ECE 305: Spring 2018

Band Structures + Quasi-Fermi Levels

Pierret, *Semiconductor Device Fundamentals* (SDF)
Chapter 3 (pp. 75-104)

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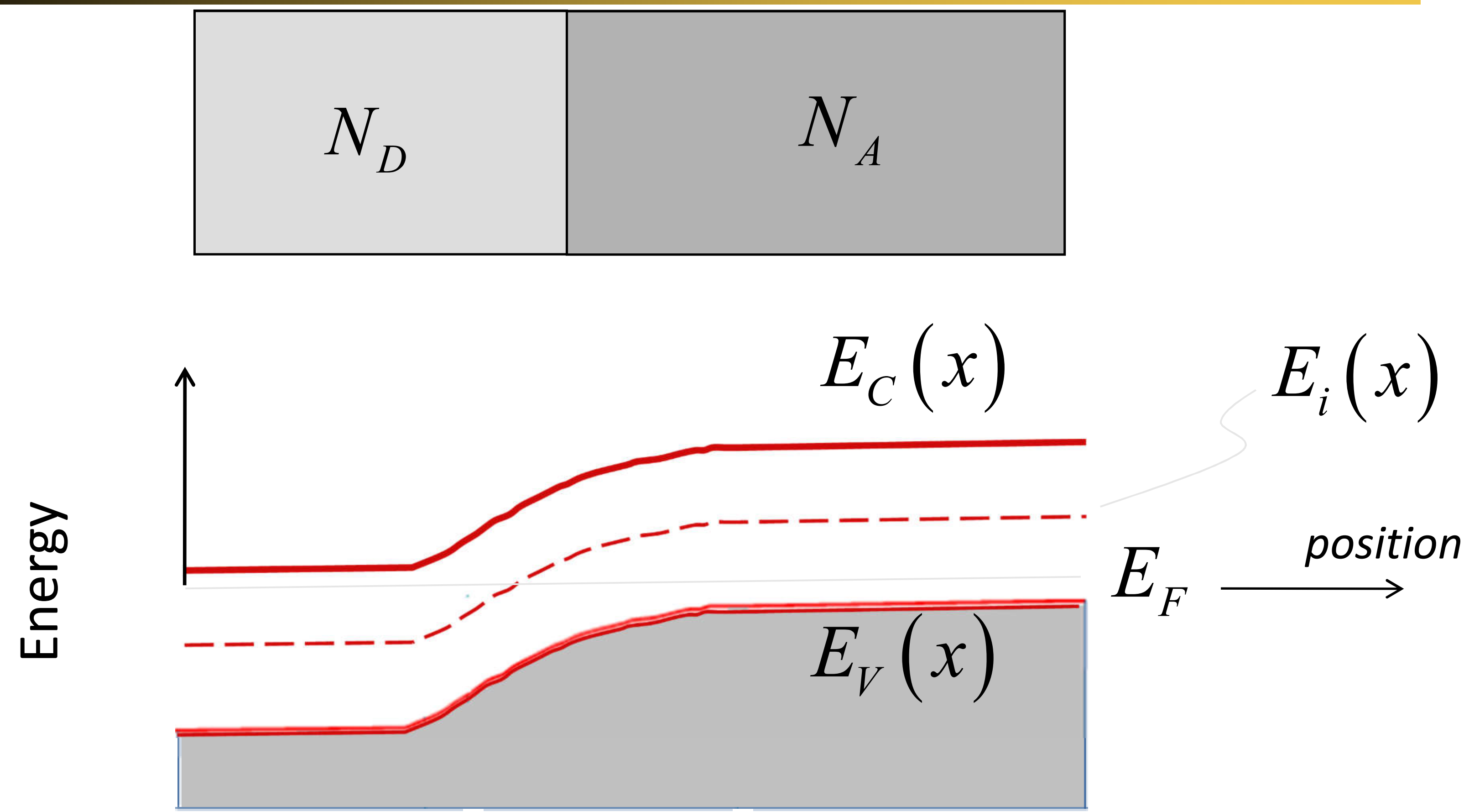
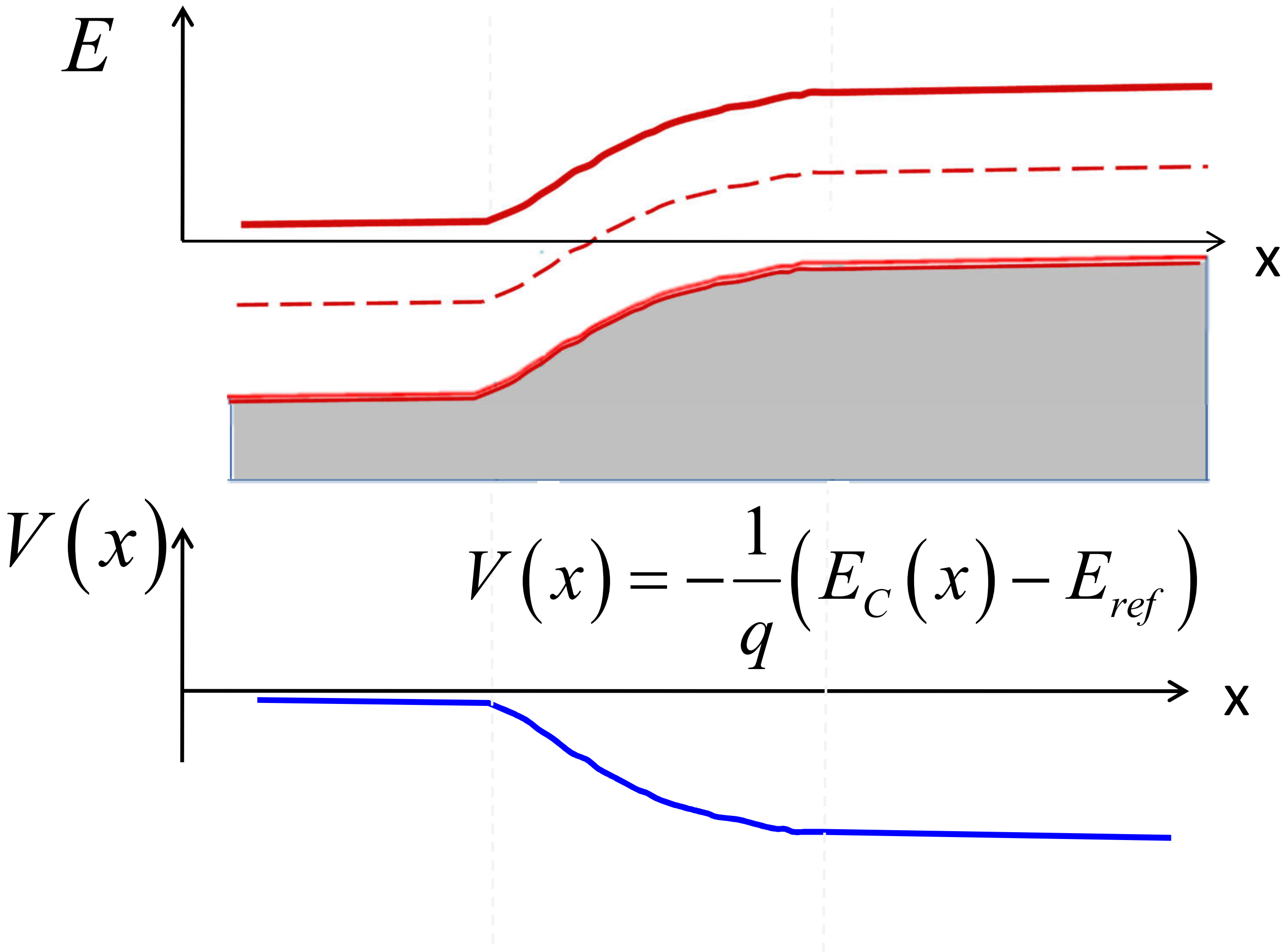
1. Junction formation
2. Poisson's equation
3. The semiconductor equations
4. Generation and recombination
5. Quasi-Fermi levels

