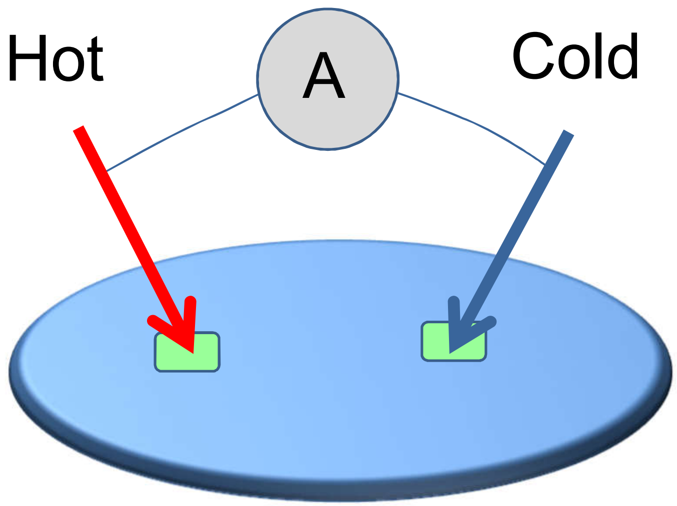
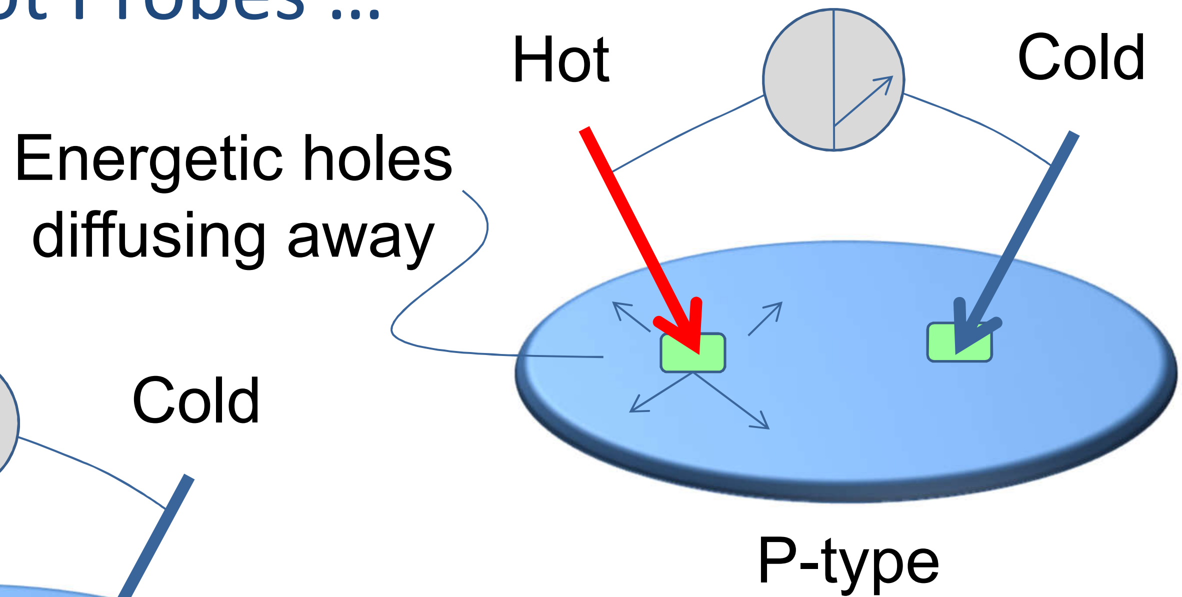
outline

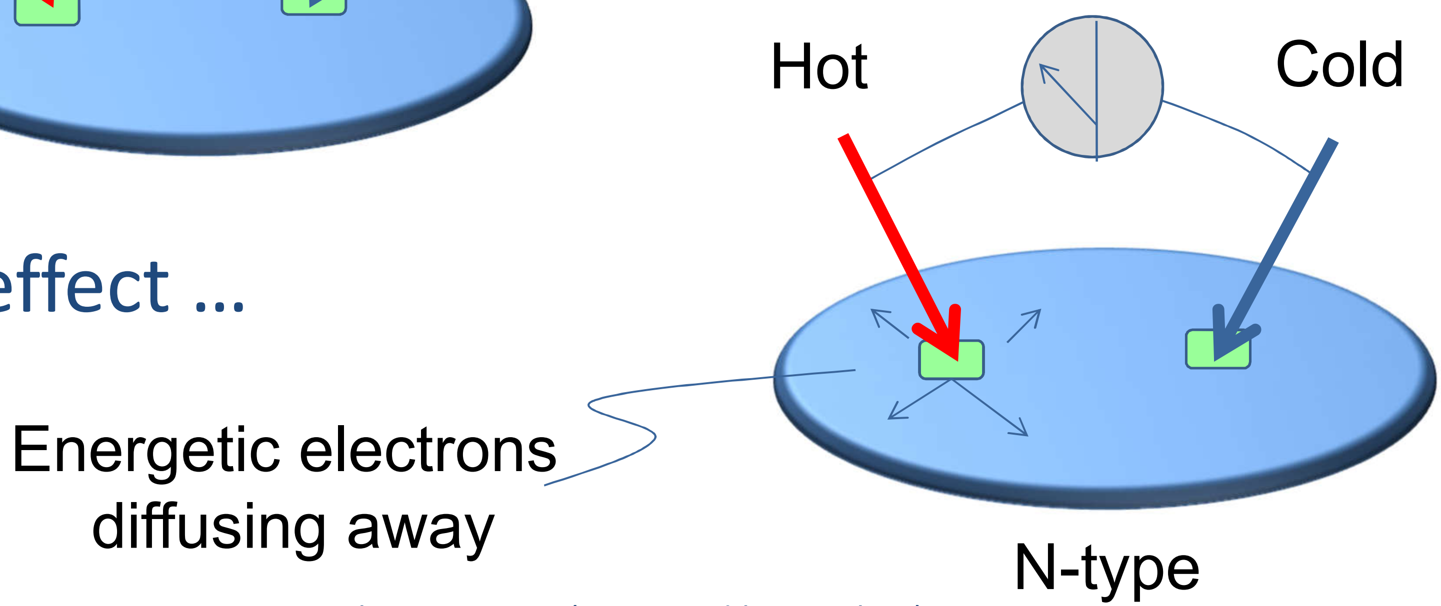
1. Carrier diffusion
2. Diffusion current
3. Energy band diagrams
4. Junction formation
5. Poisson’s equation

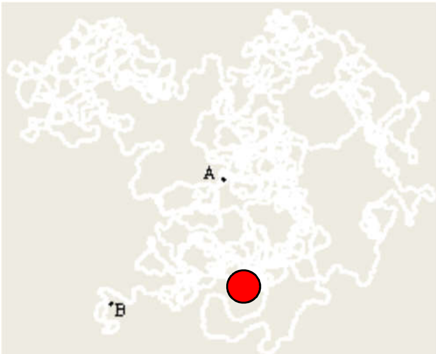
Doping by Hot Probes …





Hall effect …

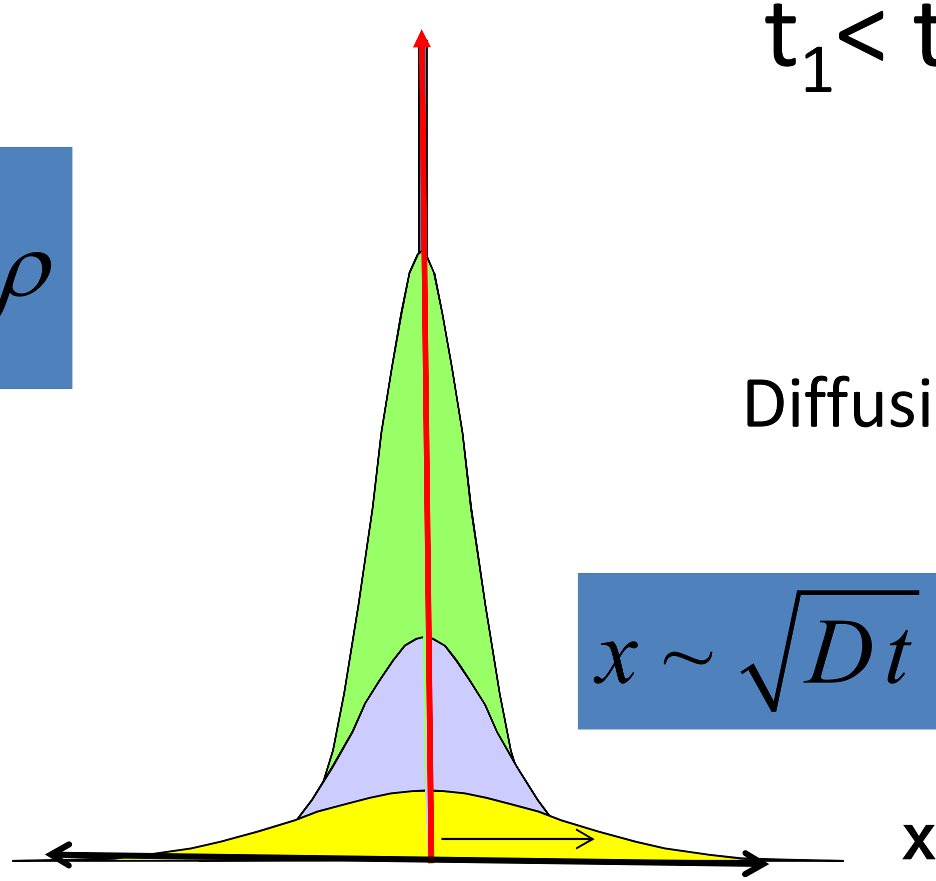




The Essence of Diffusion

$$\frac{d\rho}{dt} = D\nabla^2\rho$$

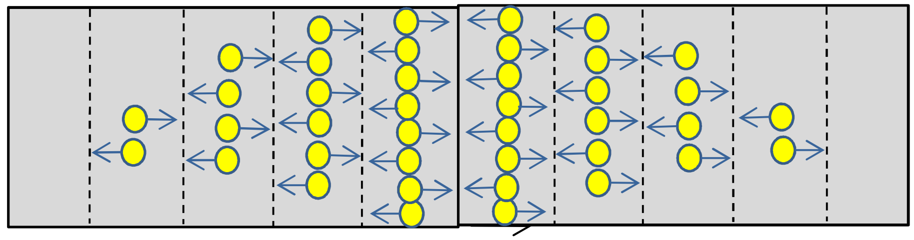
$t_1 < t_2 < t_3$

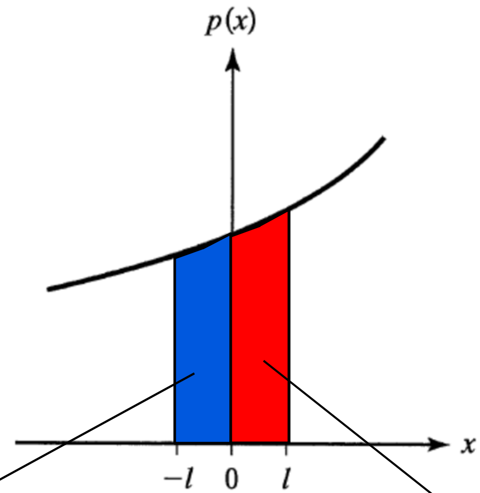


Diffusion distance

$$x \sim \sqrt{Dt}$$

Calculating Diffusion Flux ...





