ECE 305: Spring 2018

Semiconductor Equations

Pierret, *Semiconductor Device Fundamentals* (SDF)
Chapter 3 (pp. 122-138)

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outline

✓ 1. Semiconductor Equation Overview

✓ 2. MCDE Examples

✓ 3. Solving Poisson's Equation

the semiconductor equations

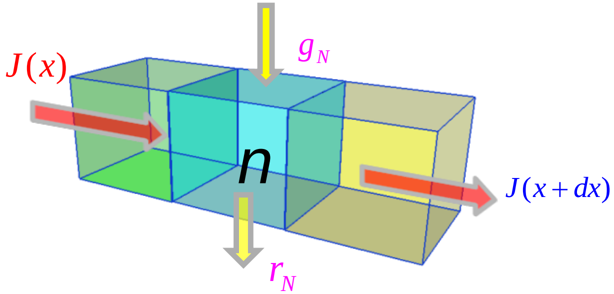
$$\frac{\partial p}{\partial t} = -\nabla \bullet \left(\frac{\vec{J}_p}{q} \right) + G_p - R_p$$

$$\frac{\partial n}{\partial t} = -\nabla \bullet \left(\frac{\vec{J}_n}{-q} \right) + G_n - R_n$$

$$0 = -\nabla \bullet \left(\varepsilon \vec{\mathcal{E}} \right) + \rho$$

Five equations in five unknowns:

$$p(\vec{r}), n(\vec{r}), V(\vec{r}), \vec{J}_p \quad \vec{J}_n$$



$$\vec{J}_p = pq\mu_p \vec{\mathcal{E}} - qD_p \vec{\nabla} p$$

$$r = q\left(p - n + N_D^+ - N_A^- \right)$$

$$\vec{J}_n = nq\mu_n \vec{\mathcal{E}} + qD_n \vec{\nabla} n$$

$$\vec{\mathcal{E}}(\vec{r}) = \nabla V(\vec{r})$$

Semiconductor equations: 2 key cases

- Diffusion problems ($\mathcal{E} = 0$): MCDE

$$\frac{\partial \Delta p}{\partial t} = D_p \frac{d^2 \Delta p}{dx^2} - \frac{\Delta p}{\tau_p} + G_L$$

- Drift problems ($\mathcal{E} \neq 0$): Drift current equations

$$\vec{J}_p = pq\mu_p \vec{\mathcal{E}} - qD_p \vec{\nabla} p = p\mu_p \vec{\nabla}\left(F_p / q\right)$$

$$\vec{J}_n = nq\mu_n \vec{\mathcal{E}} + qD_n \vec{\nabla} n = n\mu_n \vec{\nabla}\left(F_n / q\right)$$

outline

✓ 1. Semiconductor Equation Overview

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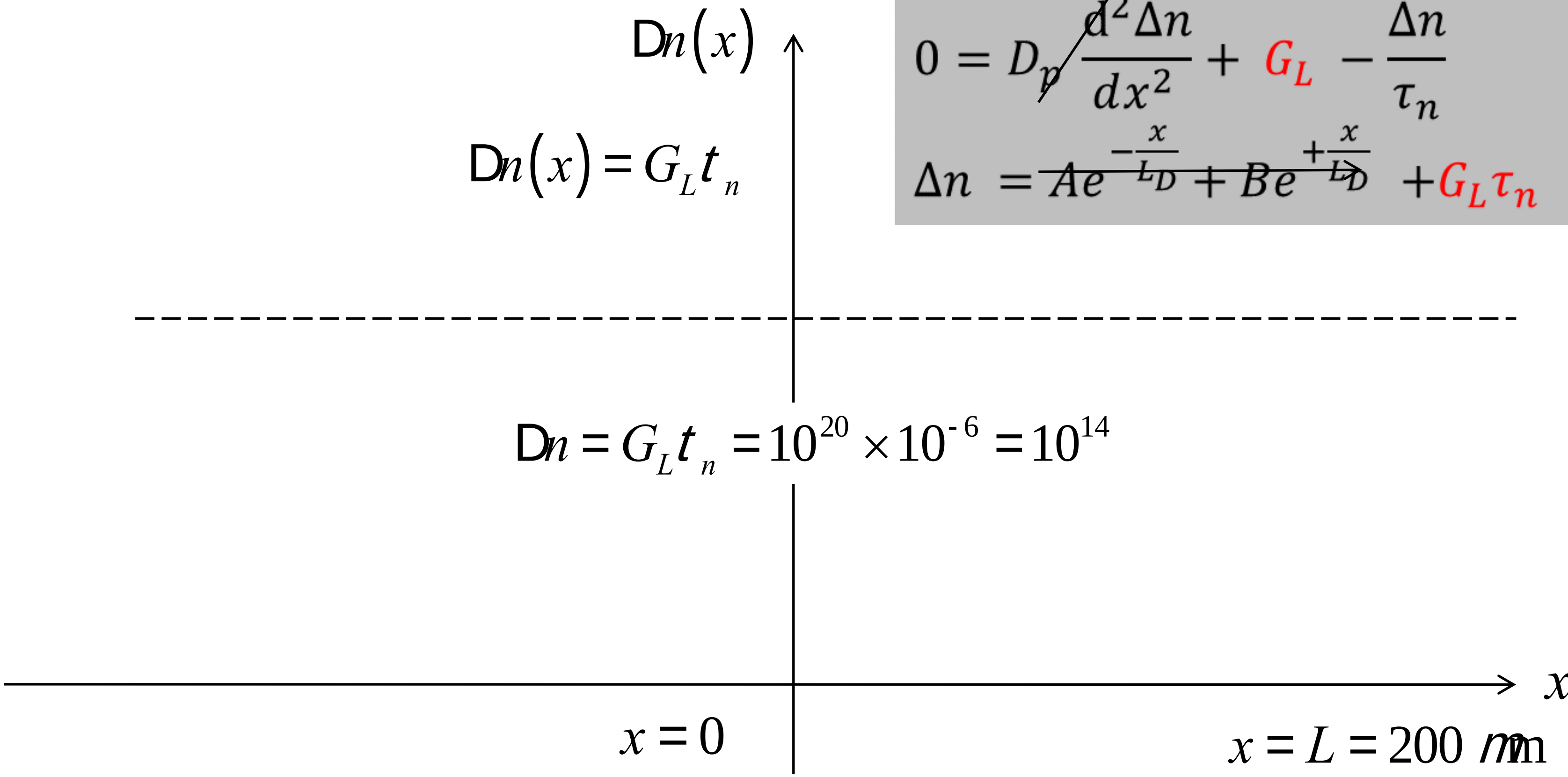
Example #1: Solution