Formation of a Junction



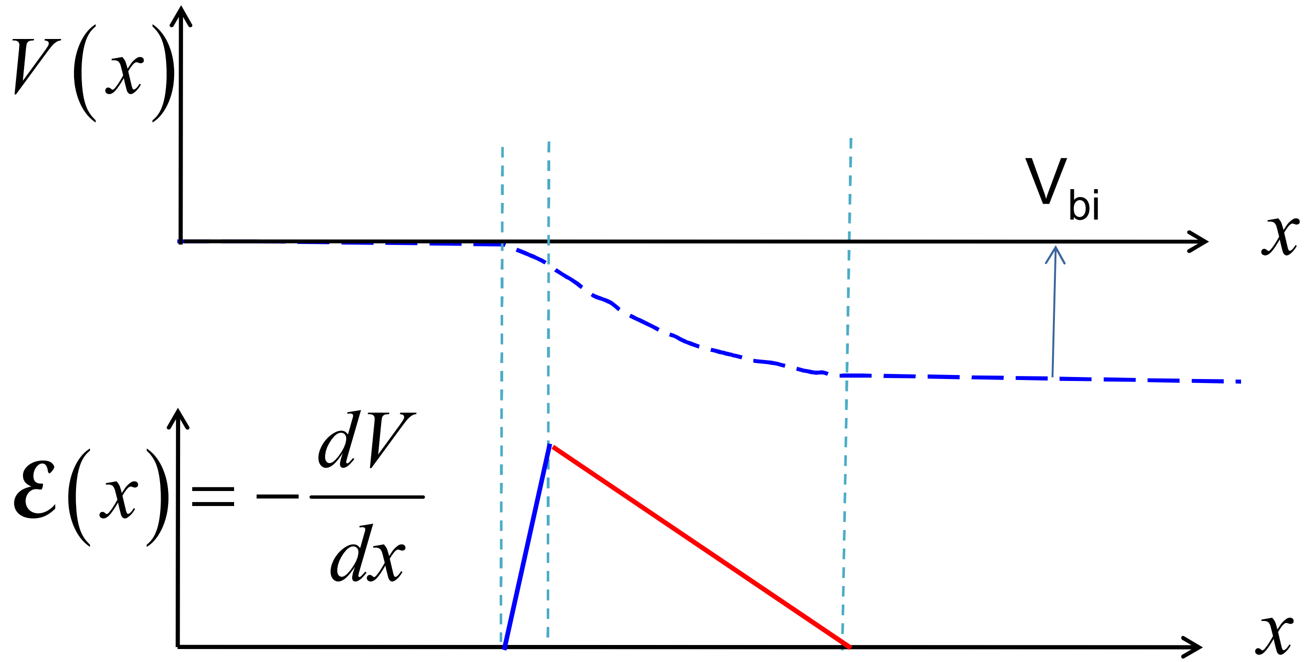
Band-diagram to Potential



$$V(x) = -\frac{1}{q}\left(E_C(x) - E_{ref}\right)$$

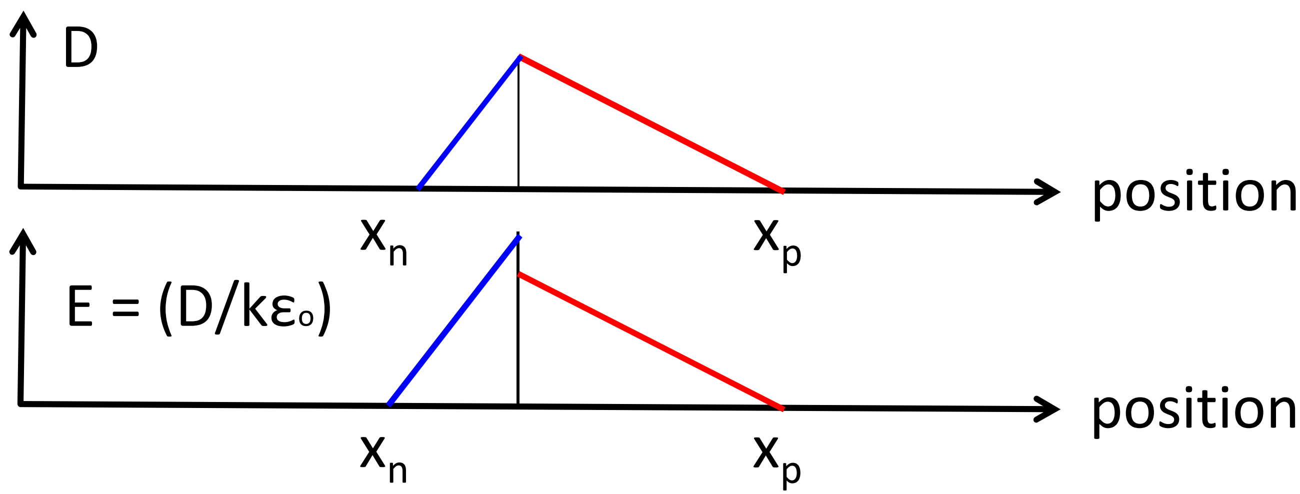
Joules vs. eV

Potential, Electric Field, Band diagram



$$\mathcal{E}(x) = -\frac{dV}{dx} = \frac{1}{q}\frac{dE_c}{dx} = \frac{1}{q}\frac{dE_\upsilon}{dx} = \frac{1}{q}\frac{dE_i}{dx}$$