$$\mathcal{F} = \left[+\frac{1}{2}\left(\frac{p(0) + p(0) - \frac{dp}{dx} l}{2} \right) \times l - \frac{1}{2}\left(\frac{p(0) + p(0) + \frac{dp}{dx} l}{2} \right) \times l \right] \Bigg/ \frac{l}{\upsilon_{th}}$$

$$= -\frac{l\upsilon}{2}\frac{dp}{dx} \equiv -D\frac{dp}{dx}$$

outline

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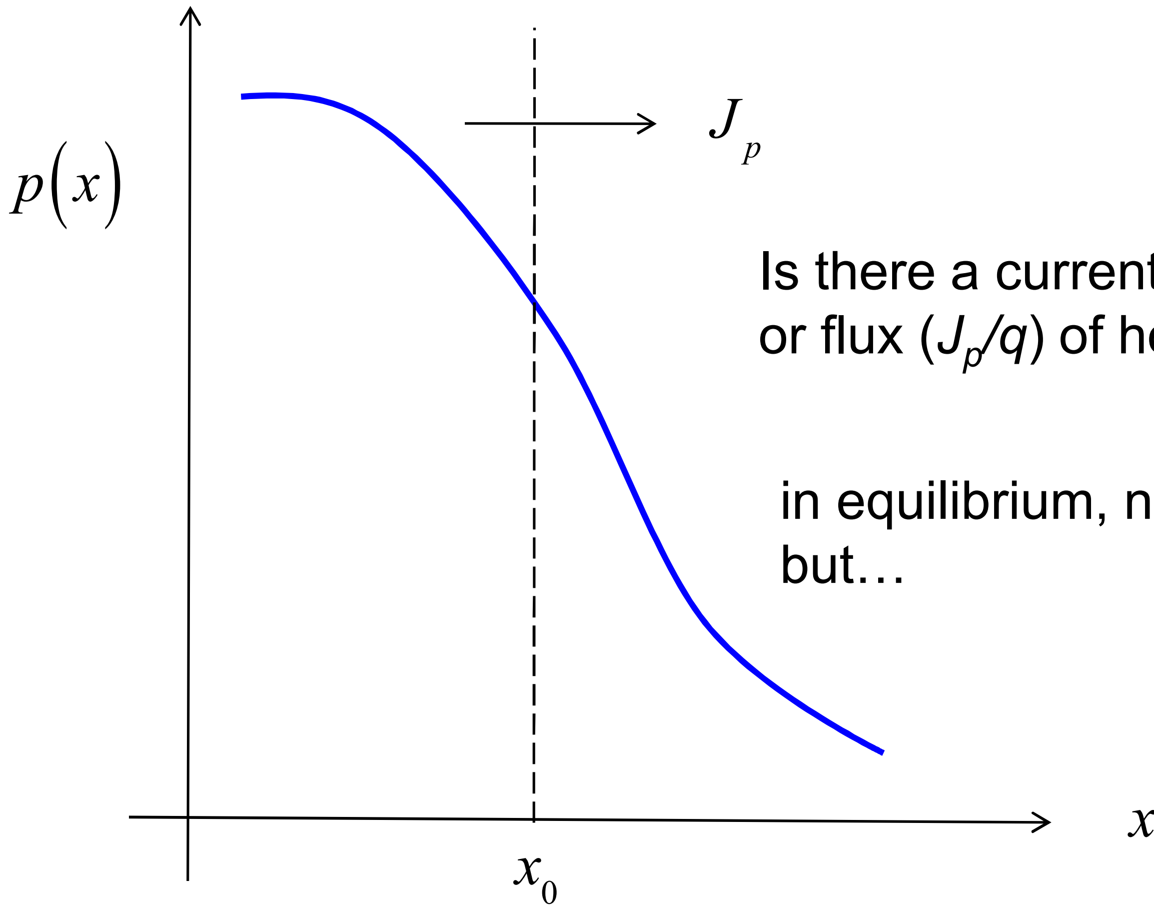
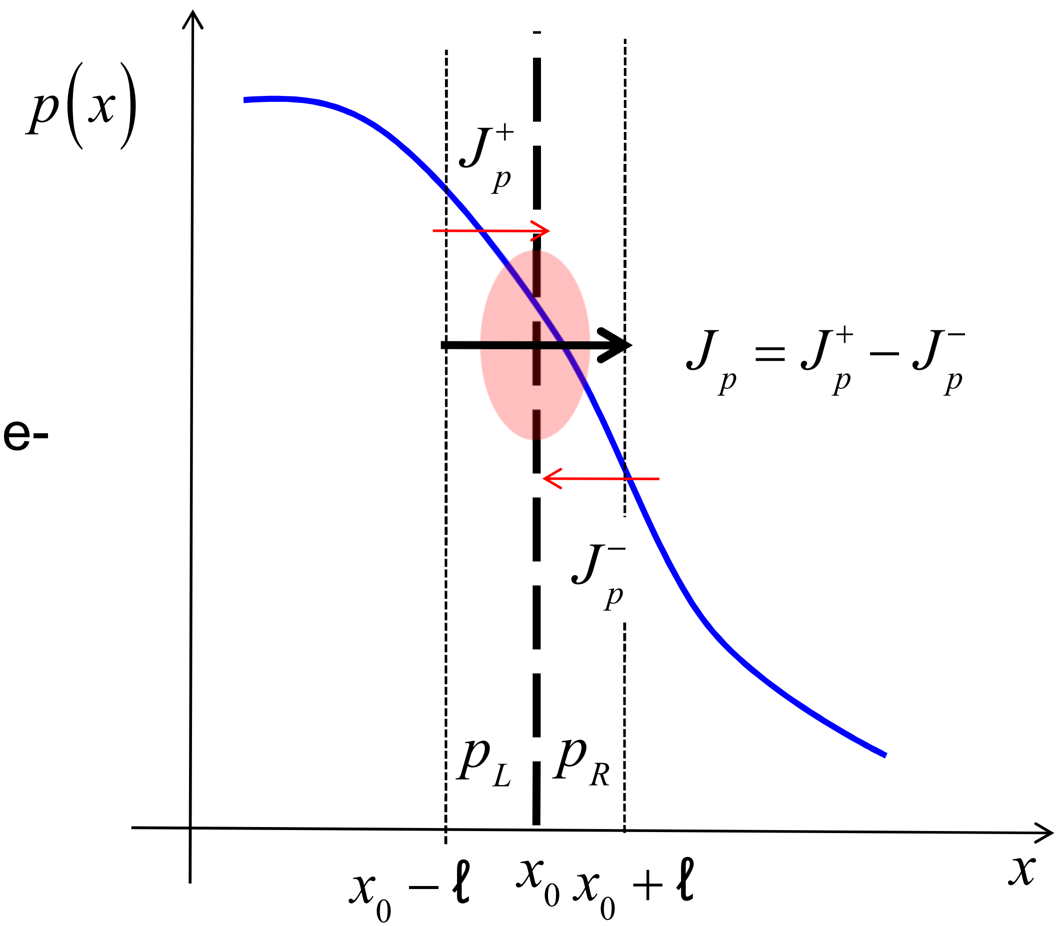
The diffusion current



Is there a current or flux (J_p/q) of holes?

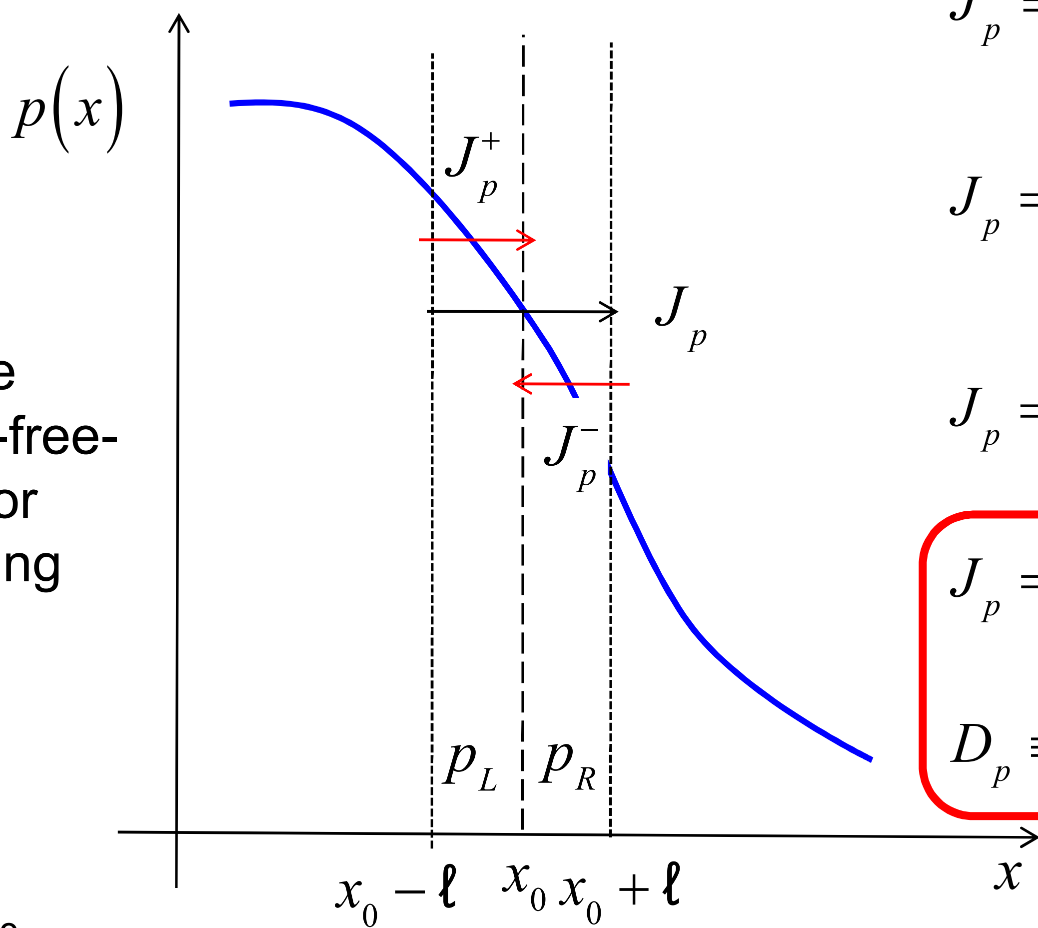
in equilibrium, no
but…

diffusion current



ℓ is the "mean-free-path" for scattering

diffusion current



ℓ is the "mean-free-path" for scattering

$$J_p^+ = q\frac{p_L}{2}\upsilon_T \qquad J_p^- = q\frac{p_R}{2}\upsilon_T$$

$$J_p = J_p^+ - J_p^- = q\frac{\upsilon_T}{2}\left(p_L - p_R\right)$$

$$J_p = q\frac{\upsilon_T \ell}{2}\frac{\left(p_L - p_R\right)}{\ell}$$

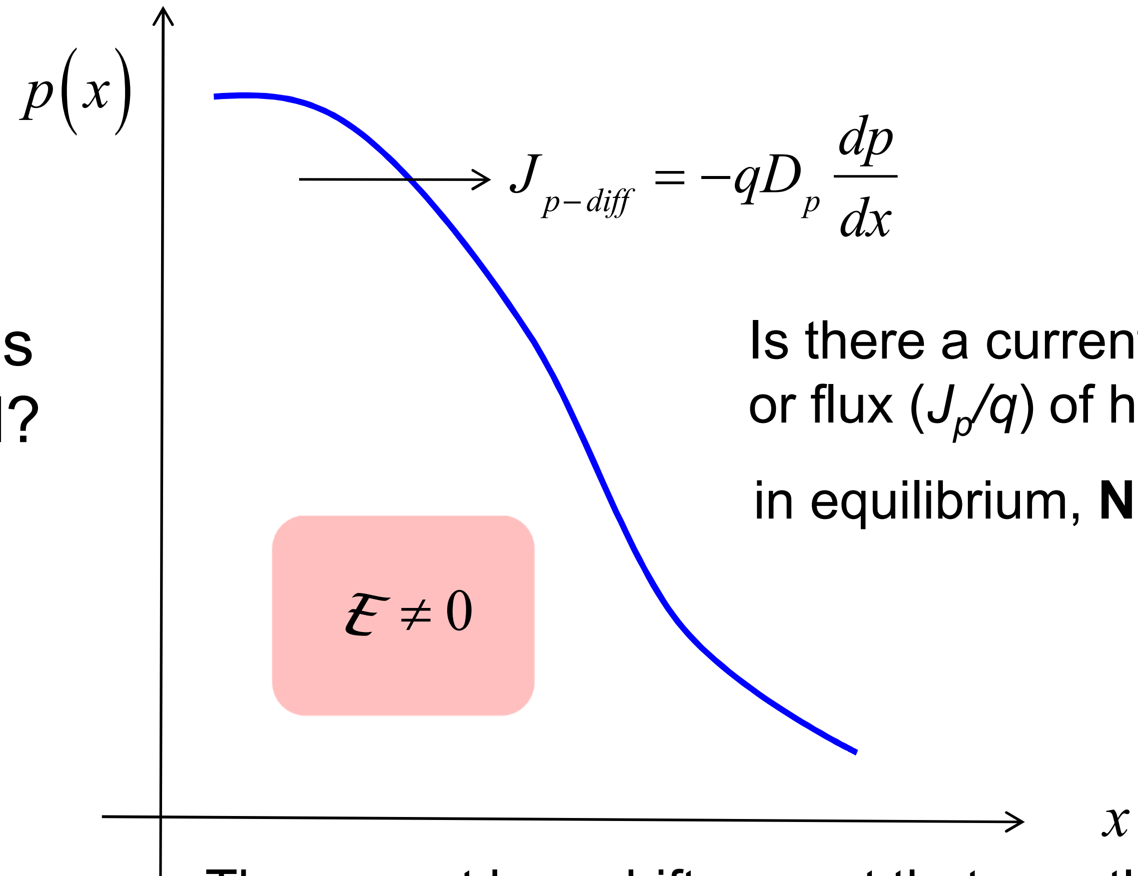
$$J_p = -qD_p\frac{dp}{dx}$$

$$D_p \equiv \frac{\upsilon_T \ell}{2}\ \mathrm{cm^2/s}$$

diffusion current

Question:
What direction is the electric field?