$$0 = D_p \frac{d^2 \Delta n}{dx^2} + G_L - \frac{\Delta n}{\tau_n}$$

$$\Delta n = Ae^{-\frac{x}{L_D}} + Be^{+\frac{x}{L_D}} + G_L \tau_n$$



Steady-state, uniform generation, no spatial variation

Example #2 Summary

Analytical solutions

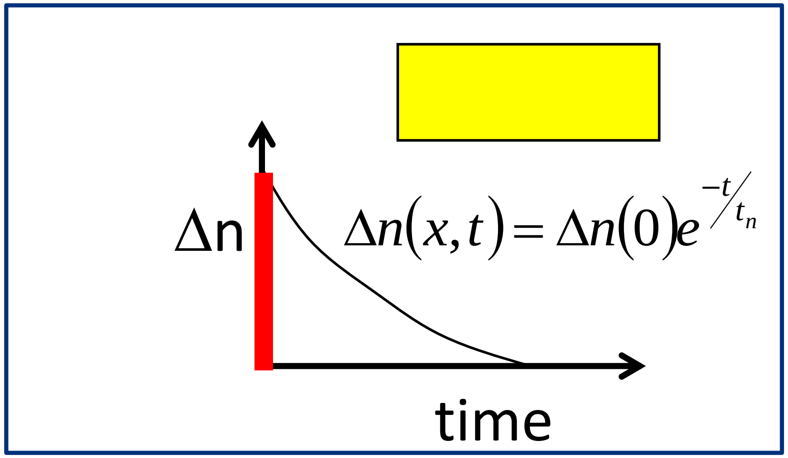
$$\frac{\partial n}{\partial t} = \frac{1}{q}\nabla \bullet \mathbf{J}_N - r_N + g_N$$

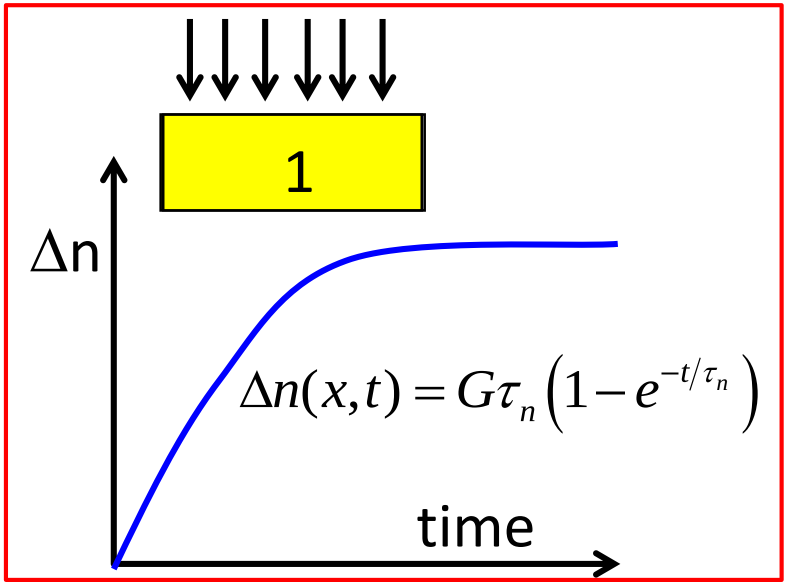
$$\mathbf{J}_N = qn\mu_N \mathcal{E} + qD_N \nabla n$$

$$\frac{\partial p}{\partial t} = \frac{-1}{q}\nabla \bullet \mathbf{J}_P - r_P + g_P$$

$$\mathbf{J}_P = qp\mu_P \mathcal{E} - qD_P \nabla p$$

$$\nabla \bullet D = q\left(p - n + N_D^+ - N_A^-\right)$$



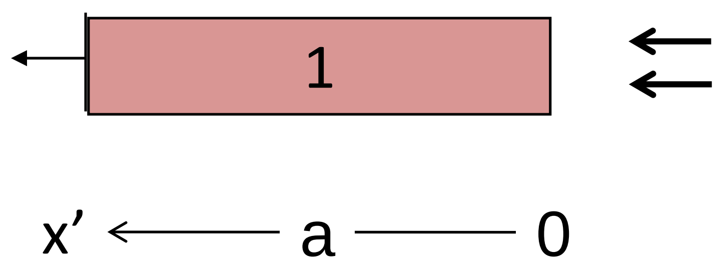


Example #3: One-sided Carrier Diffusion

Steady state, no generation/recombination

$\Delta n(x',t) = 0$

Metal contact



$$\cancel{\frac{\partial n}{\partial t}} = \frac{1}{q}\frac{dJ_n}{dx} - \cancel{r_N} + \cancel{g_N}$$

$$\mathbf{J}_N = qn\cancel{\mu_N E} + qD_N \frac{dn}{dx}$$

$$0 = D_N \frac{d^2 n}{dx^2}$$

Example 3B: One-sided carrier diffusion Long sample (steady state)

Steady-state, sample **long** compared to the diffusion length.
i.e., a short diffusion length

$\Delta p(x=0)$ fixed

1) Simplify the MCDE
2) Solve the MCDE
3) Deduce F_p from Δp

$$\frac{\partial \Delta p}{\partial t} = D_p \frac{d^2 \Delta p}{dx^2} - \frac{\Delta p}{\tau_p} + G_L$$

Example 3B: One-sided carrier diffusion Long sample (steady state)

Steady-state, sample **long** compared to the diffusion length. No generation. $\Delta n(x=0)=10^{12}\ \mathrm{cm}^{-3}$ fixed

Step 3) What type of problems are we talking about?

Step 4B) Key words: steady-state, without light, long device

ii) If I write $\frac{\partial \Delta p}{\partial t} = D_p \frac{d^2 \Delta p}{dx^2} - \frac{\Delta p}{\tau_p} + G_L$ for MCDE, would I be right?

iii) Which approximate equation should I choose:

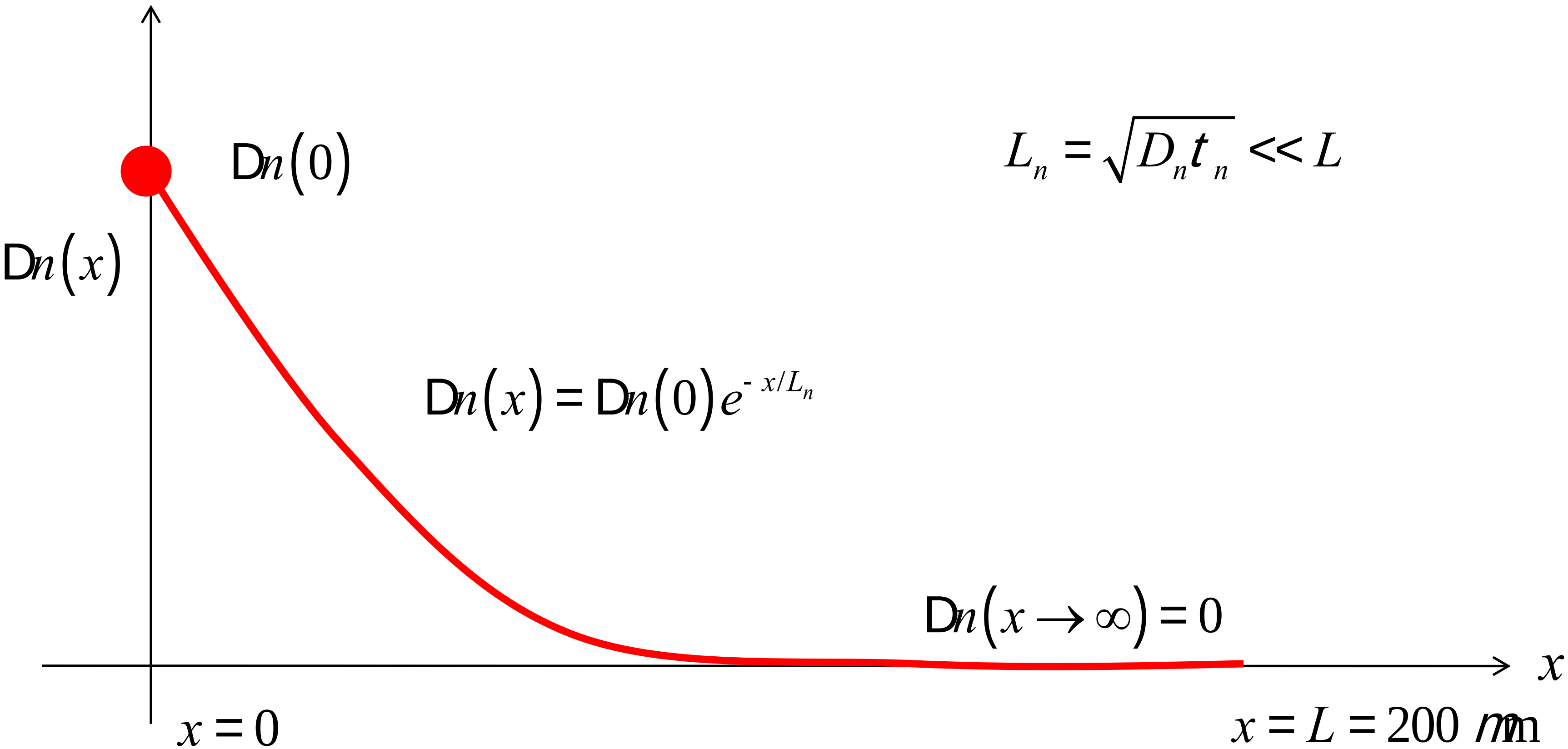
$$\frac{\partial \Delta n}{\partial t} = G_L - \frac{\Delta n}{\tau_p},$$

solution $\Delta n = G_L \tau_n + B\, e^{-\frac{t}{\tau_p}}$

$$0 = D_p \frac{d^2 \Delta n}{dx^2} + G_L - \frac{\Delta p}{\tau_p}$$

solution $\Delta p = A e^{-\frac{x}{L_D}} + B e^{+\frac{x}{L_D}} + G_L \tau_n$

Example 3B: One-sided carrier diffusion Long sample (steady state)



Steady-state, sample **long** compared to the diffusion length.

Example 3B: One-sided carrier diffusion Long sample (steady state)

Steady-state, sample is **5 micron long**. No generation.

$\Delta n(x=0) = 10^{12}\ \mathrm{cm}^{-3}$

$\Delta n(x = 5\ \mu\mathrm{m}) = 0$

Which approximate equation should I choose:

$$\frac{\partial \Delta n}{\partial t} = G_L - \frac{\Delta n}{\tau_p}, \qquad \textbf{solution} \quad \Delta n = G_L \tau_n + B\, e^{-\frac{t}{\tau_p}}$$

$$0 = D_p \frac{d^2 \Delta n}{dx^2} + G_L - \frac{\Delta p}{\tau_p} \qquad \textbf{solution} \quad \Delta p = Ae^{-\frac{x}{L_D}} + Be^{+\frac{x}{L_D}} + \text{~~}G_L\tau_n\text{~~}$$

$$\text{or} \quad \Delta p = A + Bx + \text{~~}G_L\tau_n\text{~~} \qquad L \ll L_D$$

What are my boundary conditions?

Example 3B: One-sided carrier diffusion Long sample (steady state)

$$\Delta n(x=0) = 10^{12}\ \text{cm}^{-3}$$

$$\Delta n(x >> L_D) = 0$$

$$\Delta n(x) = Ax + B$$

$$\Delta n(x=0) = 10^{12}\ \text{cm}^{-3}$$

$$\Delta n(x) = \Delta n(0)\left(1 - \frac{x}{L}\right)$$

$$\Delta n(x=L) = 0$$

Example 3C: One-sided carrier diffusion Intermediate sample (steady state)

Steady-state, sample is **30 micrometers long**. No generation.

$$\Delta n(x=0) = 10^{12}\ \text{cm}^{-3}$$ fixed

$$\Delta n(x = 30\ \mu\text{m}) = 0$$

$$\frac{\partial \Delta n}{\partial t} = D_p \frac{d^2 \Delta n}{dx^2} - \frac{\Delta}{\tau_n} + G_L$$

$$0 = D_p \frac{d^2 \Delta n}{dx^2} - \frac{\Delta}{\tau_n} + 0$$

$$\frac{d^2 \Delta n}{dx^2} - \frac{\Delta n}{L_n} = 0 \qquad L_n \equiv \sqrt{D_n \tau_n}$$

1) Simplify the MCDE
2) Solve the MCDE
3) Deduce F_p from *Δp*

$$L_n = 28\ \mu\text{m} \qquad L = 30\ \mu\text{m}$$

Example 3C: One-sided carrier diffusion Intermediate sample (steady state)

Steady-state, sample is **30 micrometers long**. No generation.