Interface Boundary Conditions

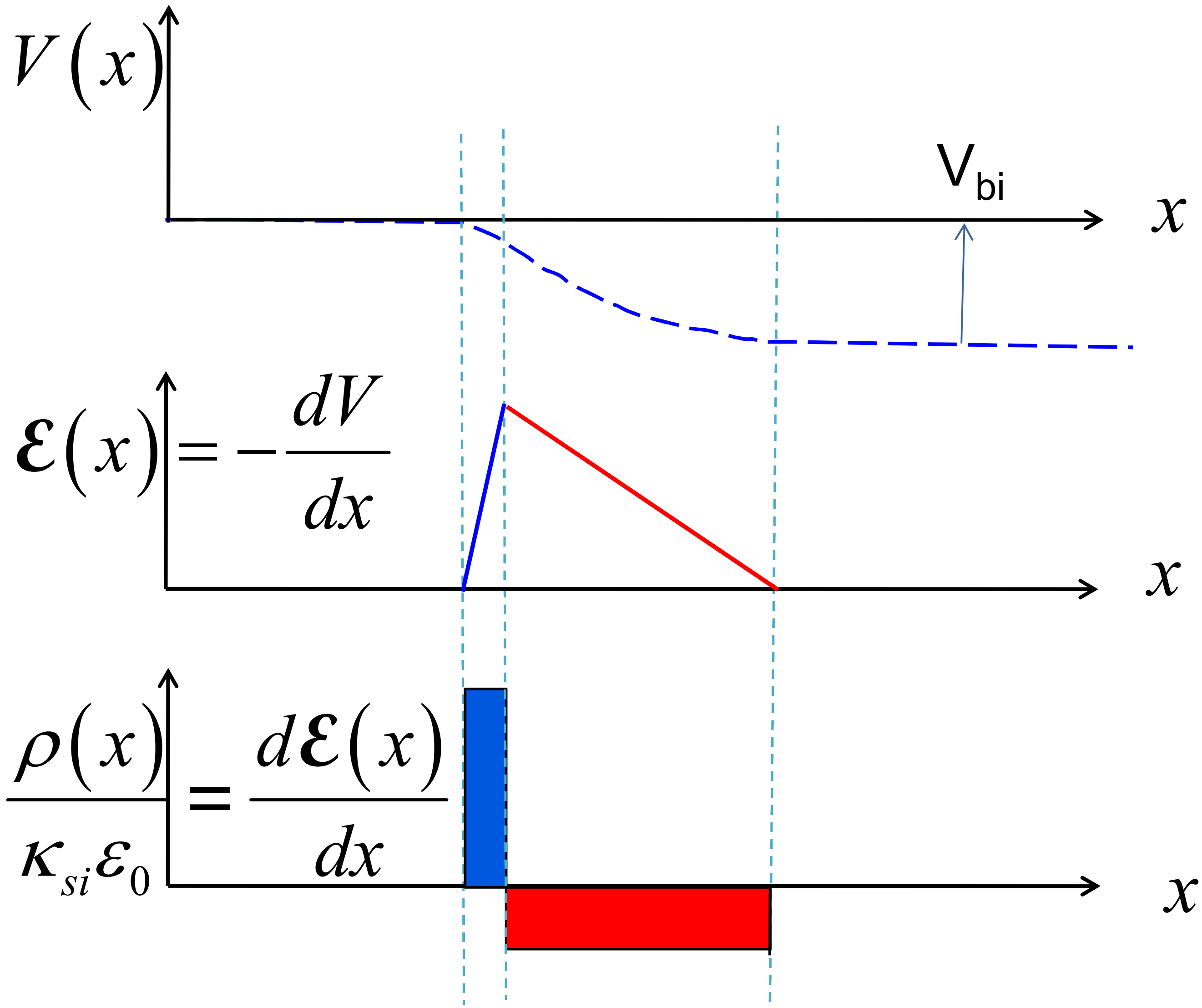


$$D_1 = \kappa_1 \varepsilon_0 \mathcal{E}(0^-) = \kappa_2 \varepsilon_0 \mathcal{E}(0^-) = D_2$$

$$\mathcal{E}(0^-) = \frac{\kappa_2}{\kappa_1} \mathcal{E}(0^+)$$

Displacement is continuous across the interface, field need not be ..

Potential, Field, Charge



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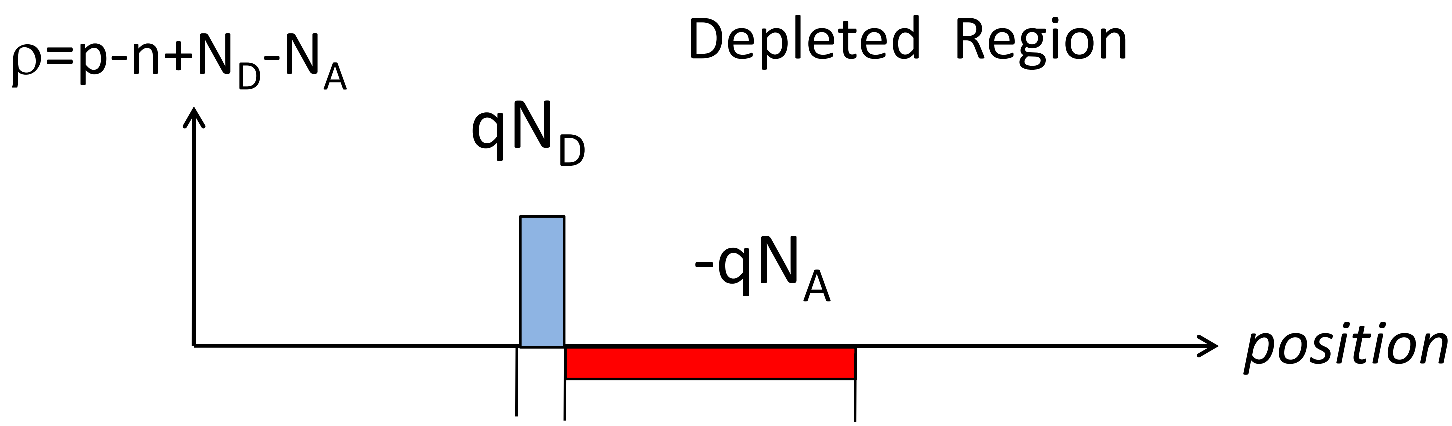
Poisson Equation



$$\nabla \cdot D = q\rho$$

In 1D, $$\kappa_S \varepsilon_0 \frac{d\mathcal{E}}{dx} = q\rho(x) \equiv q\left(p - n + N_D^+ - N_A^-\right)$$

$$\kappa_S \varepsilon_0 \frac{d^2V}{dx^2} = -q\left(p - n + N_D^+ - N_A^-\right)$$