a) To the right
b) To the left



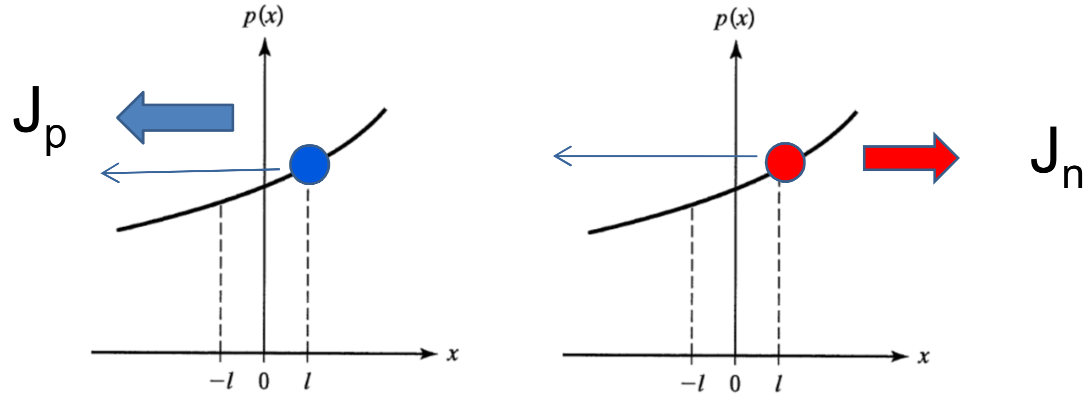
$$J_{p-diff} = -qD_p \frac{dp}{dx}$$

Is there a current or flux (J_p/q) of holes?

in equilibrium, **NO!**

There must be a drift current that exactly cancels the diffusion current.

Flux and Current



$$J_{p,diff} = +q \times \mathcal{F}_p = +q \times \left(-D_p \frac{dp}{dx}\right) = -qD_p \frac{dp}{dx}$$

$$J_{n,diff} = -q \times \mathcal{F}_n = -q \times \left(-D_n \frac{dn}{dx}\right) = qD_n \frac{dn}{dx}$$

$$\vec{\mathcal{F}} = -D\vec{\nabla} n$$

Fick's Law of Diffusion (1855)

Einstein Relationship at Equilibrium …

$$\frac{D}{\mu}=\frac{\frac{l\upsilon}{2}}{\frac{q\tau}{m_0^*}}=\frac{\frac{(\upsilon\tau)\times\upsilon}{2}}{\frac{q\tau}{m_0^*}}=\frac{\frac{1}{2}m_0^*\upsilon^2}{q}=\frac{k_BT}{q}$$

… because scattering dominates both phenomena

drift- diffusion equation

$$\mu_p = \frac{q\tau}{m_p^*}$$

$$\mu_n = \frac{q\tau}{m_n^*}$$

$$D_p = \frac{\upsilon_{Tp}\ell_p}{2}$$

$$D_n = \frac{\upsilon_{Tn}\ell_n}{2}$$

$$\vec{J}_p = \vec{J}_{p-drift} + \vec{J}_{p-diff} = pq\mu_p\vec{\mathcal{E}} - qD_p\vec{\nabla}p$$

current = drift current + diffusion current

$$\vec{J}_n = \vec{J}_{n-drift} + \vec{J}_{n-diff} = nq\mu_n\vec{\mathcal{E}} + qD_n\vec{\nabla}n$$

total current = electron current + hole current

$$\vec{J} = \vec{J}_p + \vec{J}_n$$

$$D_p/\mu_p = D_n/\mu_n = k_B T/q$$

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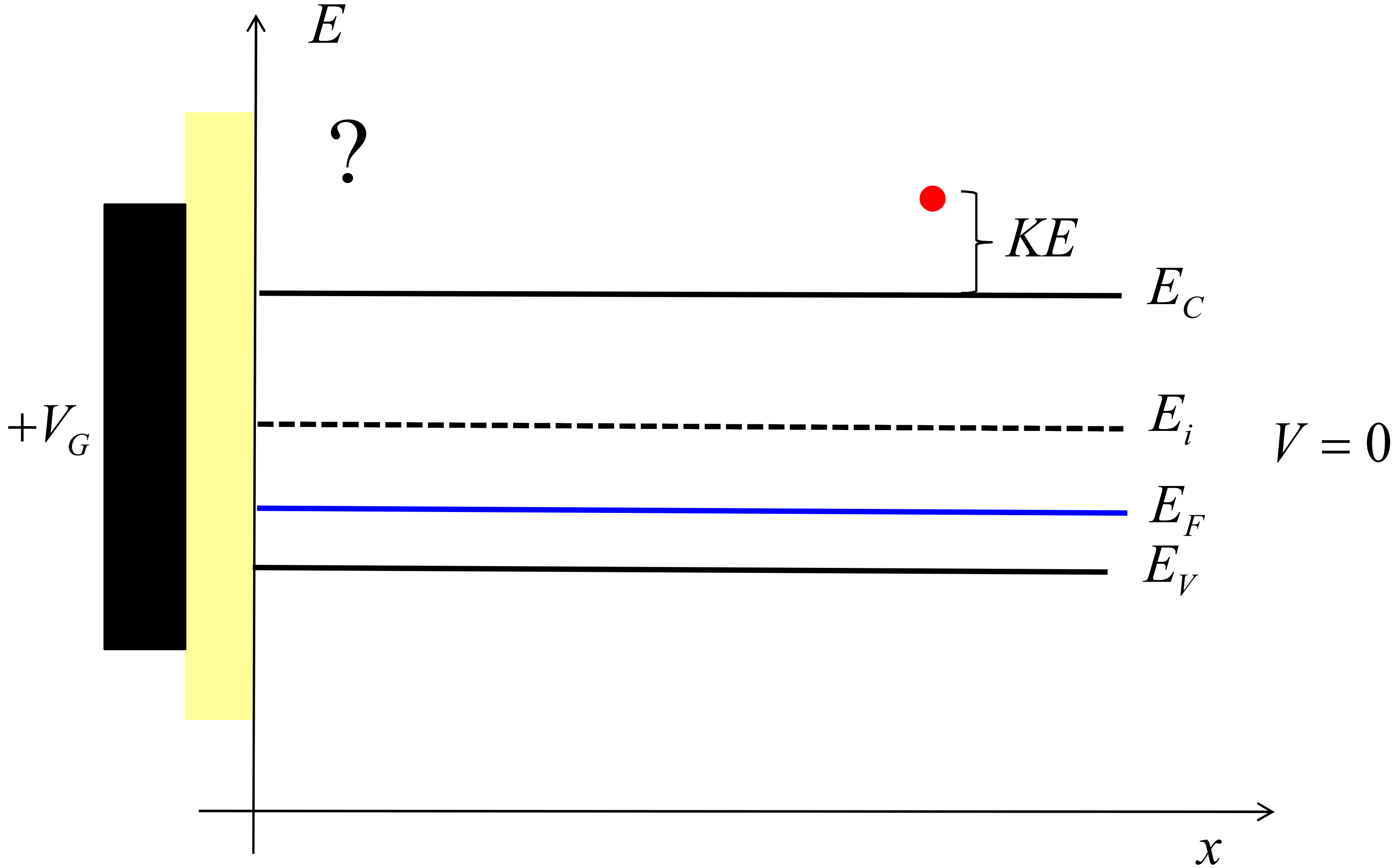
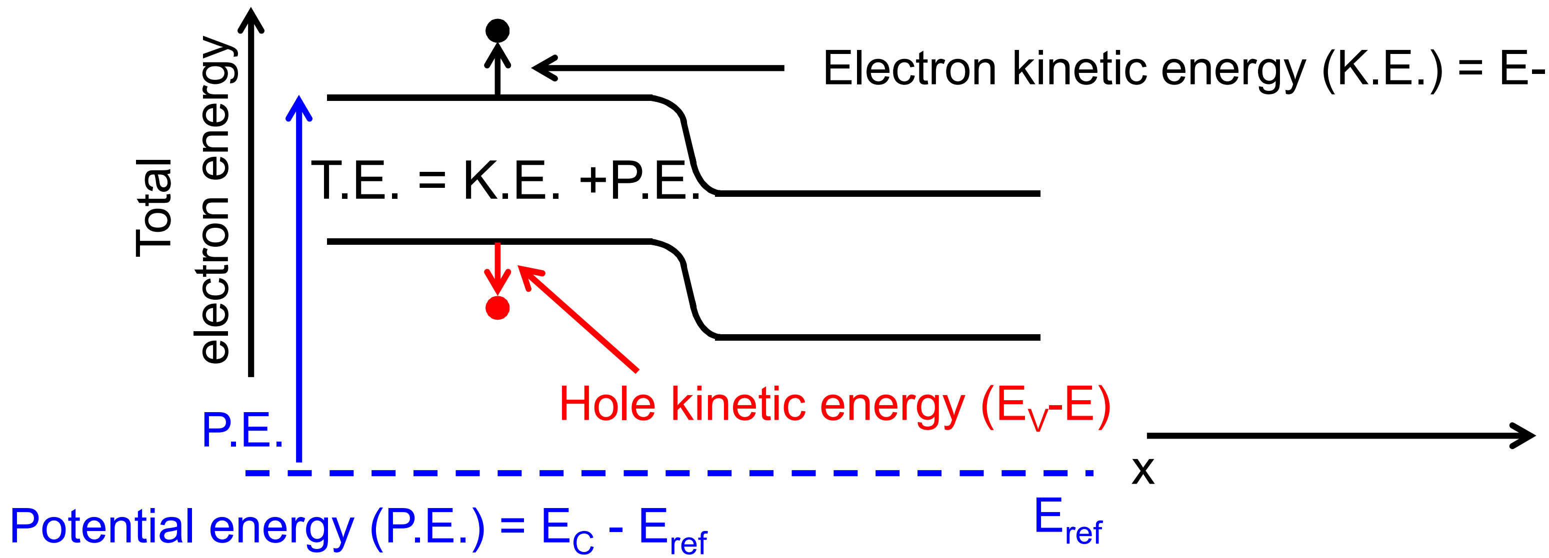
band bending