$$\Delta n(x=0) = 10^{12}\ \text{cm}^{-3}$$ fixed

$$\Delta n(x = 30\ \mu\text{m}) = 0$$

$$\frac{d^2 \Delta n}{dx^2} - \frac{\Delta n}{L_n} = 0$$

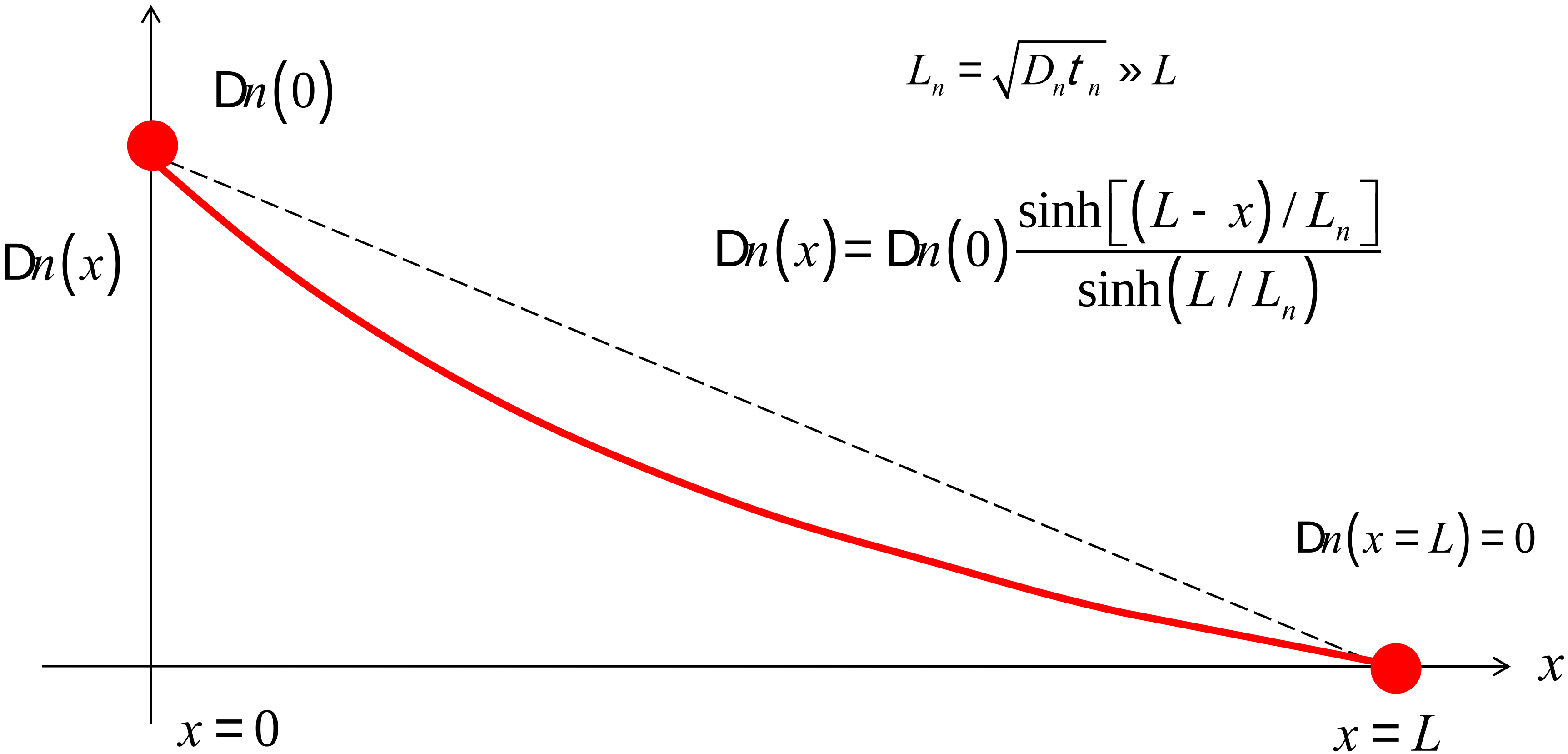
$$\Delta n(x) = Ae^{-x/L_n} + Be^{+x/L_n}$$

$$\Delta n(0) = A + B = 10^{12}$$

$$\Delta n(L) = Ae^{-L/L_n} + Be^{+L/L_n} = 0$$

1) Simplify the MCDE
2) Solve the MCDE
3) Deduce F_p from Δp

Example 3C: One-sided carrier diffusion Intermediate sample (steady state)



$$L_n = \sqrt{D_n \tau_n} \approx L$$

$$\Delta n(x) = \Delta n(0) \frac{\sinh\left[(L - x)/L_n\right]}{\sinh(L/L_n)}$$

Steady-state, sample neither long nor short compared to the diffusion length.

Example #3 summary: One-sided carrier diffusion

Length scale	Solution type
Long ($L \gg L_D$)	Decaying exponentials
Short ($L \ll L_D$)	Linear
Intermediate ($L \sim L_D$)	Hyperbolic functions

Notes:

L is length of region where MCDE applies

$L_D = \sqrt{D_{n,p}\tau_{n,p}}$ is the diffusion length for carriers

Homework #5: Fall 2017 (cf. this week's homework!)

1. Assume that a n-type region of crystalline silicon with $\mu_p = 450\ \mathrm{cm}^2/\mathrm{V}\cdot\mathrm{s}$ and lifetime $\tau_p = 10$ ms is uniformly illuminated by a photon flux $G_L = 10^{20}$ /cm³·s, reaches a steady state, which is then switched off at $t = 0$.
   a. Write down the simplest form of the minority carrier diffusion equation that accurately describes its behavior. Briefly justify your answer.
   b. Sketch the time-dependent decay of the carrier concentration.
   c. What is the value of the carrier concentration at $t = 20$ ms?

Homework #5: Fall 2017

a. Write down the simplest form of the minority carrier diffusion equation that accurately describes its behavior. Briefly justify your answer.

Homework #5: Fall 2017

b. Sketch the time-dependent decay of the carrier concentration.

Homework #5: Fall 2017

c. What is the value of the carrier concentration at $t = 20$ ms?

Homework #5: Fall 2017

2. Assume that a uniform p-type region of gallium arsenide ($T = 300$ K) has length $L = 1$ μm, lifetime $\tau_n = 5$ ns and $\mu_n = 8500\ \mathrm{cm^2/V\cdot s}$. Assume that all carriers are extracted at $x = L$ (such that $\Delta n = 0$), while $\Delta n = 10^{13}$ /cm³ at $x = 0$. Assume that $p_o \gg \Delta n$ everywhere. Now consider the system after reaching a steady state.
   a. Write down the simplest form of the minority carrier diffusion equation that accurately describes its behavior. Briefly justify your answer.
   b. Solve for and sketch the minority carrier concentration Δn as a function of position between $x = 0$ and $x = L$.
   c. How would this problem change if the minority carrier lifetime τ_n were reduced to 1 ns? Justify your answer quantitatively.

Homework #5: Fall 2017

a. Write down the simplest form of the minority carrier diffusion equation that accurately describes its behavior. Briefly justify your answer.

Homework #5: Fall 2017

b. Solve for and sketch the minority carrier concentration Δn as a function of position between $x = 0$ and $x = L$.