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recap: drift- diffusion equations

$$\vec{J}_p = pq\mu_p\vec{\mathcal{E}} - qD_p\vec{\nabla}p$$

current = drift current + diffusion current

$$\vec{J}_n = nq\mu_n\vec{\mathcal{E}} + qD_n\vec{\nabla}n$$

total current = electron current + hole current

$$\vec{J} = \vec{J}_p + \vec{J}_n$$

$$D_p/\mu_p = D_n/\mu_n = k_BT/q$$

the semiconductor equations

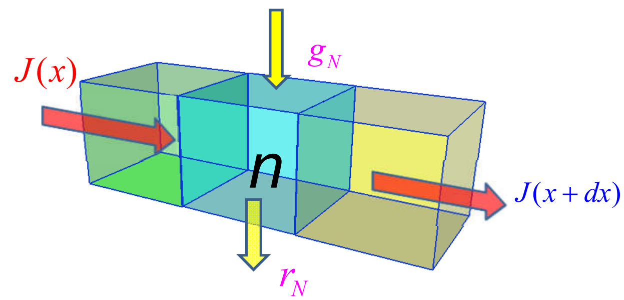
$$\frac{\partial p}{\partial t} = -\nabla \bullet \left(\frac{\vec{J}_p}{q} \right) + G_p - R_p$$

$$\frac{\partial n}{\partial t} = -\nabla \bullet \left(\frac{\vec{J}_n}{-q} \right) + G_n - R_n$$

$$0 = -\nabla \bullet \left(\varepsilon \vec{\mathcal{E}} \right) + \rho$$

Three equations in three unknowns:

$$p(\vec{r}), n(\vec{r}), V(\vec{r})$$



$$\vec{J}_p = pq\mu_p \vec{\mathcal{E}} - qD_p \vec{\nabla} p$$

$$\vec{J}_n = nq\mu_n \vec{\mathcal{E}} + qD_n \vec{\nabla} n$$

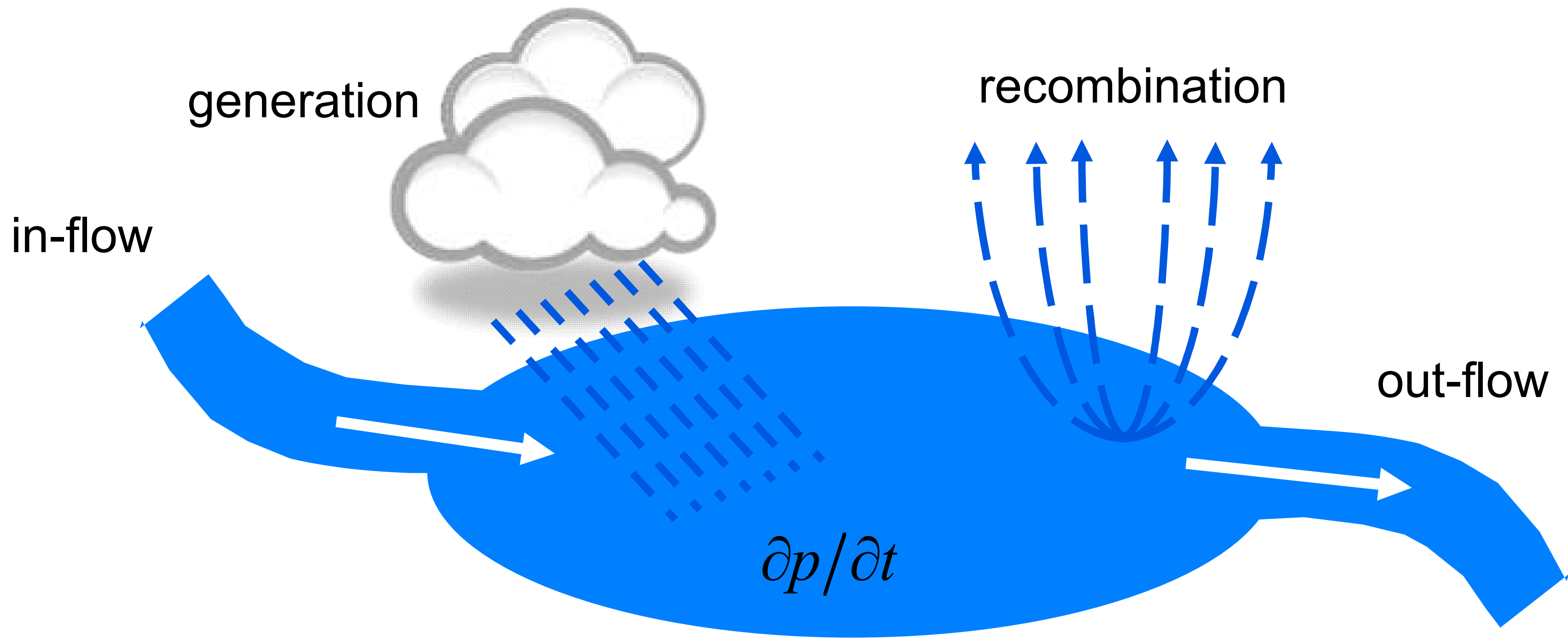
$$\rho = q\left(p - n + N_D^+ - N_A^- \right)$$