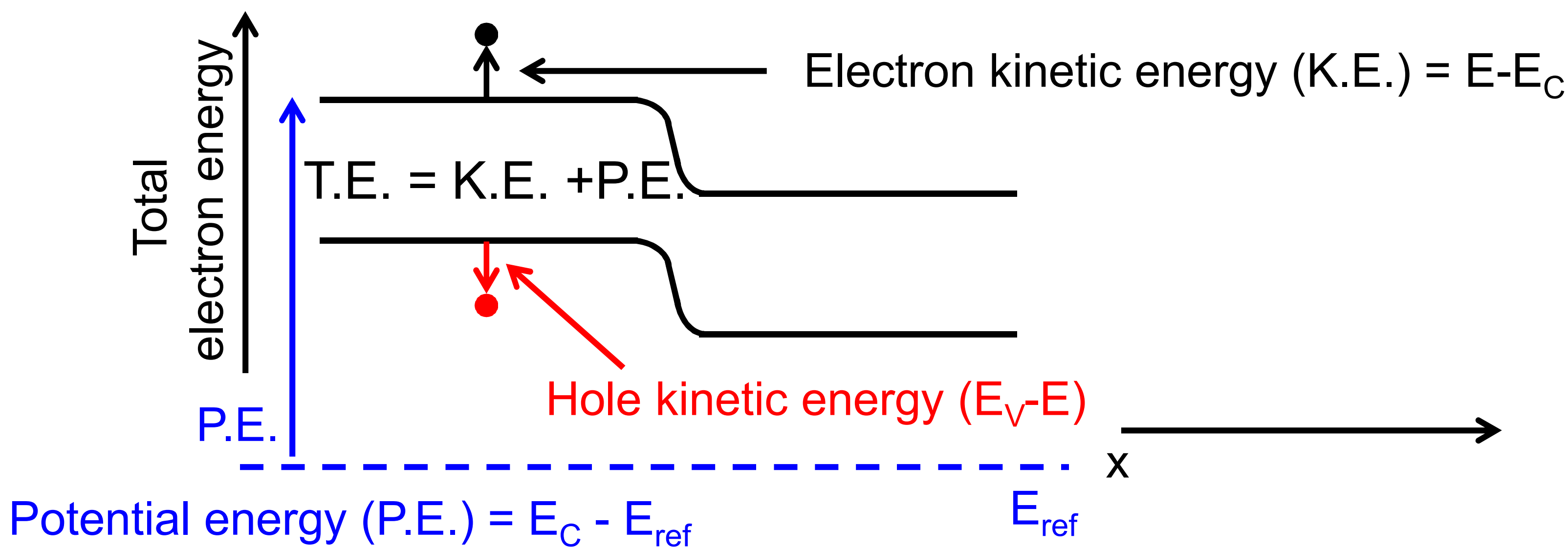


Kinetic and potential energy



- Energy bands are not constant in electric fields! Instead, they change as a function of position
- We need energy equal to the band gap to break bonds and excite carriers to the conduction band.
- If we only impart enough energy to promote an electron carrier, then it simply sits in the conduction band.
- Extra energy (kinetic energy) is needed to allow carriers to move.

Electrostatic potential

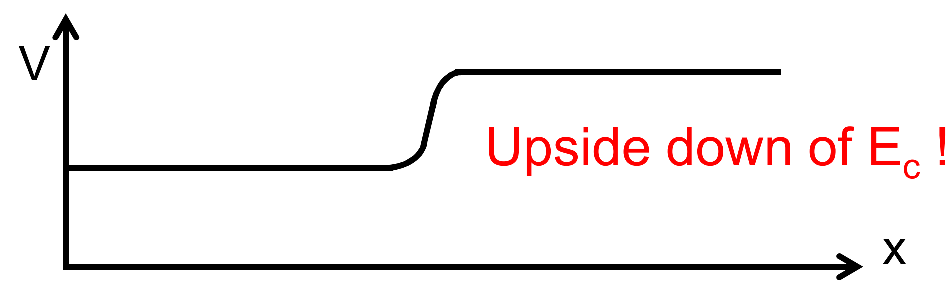


The potential energy of a charged electron: $-qV$

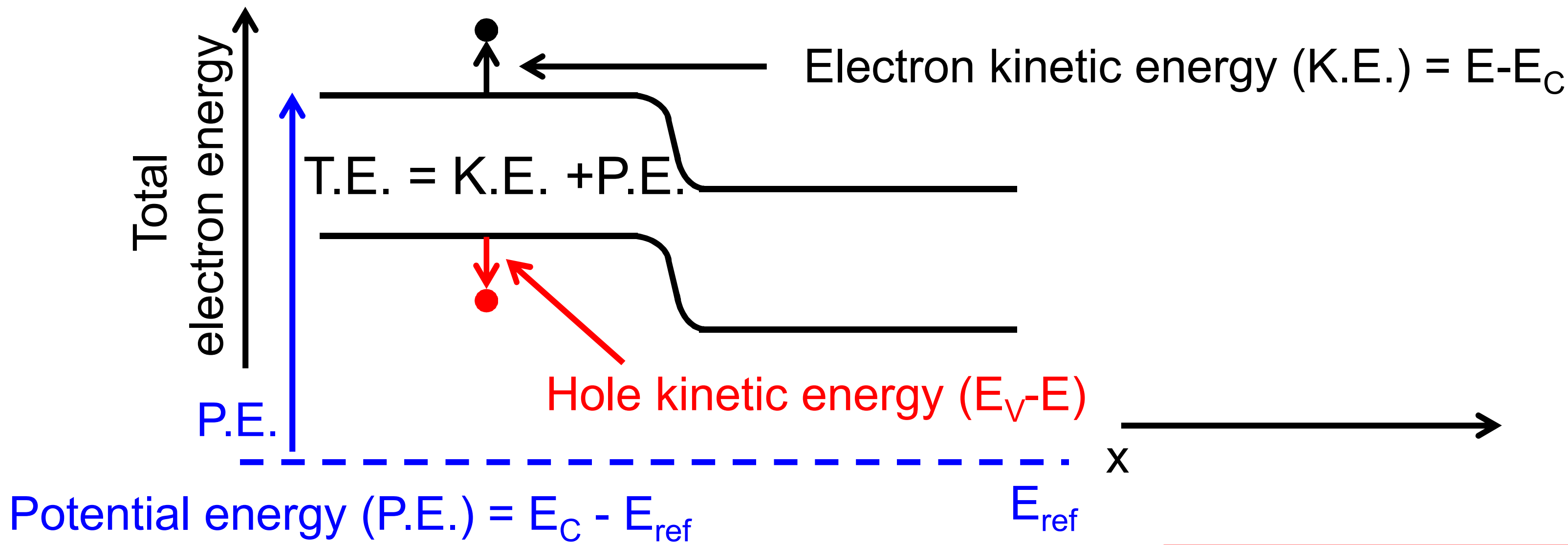
But we just defined: $P.E. = E_C - E_{ref}$

$$V = -\frac{1}{q}(E_C - E_{ref})$$

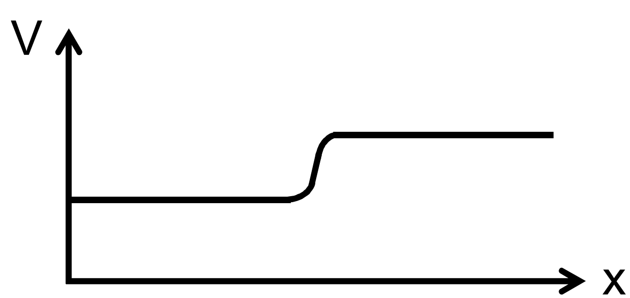
Electrostatic potential:



Electric field within the semicondctutor

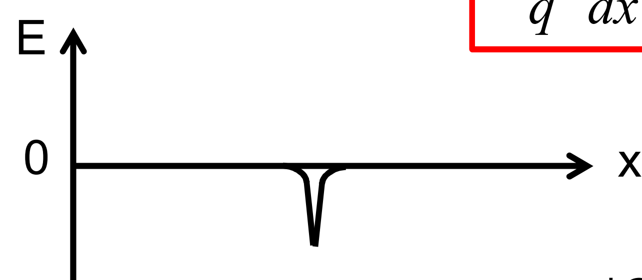


Electrostatic potential:



Electric field:

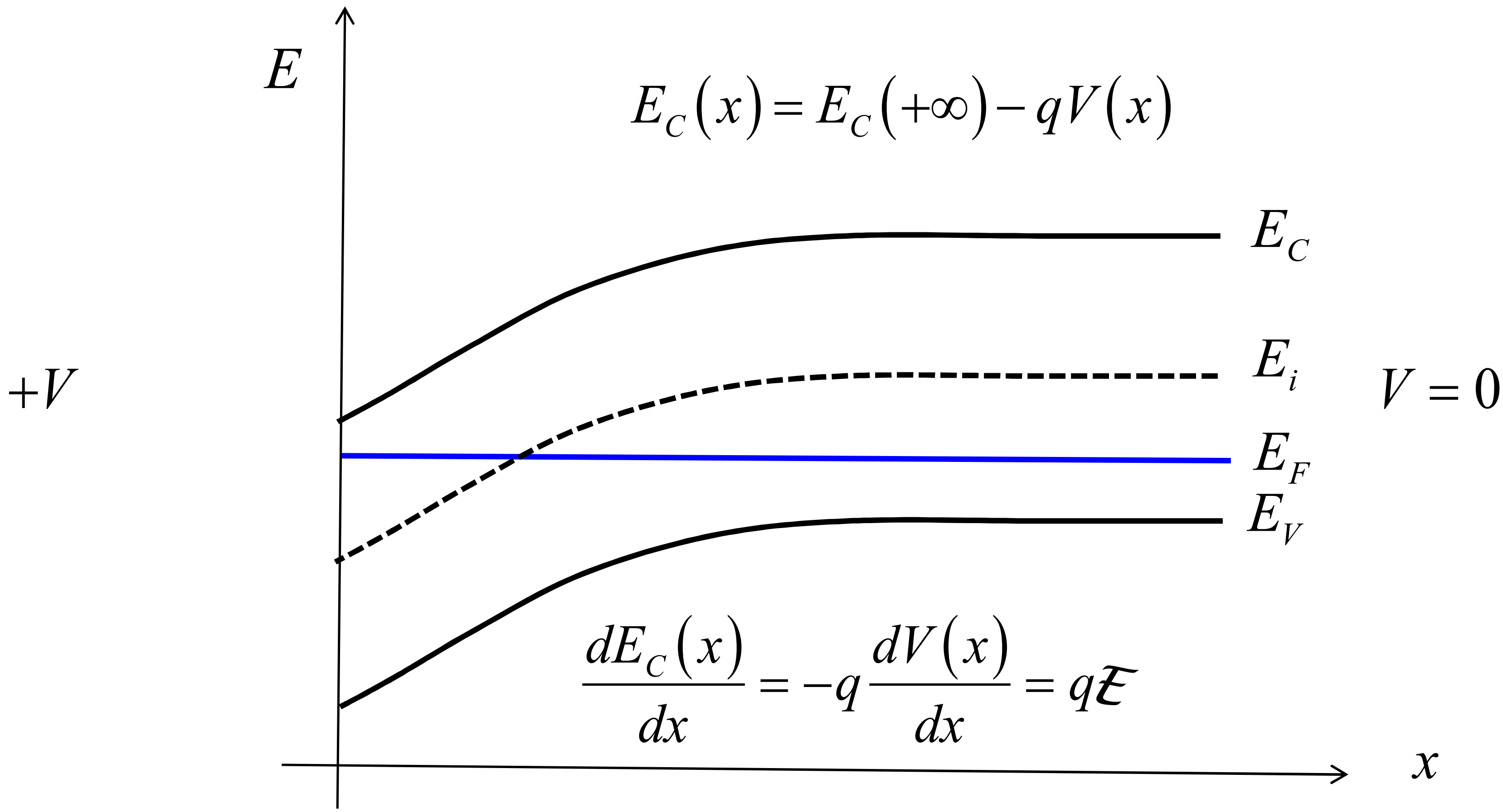
$$E = -\nabla V$$
$$= \frac{1}{q}\frac{dE_C}{dx} = \frac{1}{q}\frac{dE_V}{dx} = \frac{1}{q}\frac{dE_i}{dx}$$



*Check ex. 3.2 on p. 92

Observe how electric field changes with band bending:
https://en.wikipedia.org/wiki/Depletion_region#/media/File:PN_band.gif