Homework #5: Fall 2017

c. How would this problem change if the minority carrier lifetime τ_n were reduced to 1 ns? Justify your answer quantitatively.

outline

✓ 1. Semiconductor Equation Overview

✓ 2. MCDE Examples

3. Solving Poisson's Equation

"the semiconductor equations"

How do we **calculate** rho(*x*), $\mathcal{E}$(*x*), and V(*x*)?

$$\frac{\partial p}{\partial t} = -\nabla \bullet \left(\frac{\vec{J}_p}{q} \right) + G_p - R_p$$

$$\frac{\partial n}{\partial t} = -\nabla \bullet \left(\frac{\vec{J}_n}{-q} \right) + G_n - R_n$$

$$0 = -\nabla \bullet \left(\varepsilon \vec{\mathcal{E}} \right) + \rho$$

Three equations in three unknowns:

$$p(\vec{r}), n(\vec{r}), V(\vec{r})$$

In steady state equilibrium, we *only* need to solve the Poisson equation

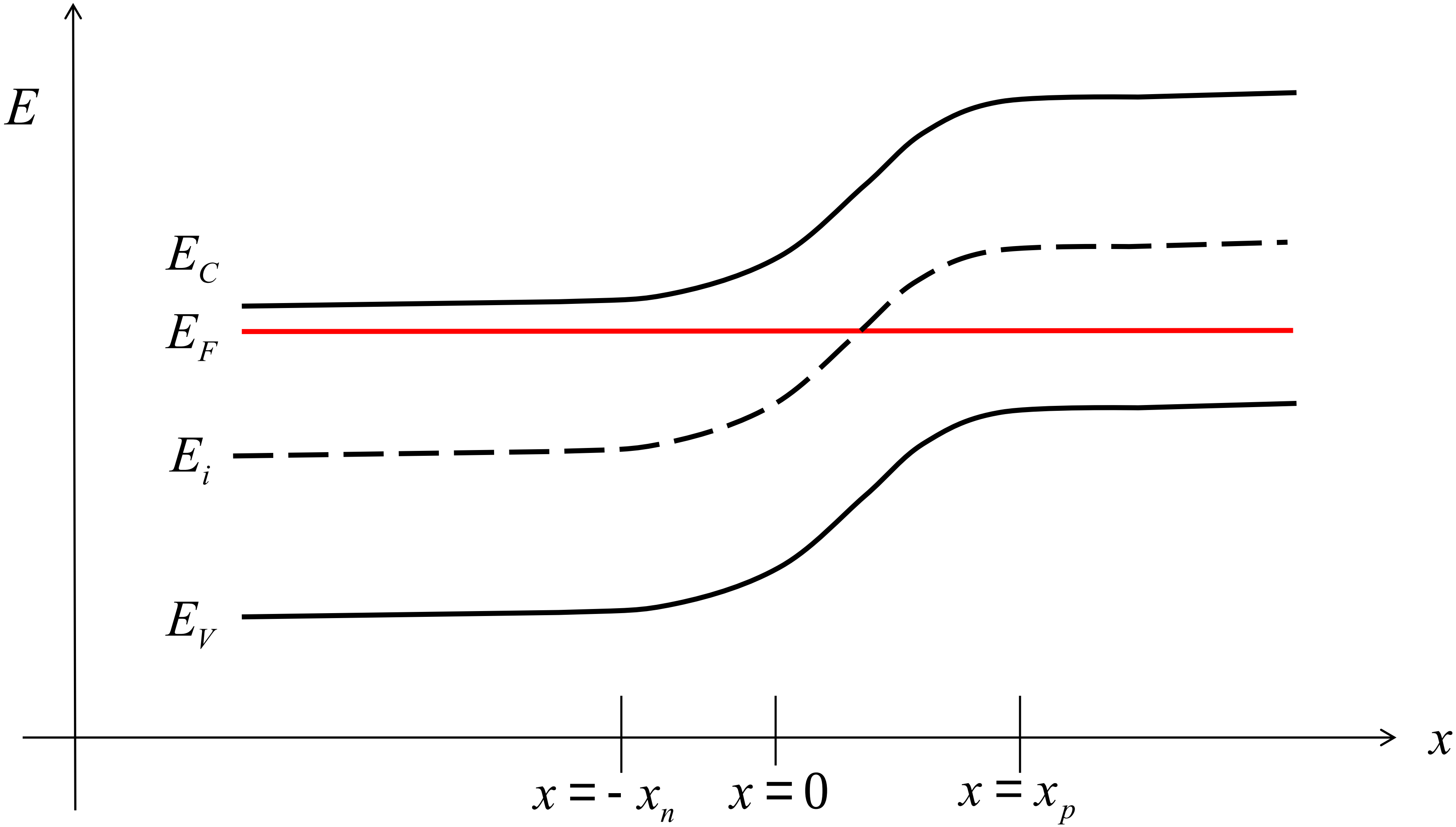
equilibrium energy band diagrams solve Poisson's equation – and more

1) Begin with E_F
2) Draw the E-bands where you know the carrier density
3) Electrostatic potential by flipping E-band upside down.
4) E-field from slope
5) n(x), p(x) from the E-band diagram
6) rho(x) from n(x) and p(x)
7) diffusion current from (5) or from (6)