$$\vec{\mathcal{E}}\left(\vec{r}\right) = \nabla V\left(\vec{r}\right)$$

continuity equation for holes

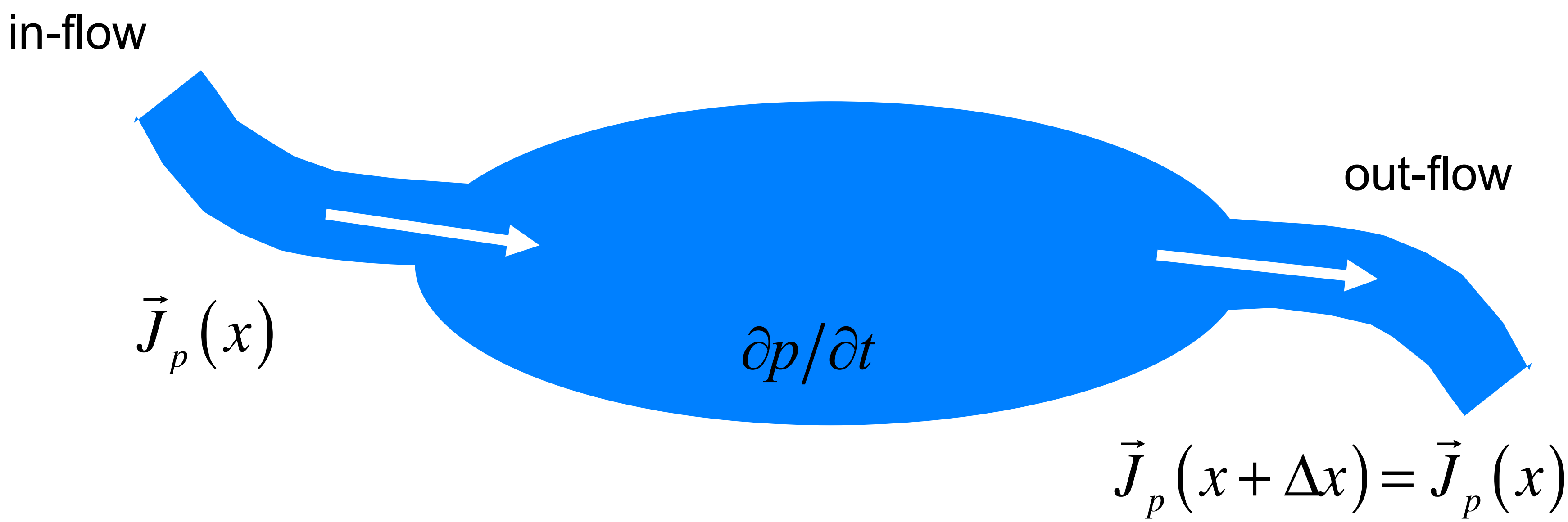
$$\frac{\partial p}{\partial t} = \text{in-flow - out-flow} + G - R$$

$$\frac{\partial p}{\partial t} = -\nabla \bullet \frac{\vec{J}_p}{q} + G_p - R_p$$



equilibrium (no G-R)

$$\frac{\partial p}{\partial t} = -\nabla \bullet \frac{\vec{J}_p}{q} + G_p - R_p \rightarrow \nabla \bullet \vec{J}_p = 0 \rightarrow \vec{J}_p \text{ is constant}$$



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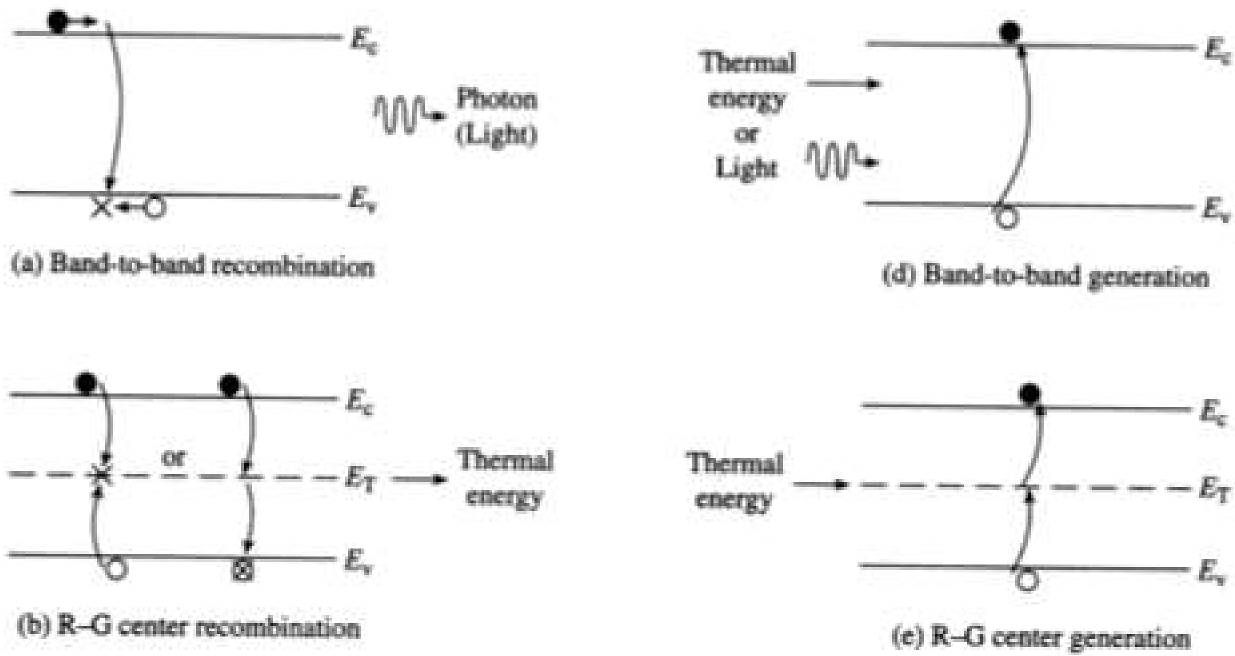
generation & recombination processes



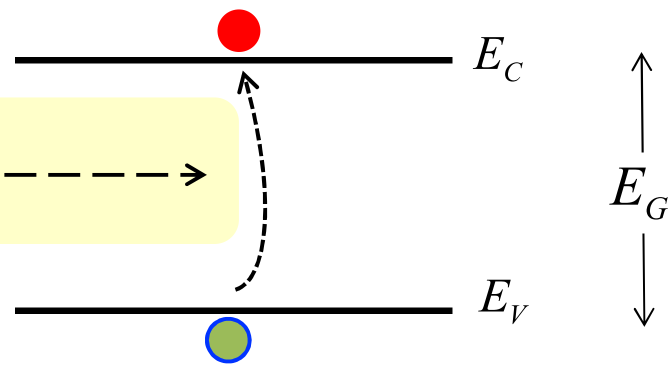
Shockley-Read-Hall (SRH)