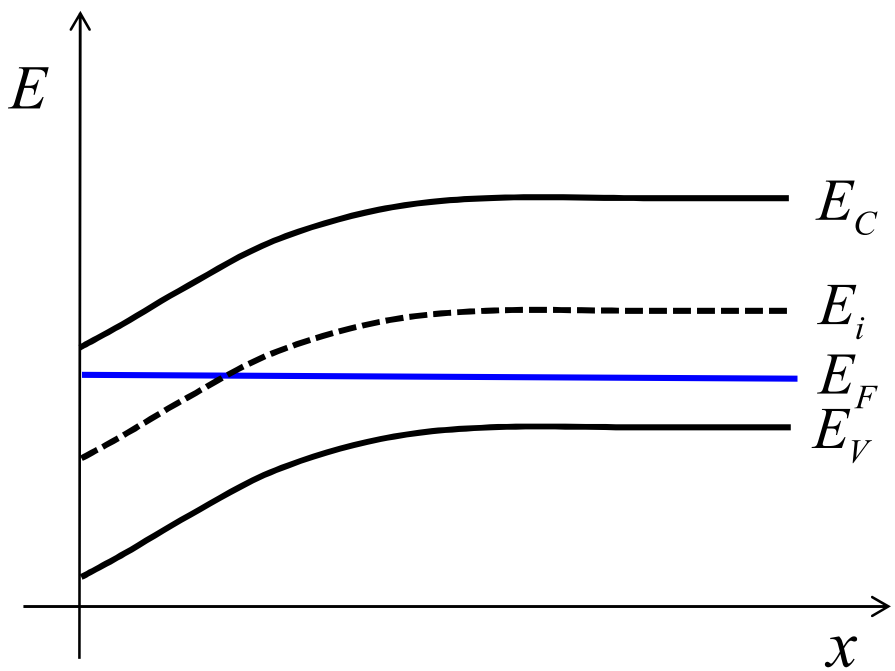
band bending



$$E_C(x) = E_C(+\infty) - qV(x)$$

$$\frac{dE_C(x)}{dx} = -q\frac{dV(x)}{dx} = q\mathcal{E}$$

band bending

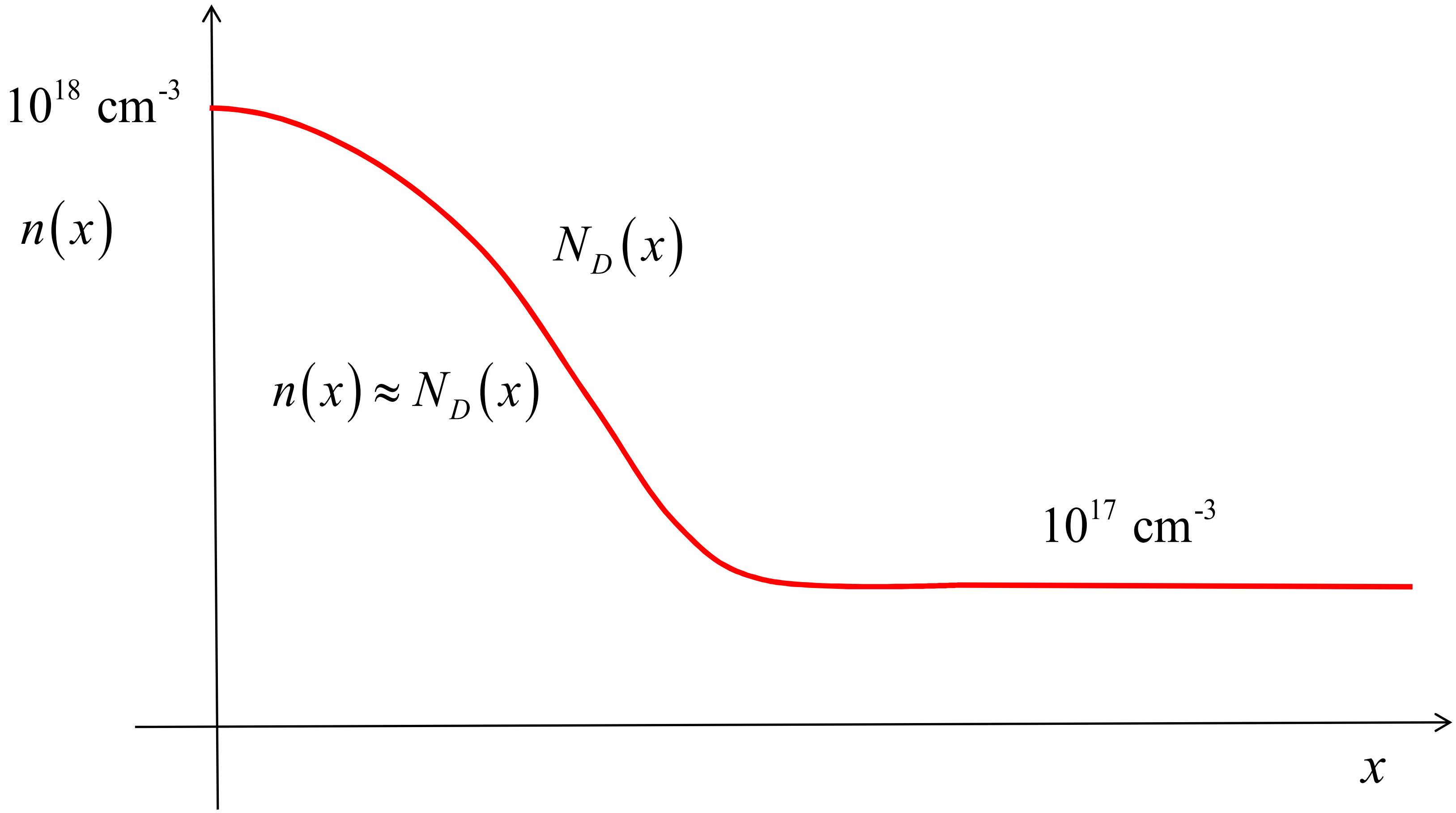


Sketch:

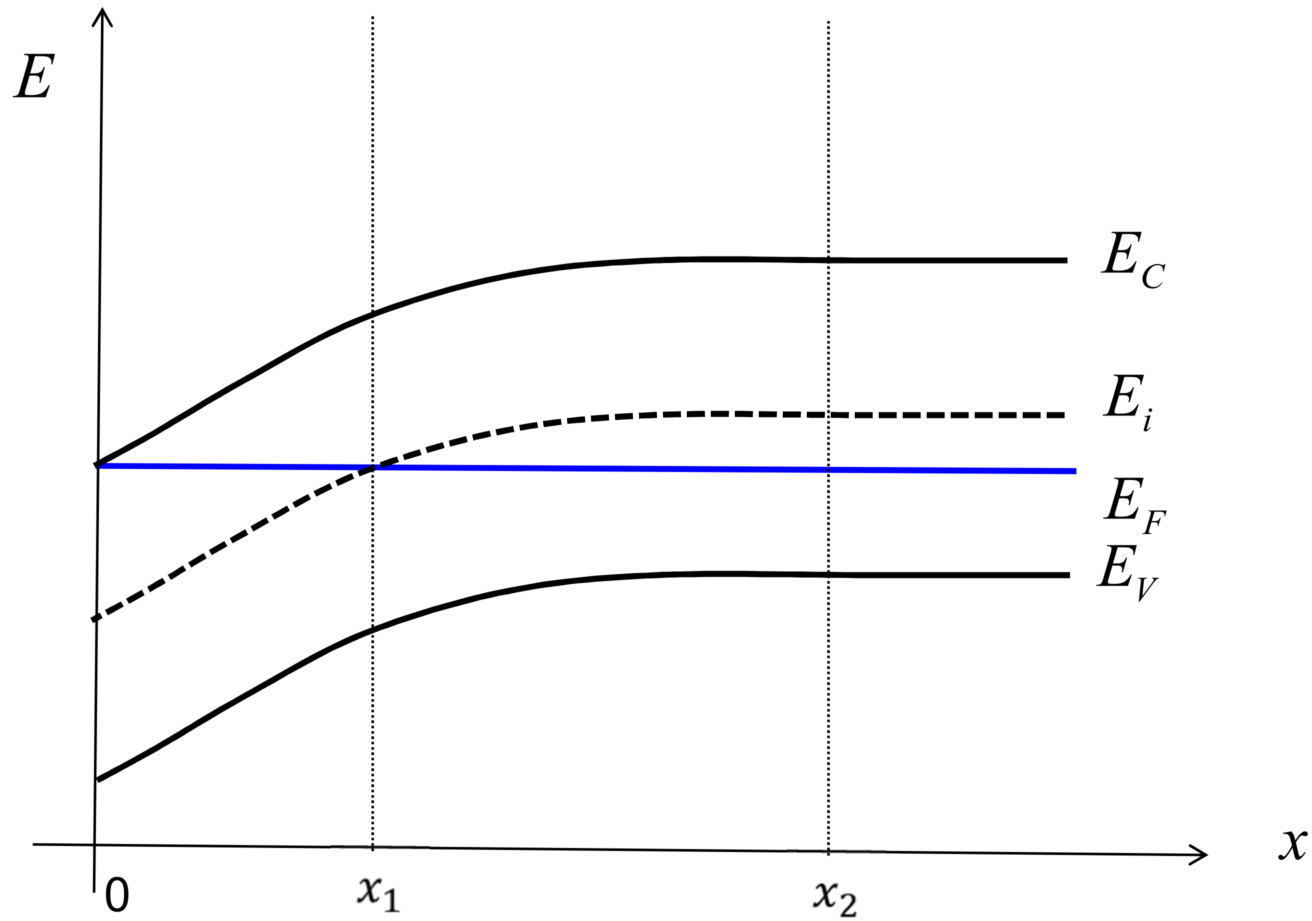
- Electrostatic potential
- Electric field
- Electron density
- Hole density

Very important point:
The Fermi level is constant in equilibrium.