$$E_C(x) = E_{C-ref} - qV(x)$$

$$\mathcal{E}(x) = \frac{1}{q} dE_C(x)/dx$$

energy band diagram



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“read” the e-band diagram

1) Electrostatic potential vs. position

2) Electric field vs. position

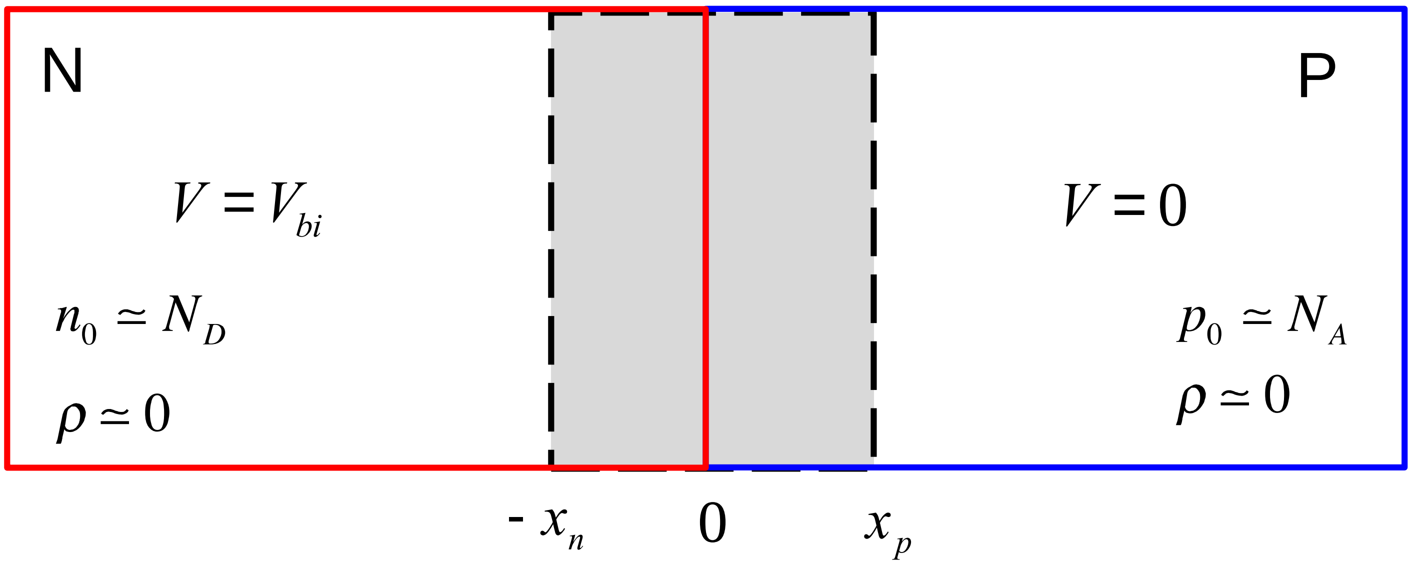
3) Electron and hole densities vs. position

4) Space-charge density vs. position

NP junction (equilibrium)

$$V_{bi} = \frac{k_B T}{q} \ln\left(\frac{N_D N_A}{n_i^2}\right)$$

"transition (depletion) region"



1) What is the width of the depletion region?
2) What is the maximum electric field?

the Poisson equation

$$0 = -\nabla \bullet \left(\varepsilon_S \vec{\mathcal{E}} \right) + \rho$$

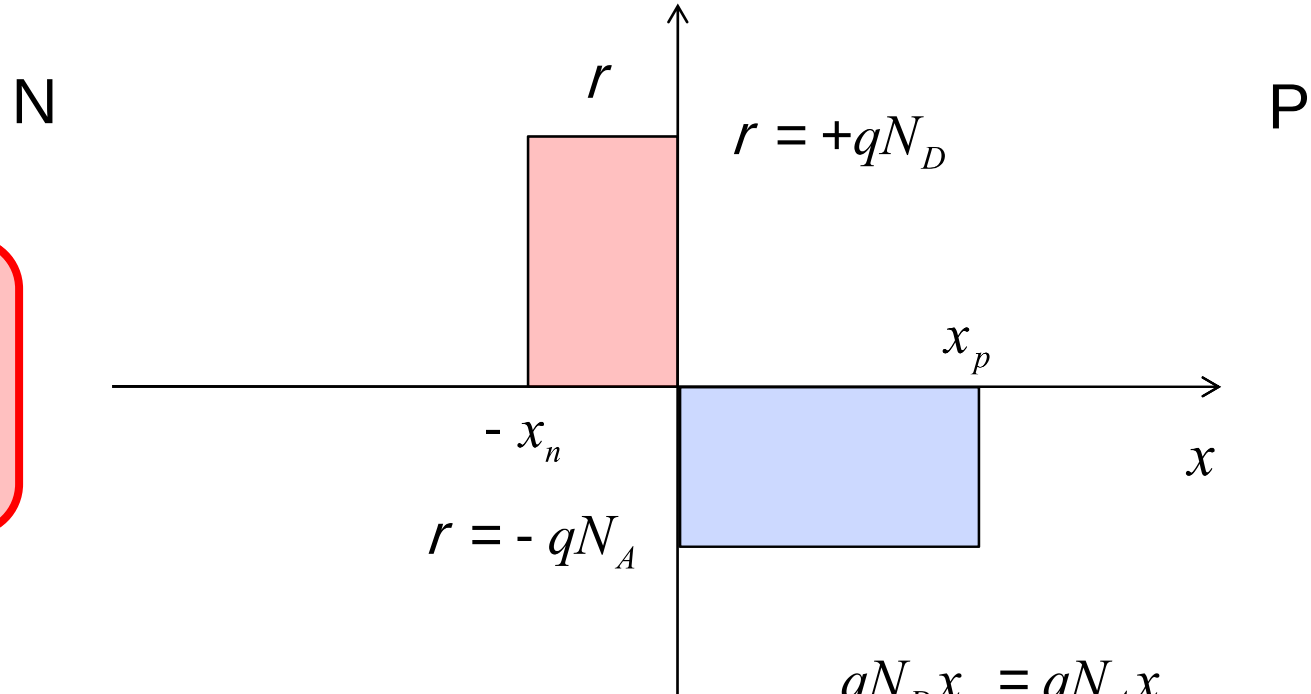
$$\nabla \bullet \left(\varepsilon_S \vec{\mathcal{E}} \right) = \rho(x)$$

$$\frac{d}{dx}\left(\varepsilon_S \mathcal{E} \right) = \rho(x)$$

$$\frac{d\mathcal{E}}{dx} = \frac{\rho(x)}{\varepsilon_S} = \frac{\rho(x)}{K_S \varepsilon_0}$$

$$\frac{d\mathcal{E}}{dx} = \frac{\rho(x)}{K_S \varepsilon_0}$$

the “depletion approximation”