Fig. 3.15a from R.F. Pierret, *Semiconductor Device Fundamentals*

optical generation

$$\lambda < \frac{hc}{E_G}$$

$$E = hf > E_G$$



photovoltaic effect

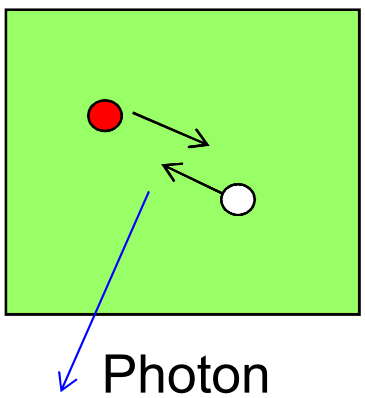
$$f\lambda = c \qquad f = \frac{c}{\lambda} \qquad E = hf = \frac{hc}{\lambda}$$

$$\frac{\partial n}{\partial t} = G_L$$

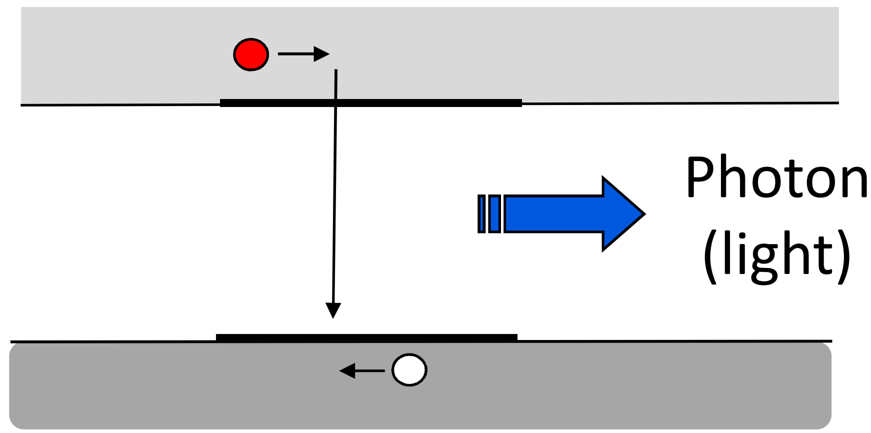
G_L is the optical generation rate (units: $cm^{-3}\ s^{-1}$)

Direct Band-to-band Recombination

In real space …



In energy space …

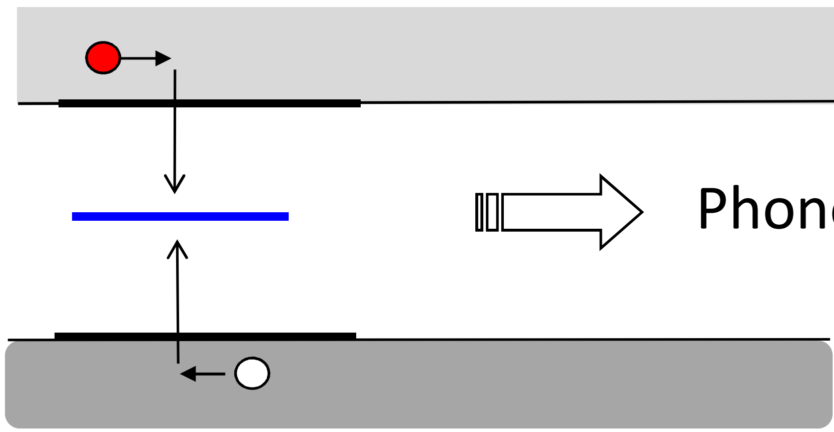


GaAs, InP, InSb (3D)

Lasers, LEDs, etc.

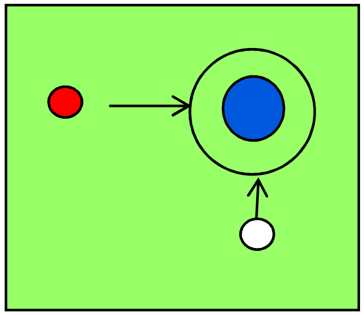
$$R_p = \left.\frac{\partial p}{\partial t}\right|_{R-G} = -\frac{\Delta p}{\tau_p}$$

Indirect Recombination (Trap-assisted)



Phonon (Thermal Energy)

Ge, Si, ….

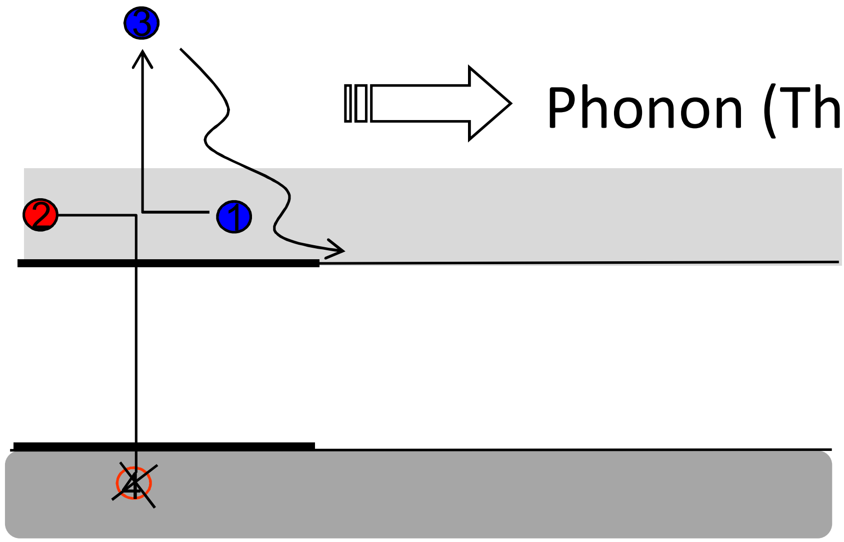


Transistors, Solar cells, etc.

Again, $R_p = \left.\frac{\partial p}{\partial t}\right|_{R-G} = -\frac{\Delta p}{\tau_p}$

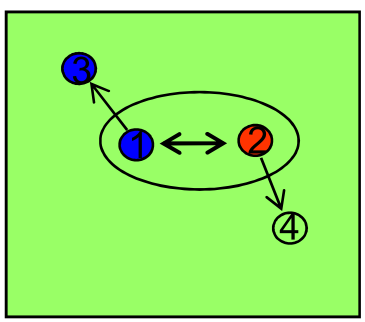
But τ_p is usually larger

Auger Recombination



Phonon (Thermal Energy)

$$R_p = \frac{\partial p}{\partial t} = \quad Bp^2 n$$



InP, GaAs, …

Lasers, etc.