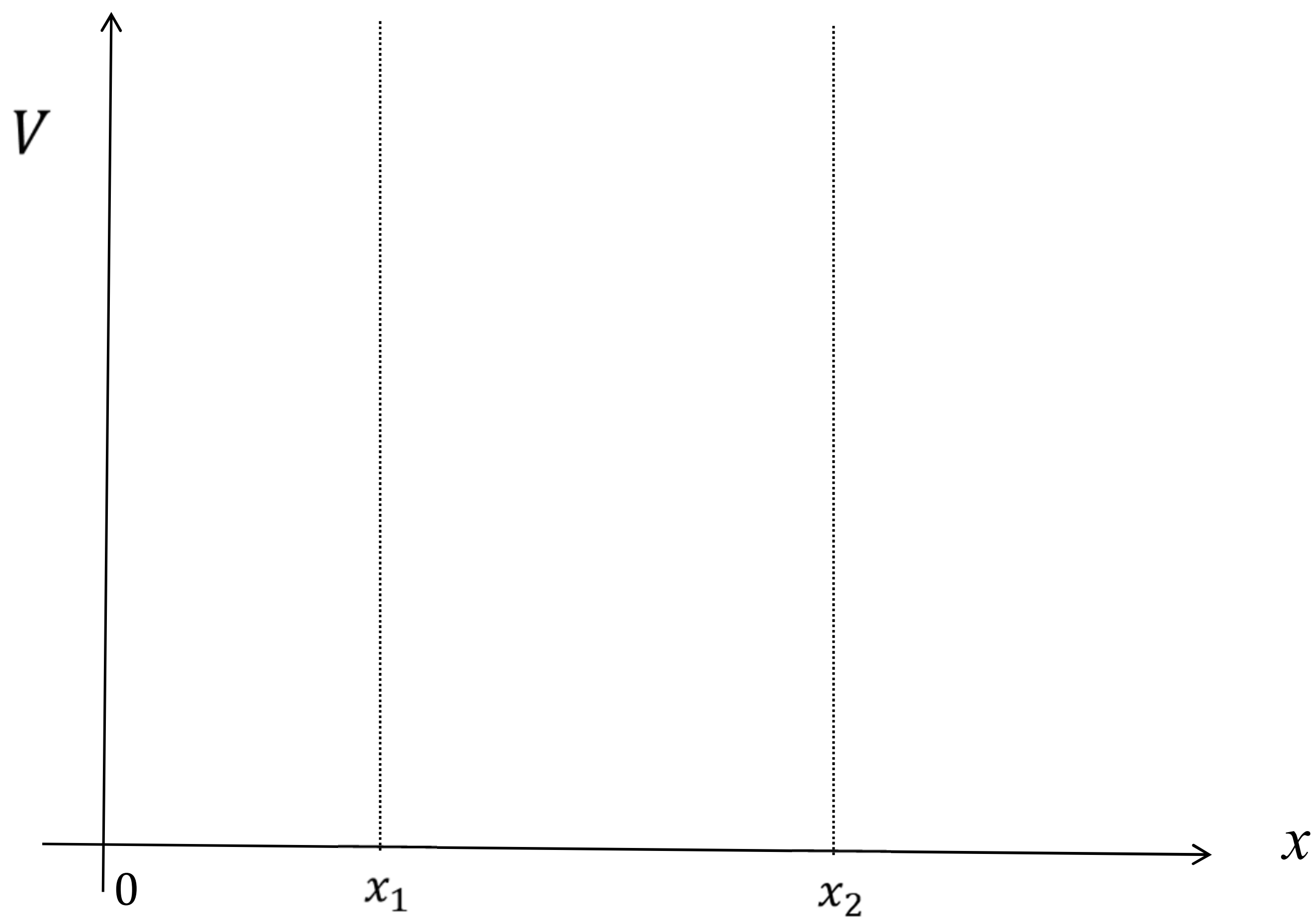
Electron density



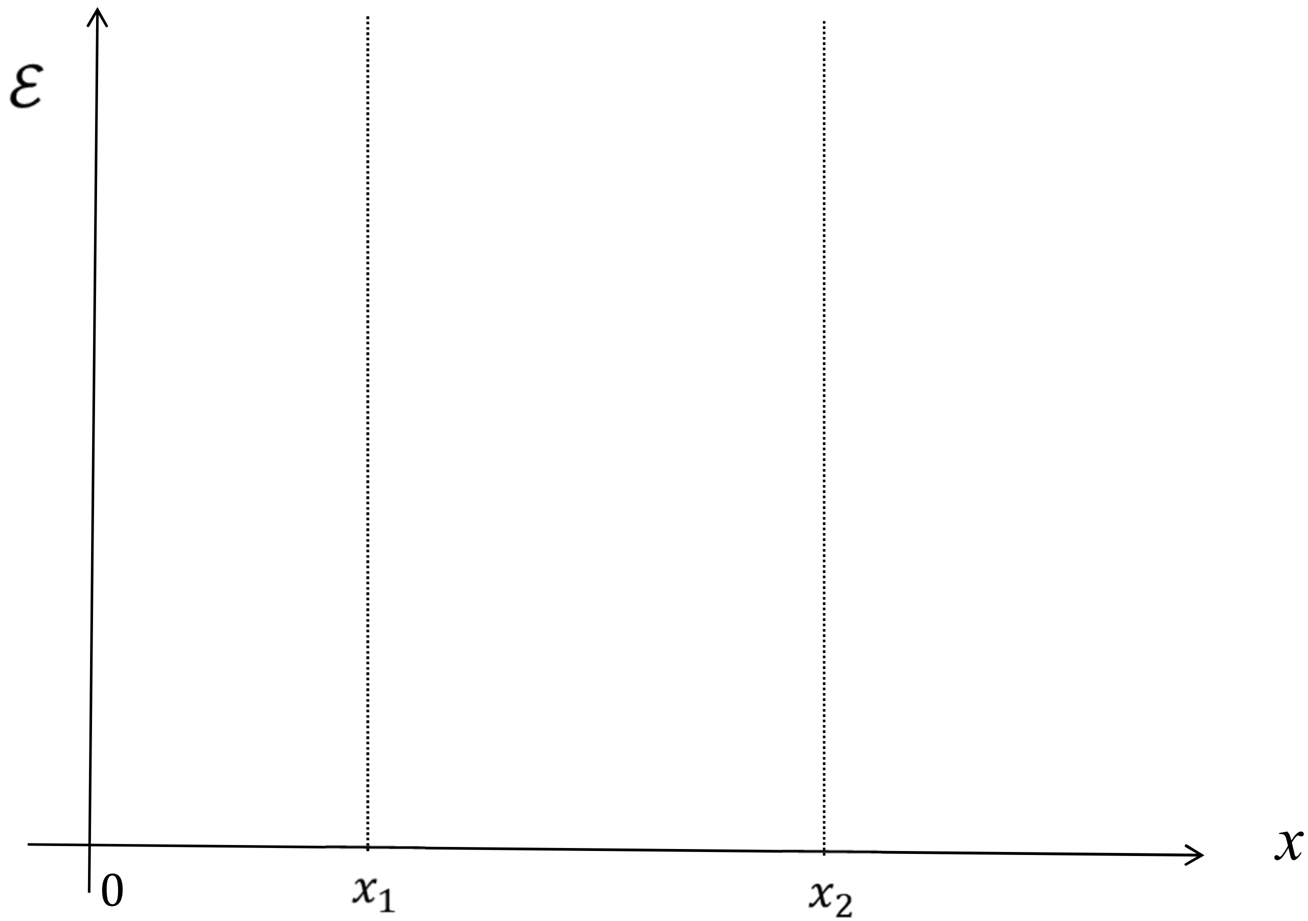
band bending



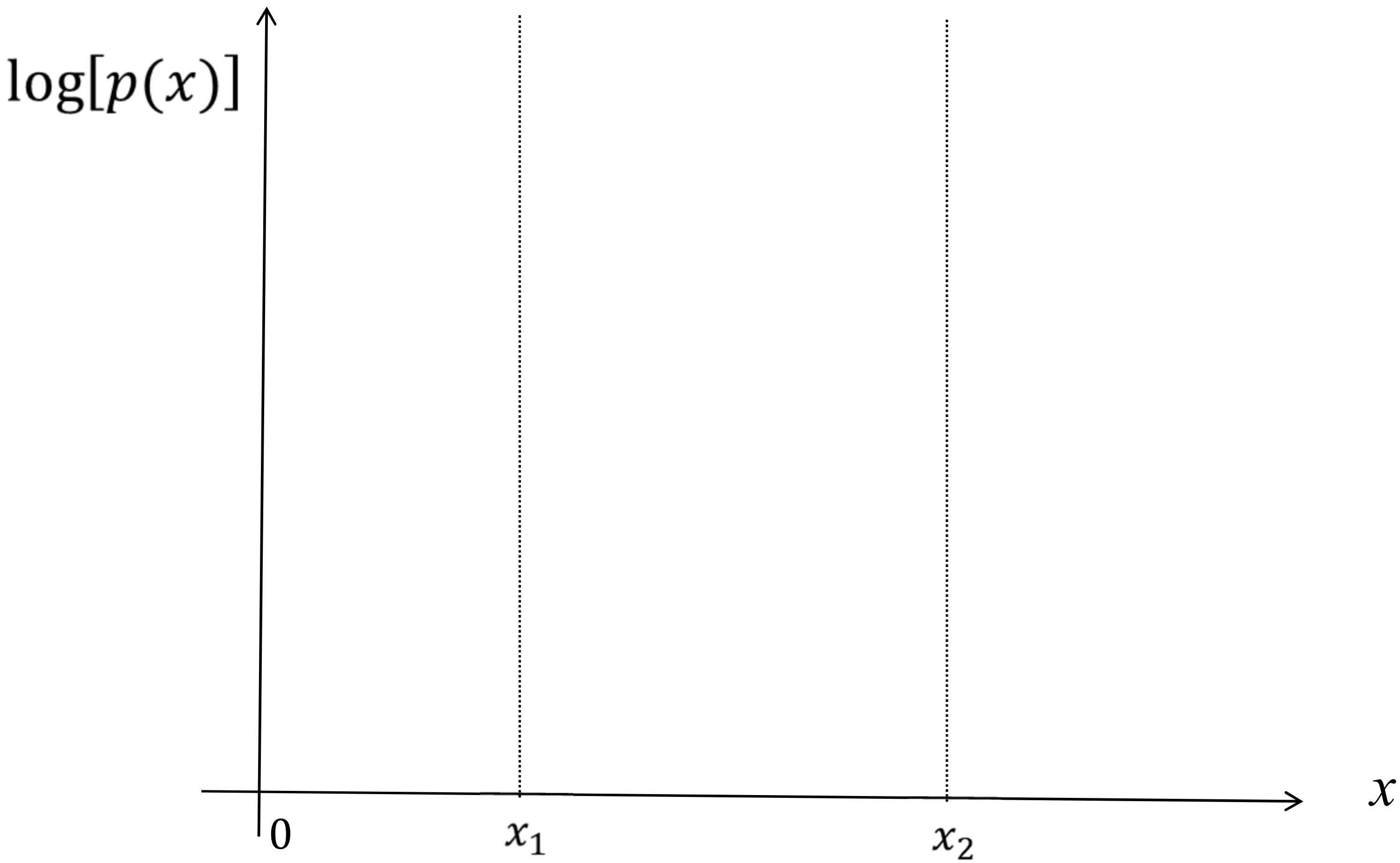
band bending



band bending



band bending



Kroemer's lemma of proven ignorance

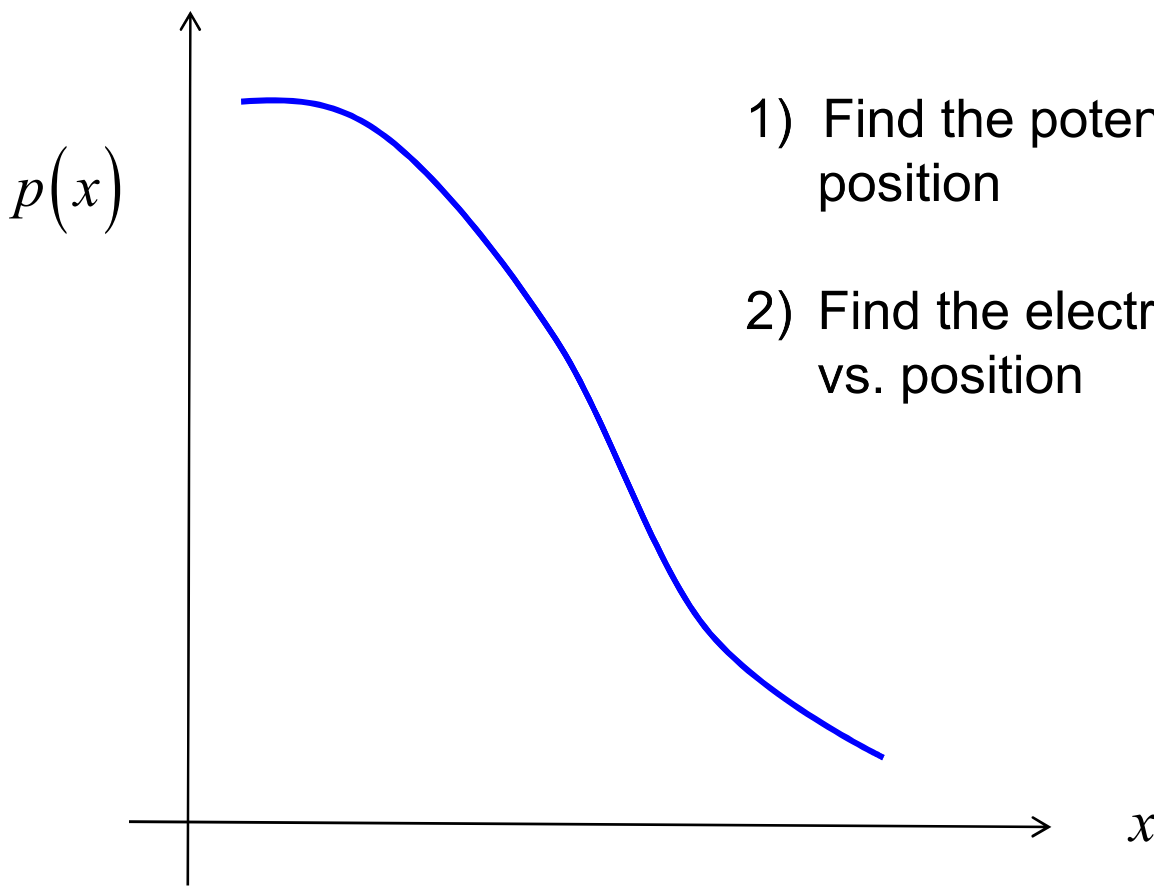
"Whenever I teach my semiconductor device physics course, one of the central messages I try to get across early is the importance of energy band diagrams. I often put this in the form of "Kroemer's lemma of proven ignorance":

> If, in discussing a semiconductor problem, you cannot draw an **Energy Band Diagram**, this shows that you don't know what you are talking about.