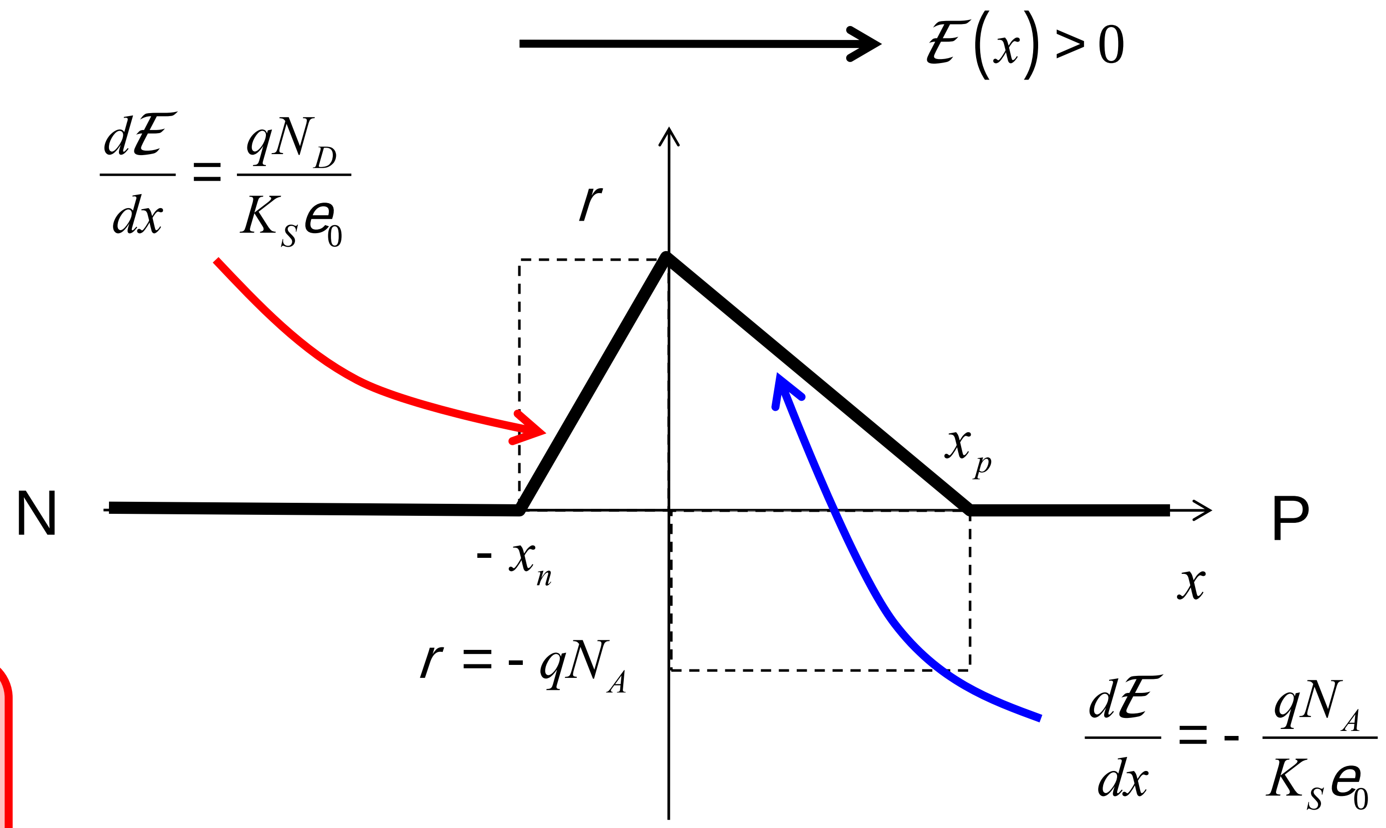


$$\frac{d\mathcal{E}}{dx} = \frac{\rho(x)}{K_S\varepsilon_0}$$

$$qN_D x_n = qN_A x_p$$

$$N_D x_n = N_A x_p$$

the electric field



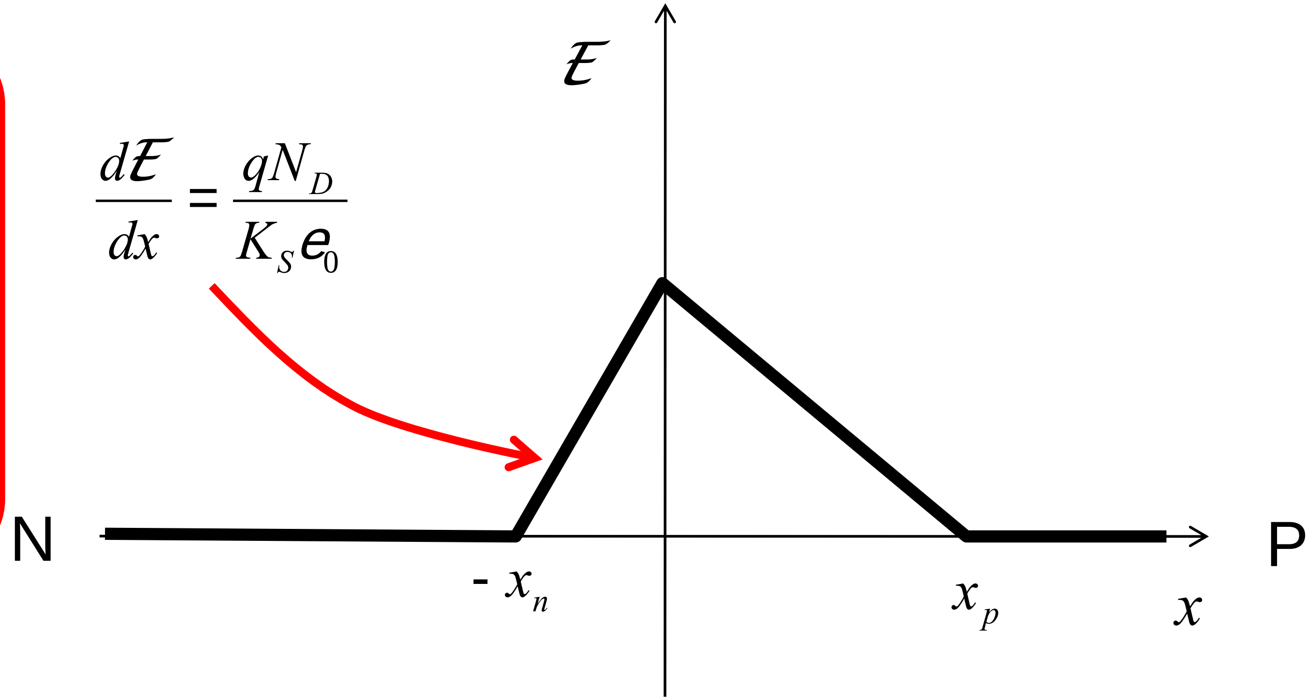
$$\frac{d\mathcal{E}}{dx} = \frac{\rho(x)}{K_S\varepsilon_0}$$