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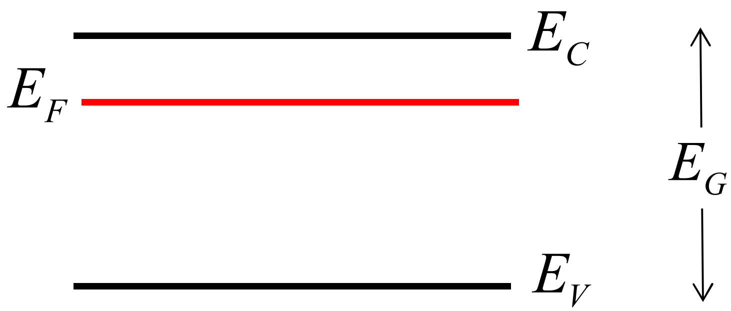
example: N-type sample

before we turn on the light: equilibrium



$$N_D = 10^{17} \text{ cm}^{-3}$$

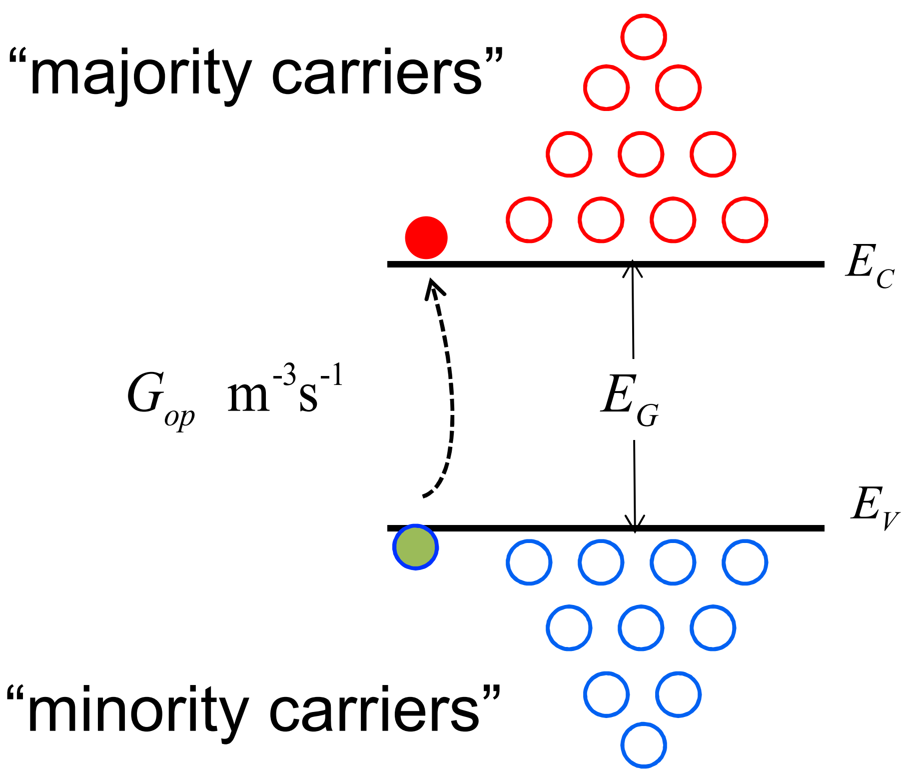
$$n_0 = 10^{17} \text{ cm}^{-3}$$

$$n_0 p_0 = n_i^2$$

$$p_0 = \frac{n_i^2}{n_0} = 10^3 \text{ cm}^{-3}$$

turn the light on: “excess carriers”

$$\Delta n = 10^{10} \ \ \text{cm}^{-3}$$



$$\Delta p = 10^{10} \ \ \text{cm}^{-3}$$

$$N_D = 10^{17} \ \ \text{cm}^{-3}$$

$$n_0 = 10^{17} \ \ \text{cm}^{-3}$$

$$n_0 p_0 = n_i^2$$

$$p_0 = \frac{n_i^2}{n_0} = 10^3 \ \ \text{cm}^{-3}$$

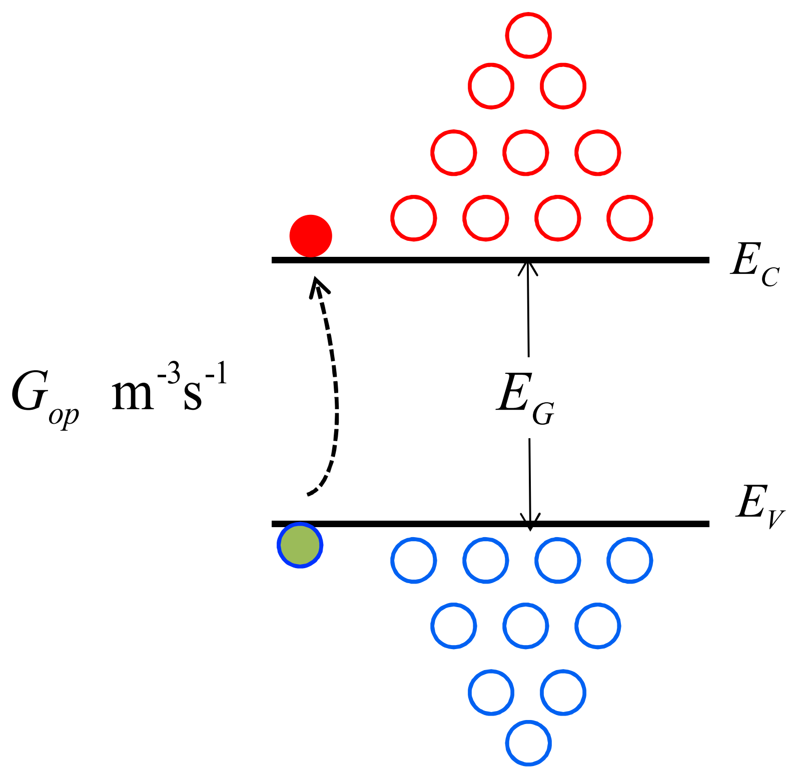
$$p = \Delta p \approx 10^{10} \ \ \text{cm}^{-3}$$

$$n = n_0 \approx 10^{17} \ \ \text{cm}^{-3}$$

“Low-level injection”

the np product

$$\Delta n = 10^{10} \ \ \text{cm}^{-3}$$



$$\Delta p = 10^{10} \ \ \text{cm}^{-3}$$

$$p = \Delta p \approx 10^{10} \ \ \text{cm}^{-3}$$

$$n = n_0 \approx 10^{17} \ \ \text{cm}^{-3}$$

"Low-level injection"

$$np = 10^{27} \ \ \text{cm}^{-3}$$

$$n_0 p_0 = n_i^2$$

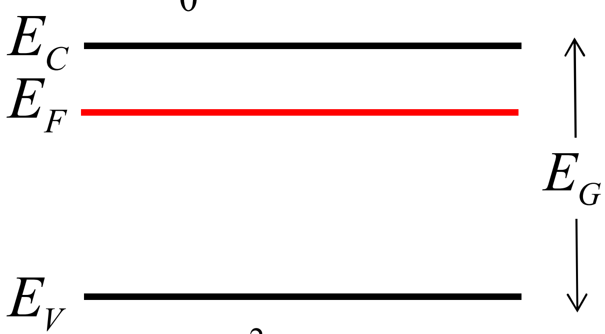
$$np \neq n_i^2$$

away from equilibrium

where is the Fermi level?