Corollary: (Nobel Prize Lecture, 2000)

> If you can draw one, but don't, then your audience won't know what you are talking about.

draw the energy band diagram



1) Find the potential vs. position

2) Find the electric field vs. position

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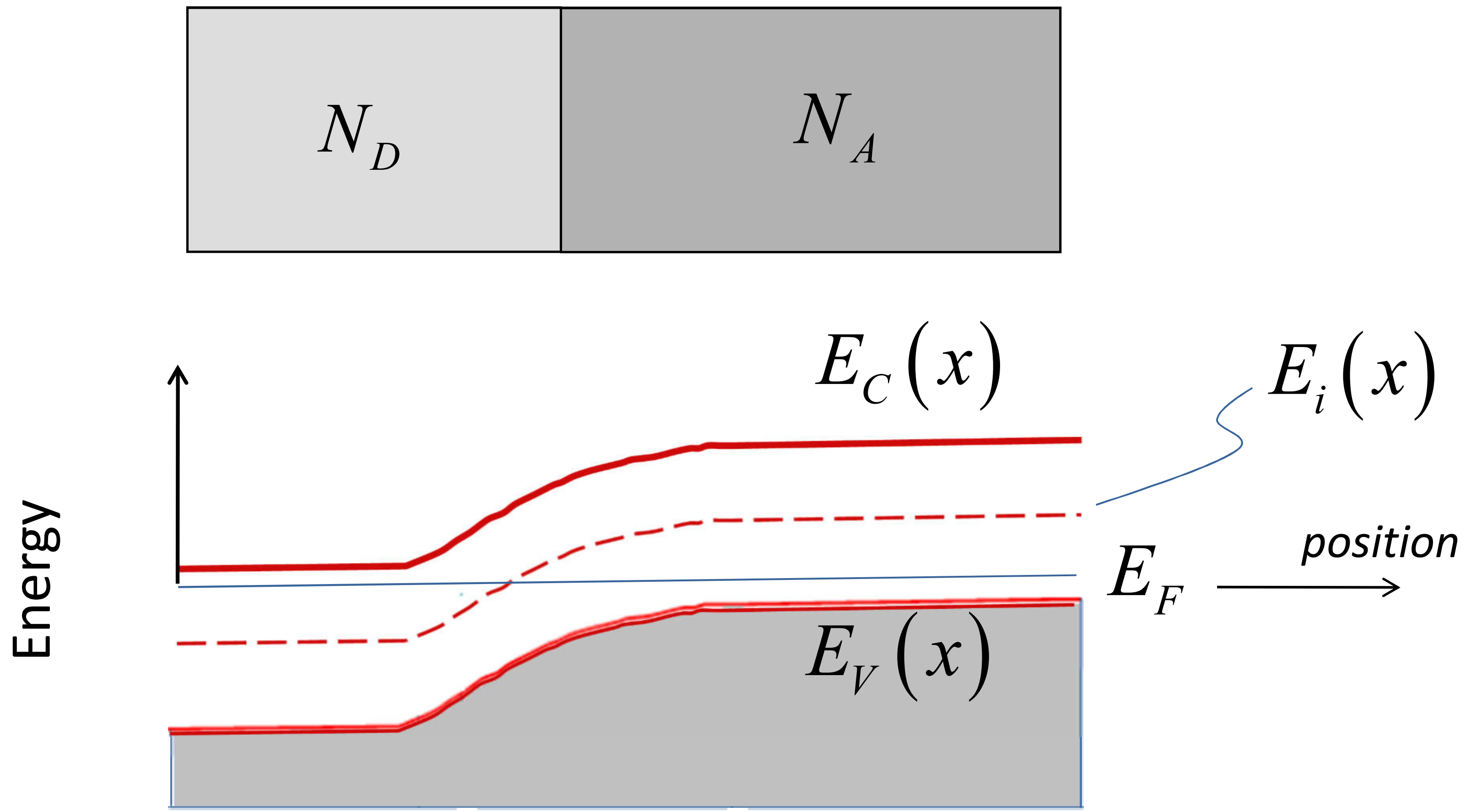
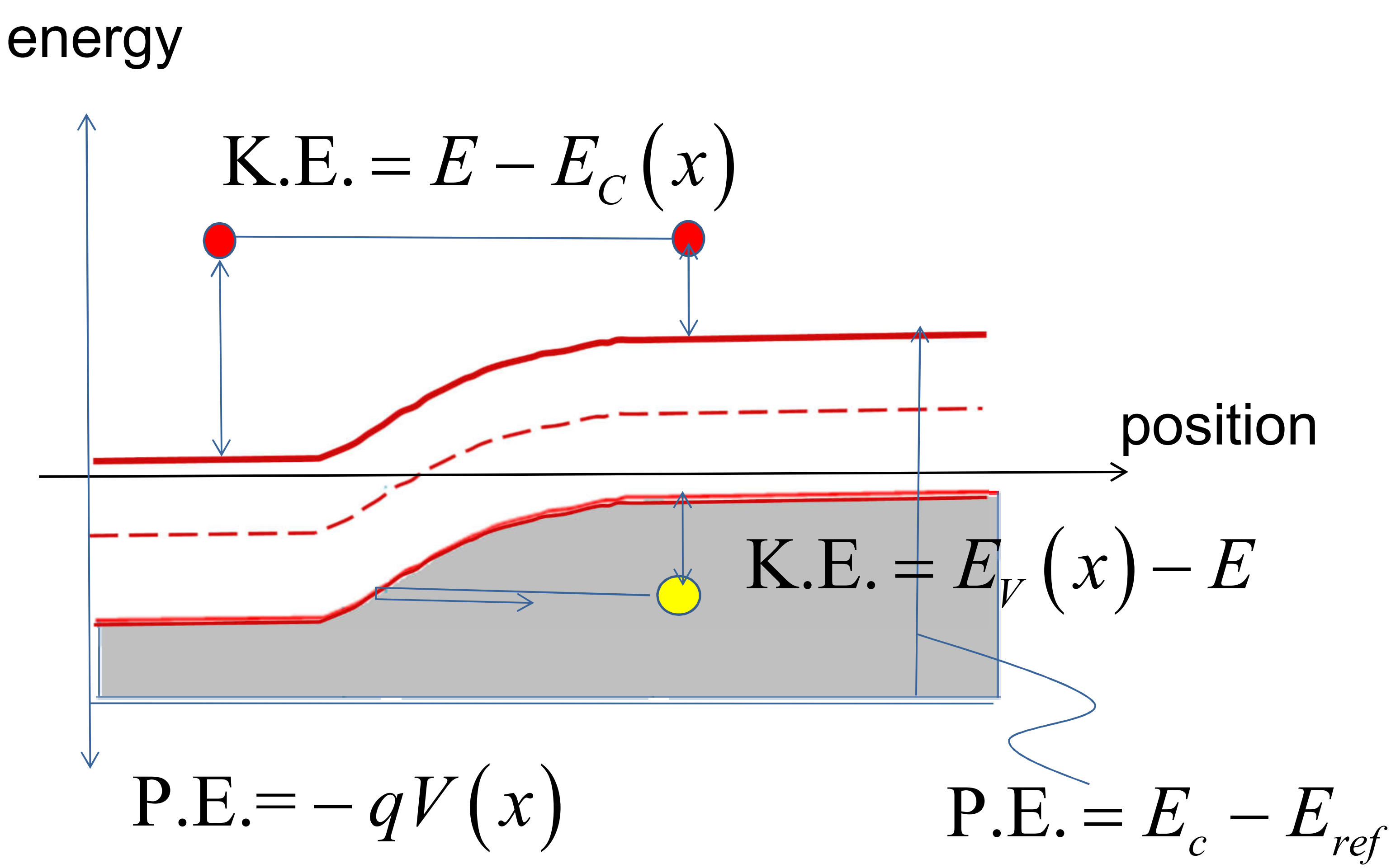
Formation of a Junction