Before we created the excess holes

$$n_0 = 10^{17} \ \text{cm}^{-3}$$

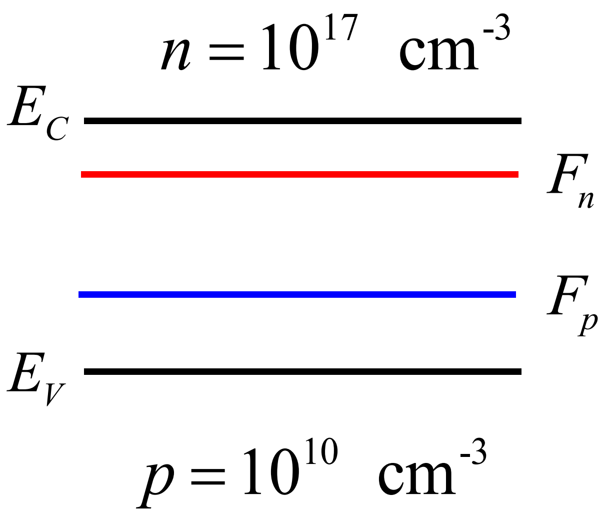


$$p_0 = \frac{n_i^2}{n_0} = 10^3 \ \text{cm}^{-3}$$

$$n_0 = n_i e^{(E_F - E_i)/k_B T}$$

$$p_0 = n_i e^{(E_i - E_F)/k_B T}$$

quasi-Fermi levels



$$n_0 = n_i e^{(E_F - E_i)/k_B T}$$

$$p_0 = n_i e^{(E_i - E_F)/k_B T}$$

$$n = n_i e^{(F_n - E_i)/k_B T}$$

$$F_n = E_F$$

$$p = n_i e^{(E_i - F_p)/k_B T}$$

$$F_p < E_F$$

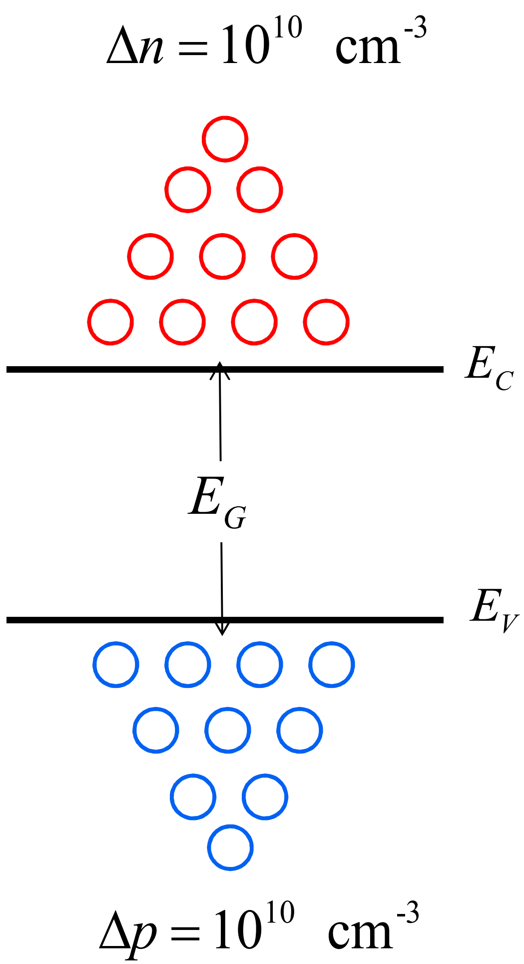
The QFL's are split

in equilibrium: $F_n = F_p = E_F$

equilibrium vs. non-equilibrium

equilibrium	non-equilibrium
$n_0 = n_i e^{(E_F - E_i)/k_B T}$	$n = n_i e^{(F_n - E_i)/k_B T}$
$p_0 = n_i e^{(E_i - E_F)/k_B T}$	$p = n_i e^{(E_i - F_p)/k_B T}$
$n_0 p_0 = n_i^2$	$np \neq n_i^2$
$f_0 = \frac{1}{1 + e^{(E - E_F)/k_B T}}$	$f_c = \frac{1}{1 + e^{(E - F_n)/k_B T}}$
	$1 - f_v = 1 - \frac{1}{1 + e^{(E - F_p)/k_B T}}$

turn the light off



Question: What happens?

Answer: The system returns to equilibrium.

How long does it take?
A time known as the "minority carrier lifetime". τ_p sec

$$R_p = \left.\frac{\partial p}{\partial t}\right|_{R-G} = -\frac{\Delta p}{\tau_p}$$

(low-level injection)

current and QFL's

$$\vec{J}_p = pq\mu_p\vec{\mathcal{E}} - qD_p\vec{\nabla}p = p\mu_p\vec{\nabla}\left(F_p/q\right)$$

$$\vec{J}_n = nq\mu_n\vec{\mathcal{E}} + qD_n\vec{\nabla}n = n\mu_n\vec{\nabla}\left(F_n/q\right)$$

$$p = n_i e^{\left(E_i - F_p\right)/k_BT}$$

$$n = n_i e^{\left(F_n - E_i\right)/k_BT}$$

$$\frac{dp}{dx} = n_i e^{\left(E_i - F_p\right)/k_BT} \times \frac{1}{k_BT}\left(\frac{dE_i}{dx} - \frac{dF_p}{dx}\right) = \frac{p}{k_BT}\left(\frac{dE_i}{dx} - \frac{dF_p}{dx}\right)$$

$$\frac{dE_i}{dx} = q\mathcal{E}_x$$

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conclusions

- Energy band diagrams (as a function of position) help us calculate many key quantities of importance, such as electrostatic potential, electric field, and carrier concentration
- They are based on the Poisson equation, $\nabla \cdot D = q\rho$
- Can be combined with the depletion approximation to calculate the behavior at the boundary of two materials with different doping in equilibrium
- They will be crucial for the remainder of the class; we'll come back to them many times