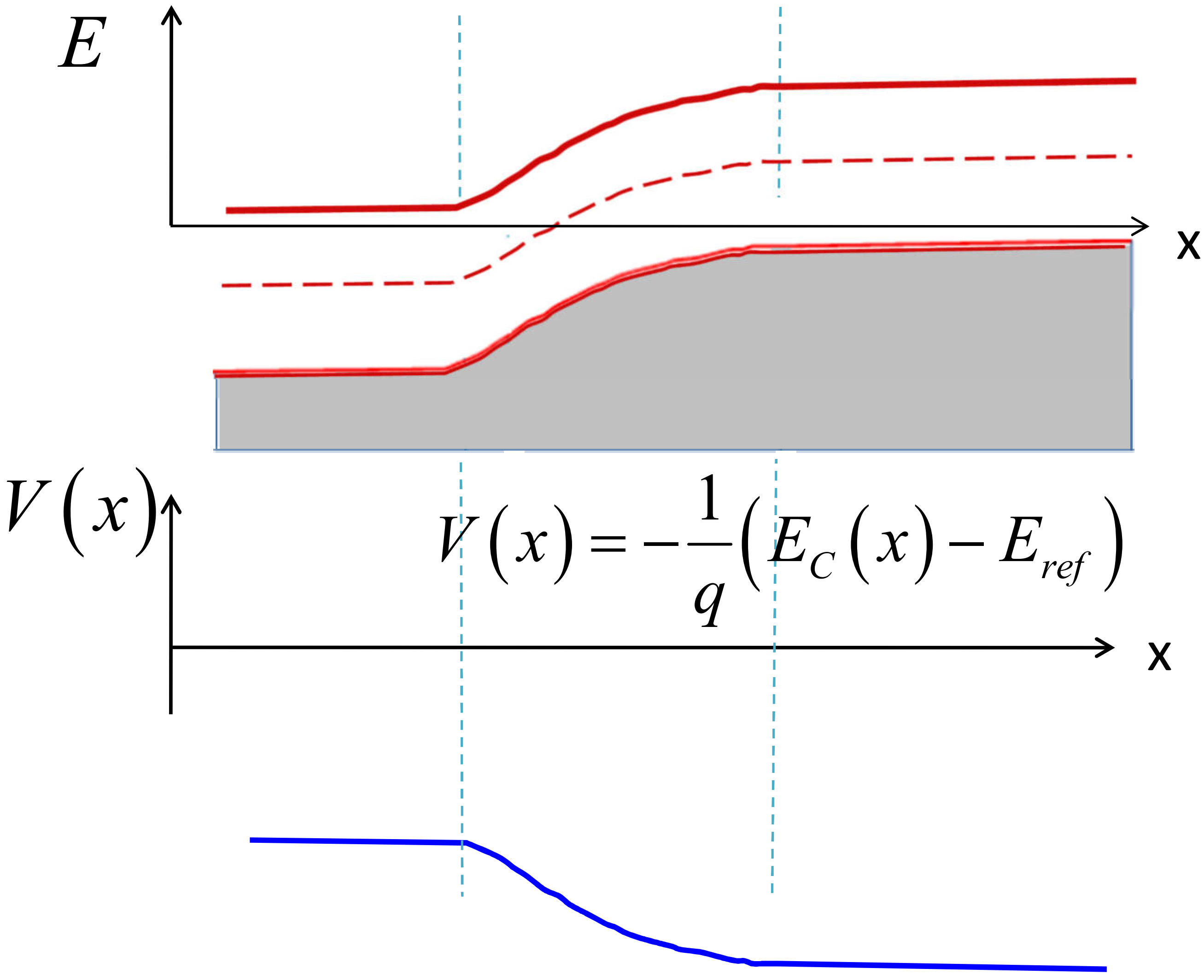


Kinetic and Potential Energies



$$V(x) = -\frac{1}{q}\left(E_C(x) - E_{ref}\right)$$

Band-diagram to Potential



$$V(x) = -\frac{1}{q}\left(E_C(x) - E_{ref}\right)$$

Joules vs. eV

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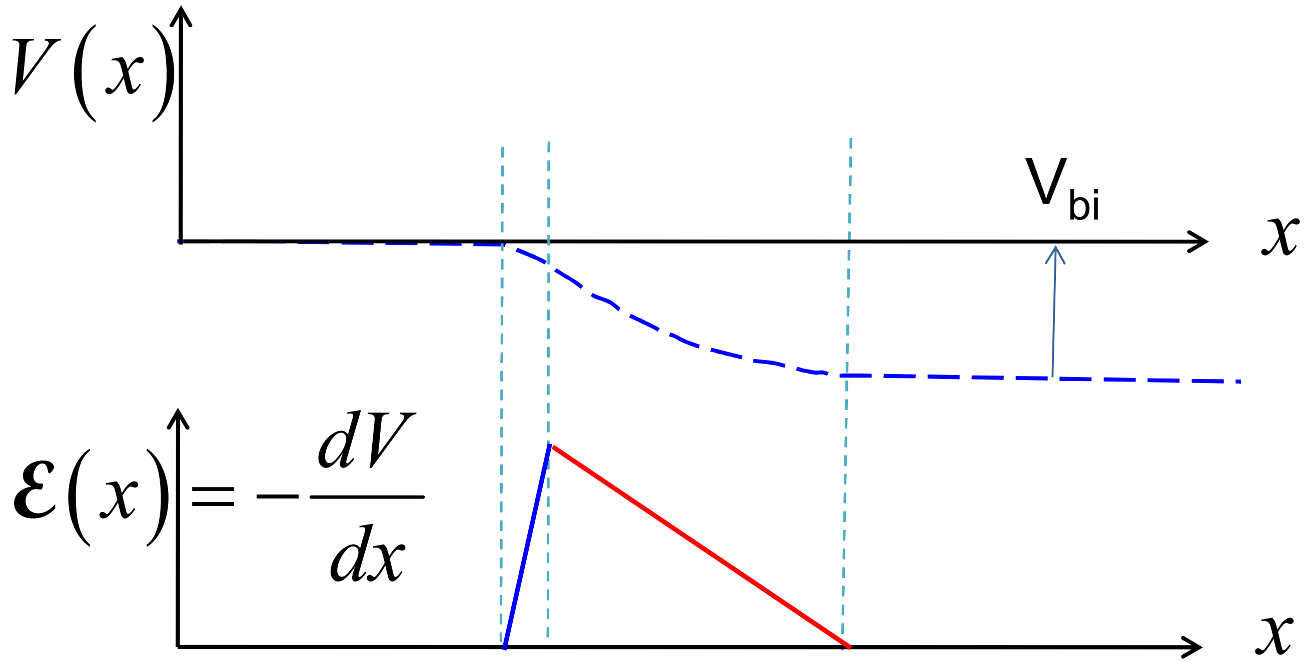
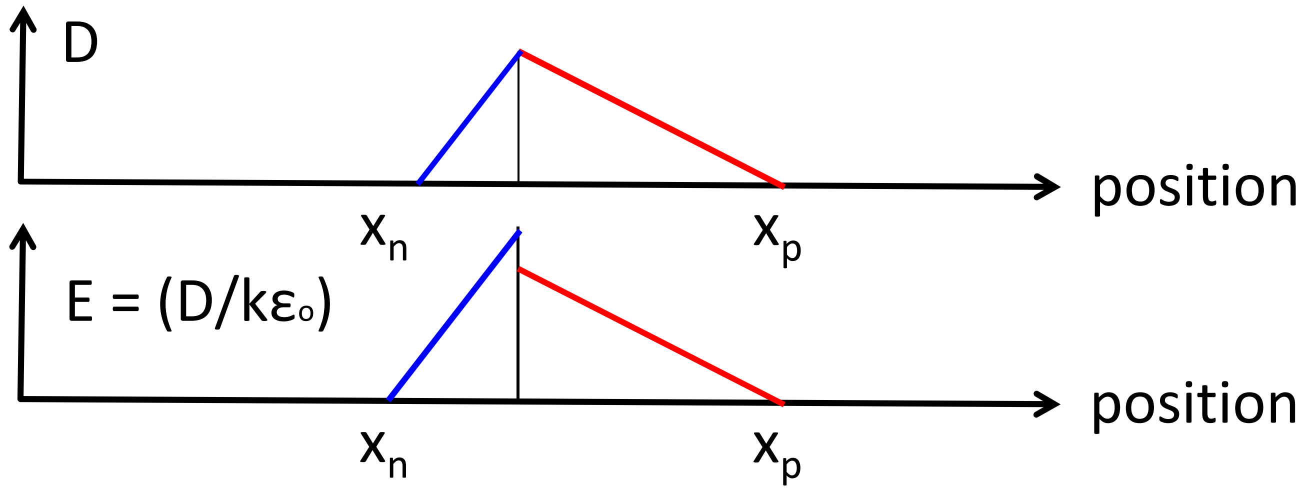
Potential, Electric Field, Band diagram



$$\mathcal{E}(x) = -\frac{dV}{dx} = \frac{1}{q}\frac{dE_c}{dx} = \frac{1}{q}\frac{dE_\upsilon}{dx} = \frac{1}{q}\frac{dE_i}{dx}$$

Interface Boundary Conditions

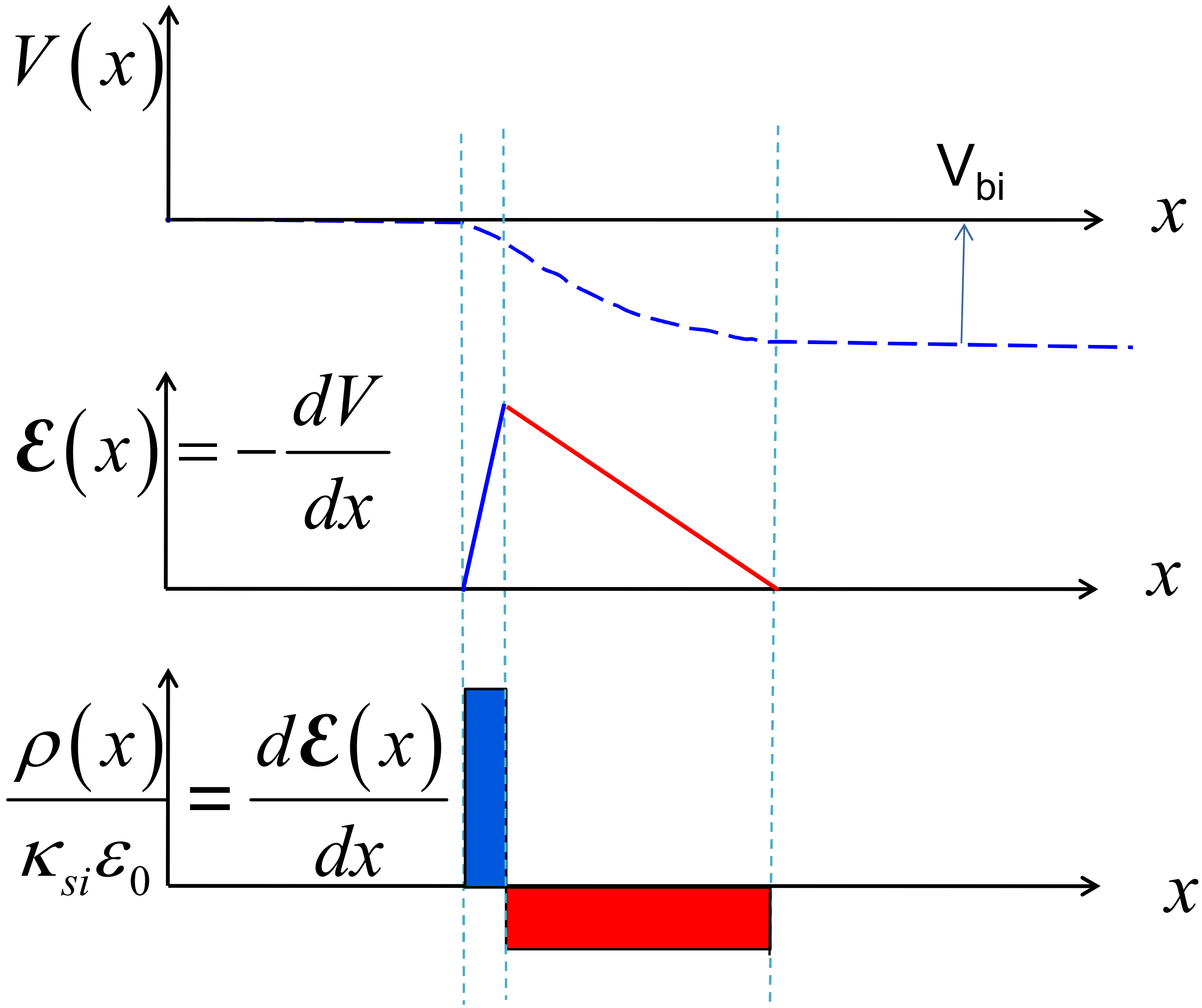


$$D_1 = \kappa_1 \varepsilon_0 \mathcal{E}(0^-) = \kappa_2 \varepsilon_0 \mathcal{E}(0^-) = D_2$$

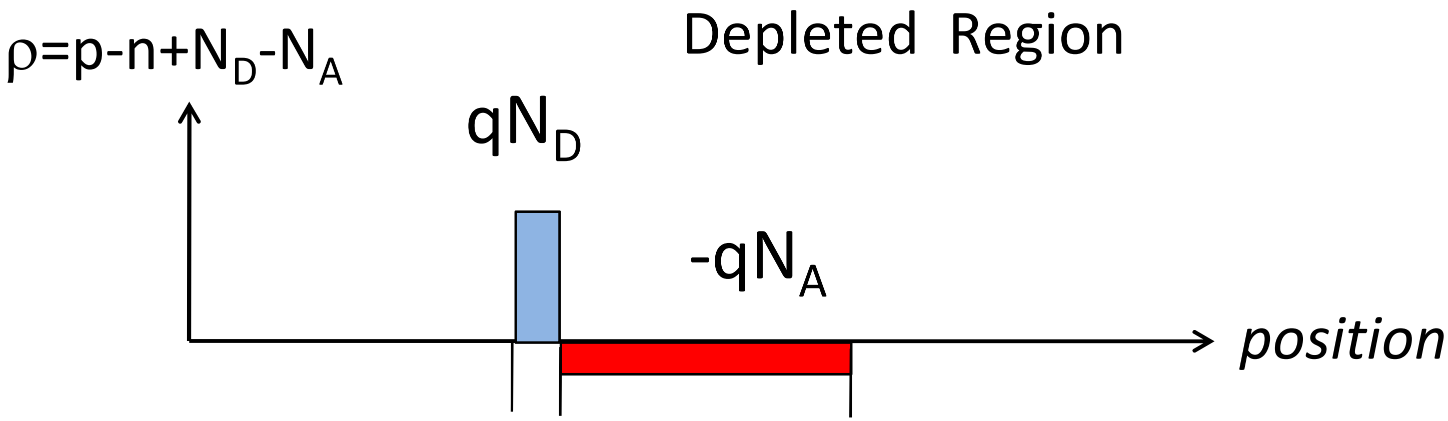
$$\mathcal{E}(0^-) = \frac{\kappa_2}{\kappa_1} \mathcal{E}(0^+)$$

Displacement is continuous across the interface, field need not be ..

Potential, Field, Charge



Poisson Equation



$$\nabla \cdot D = q\rho$$

In 1D, $$\kappa_S \varepsilon_0 \frac{d\mathcal{E}}{dx} = q\rho(x) \equiv q\left(p - n + N_D^+ - N_A^-\right)$$

$$\kappa_S \varepsilon_0 \frac{d^2V}{dx^2} = -q\left(p - n + N_D^+ - N_A^-\right)$$

conclusions

- Energy band diagrams (as a function of position) help us calculate many key quantities of importance, such as electrostatic potential, electric field, and carrier concentration
- They are based on the Poisson equation, $\nabla \cdot D = q\rho$
- Can be combined with the depletion approximation to calculate the behavior at the boundary of two materials with different doping in equilibrium
- They will be crucial for the remainder of the class; we'll come back to them many times