the electrostatic potential

$$\frac{d\mathcal{E}}{dx} = \frac{\rho(x)}{K_S \varepsilon_0}$$

$$\mathcal{E}(x) = -\frac{dV}{dx}$$

$$\frac{d\mathcal{E}}{dx} = \frac{qN_D}{K_S \varepsilon_0}$$



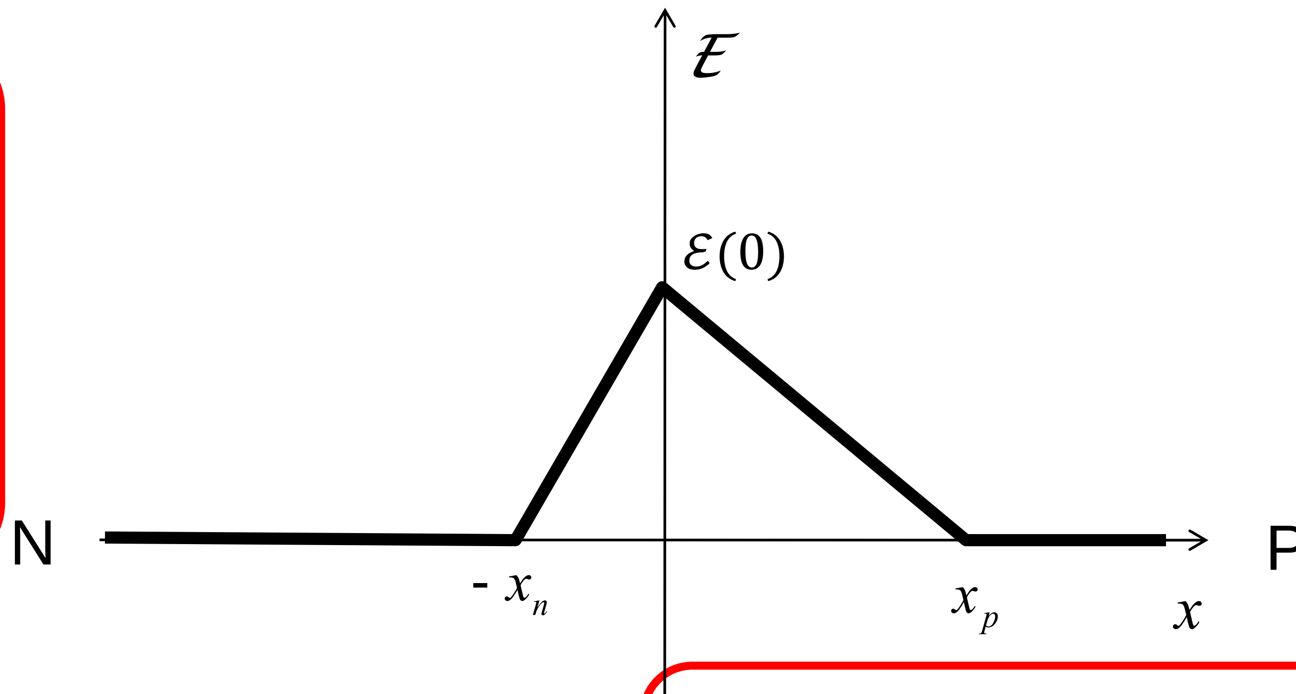
$$\mathcal{E}(0) = \frac{qN_D}{K_S \varepsilon_0} x_n \qquad W = x_n + x_P$$

$$x_n = \frac{N_A}{N_A + N_D} W \qquad N_D x_n = N_A x_P$$

the electrostatic potential

$$\frac{d\mathcal{E}}{dx} = \frac{\rho(x)}{K_S \epsilon_0}$$

$$\mathcal{E}(x) = -\frac{dV}{dx}$$



$$V_{bi} = \int_{-x_n}^{x_p} \mathcal{E}(x)\, dx$$

$$V_{bi} = \frac{1}{2}\mathcal{E}(0) W$$

$$\mathcal{E}(0) = \frac{qN_D}{K_S \epsilon_0}\frac{N_A}{N_A + N_D} W$$

$$W = \left[\frac{2K_S \epsilon_0}{q}\left(\frac{N_A + N_D}{N_D N_A}\right) V_{bi}\right]^{1/2}$$

$$\mathcal{E}(0) = \left[\frac{2qV_{bi}}{K_s \epsilon_0}\left(\frac{N_D N_A}{N_A + N_D}\right)\right]^{1/2}$$

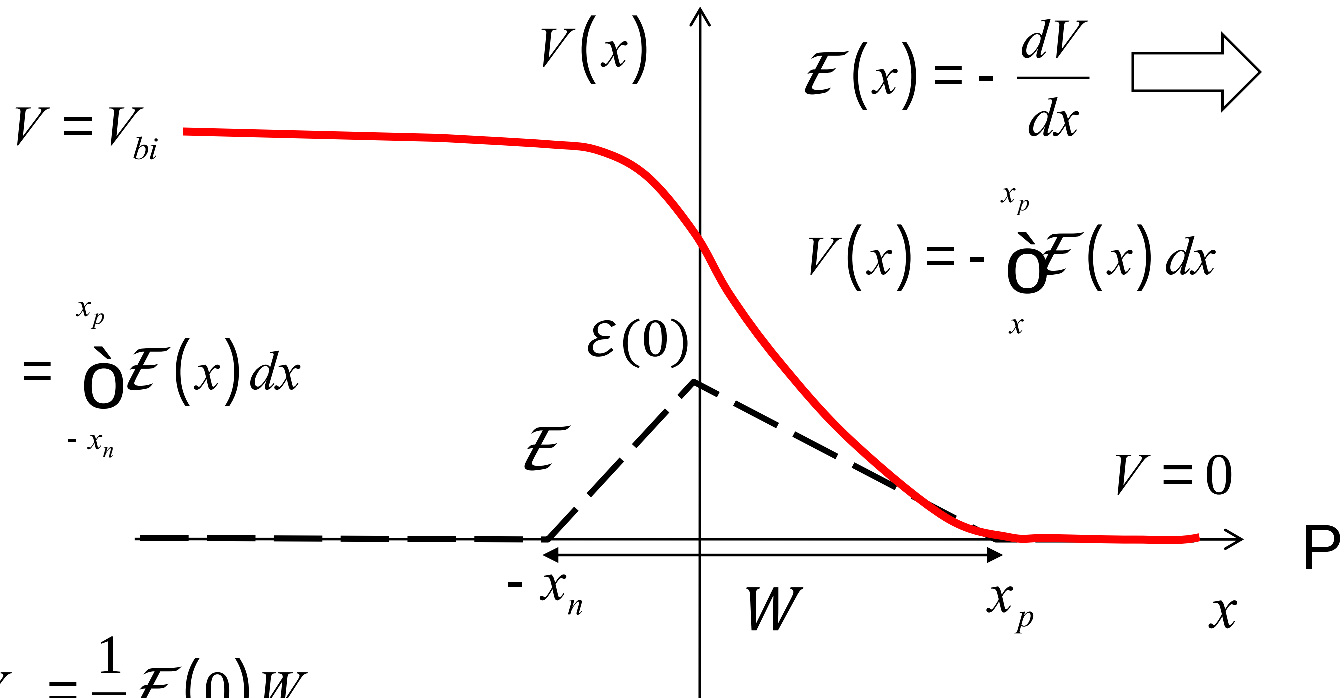
calculating $V(x)$ from $\mathcal{E}(x)$

$$\mathcal{E}(x) = -\frac{dV}{dx} \Rightarrow$$

$$V(x) = -\int_{x}^{x_p} \mathcal{E}(x)\,dx$$

$$V_{bi} = \int_{-x_n}^{x_p} \mathcal{E}(x)\,dx$$

$$V_{bi} = \frac{1}{2}\mathcal{E}(0)W$$

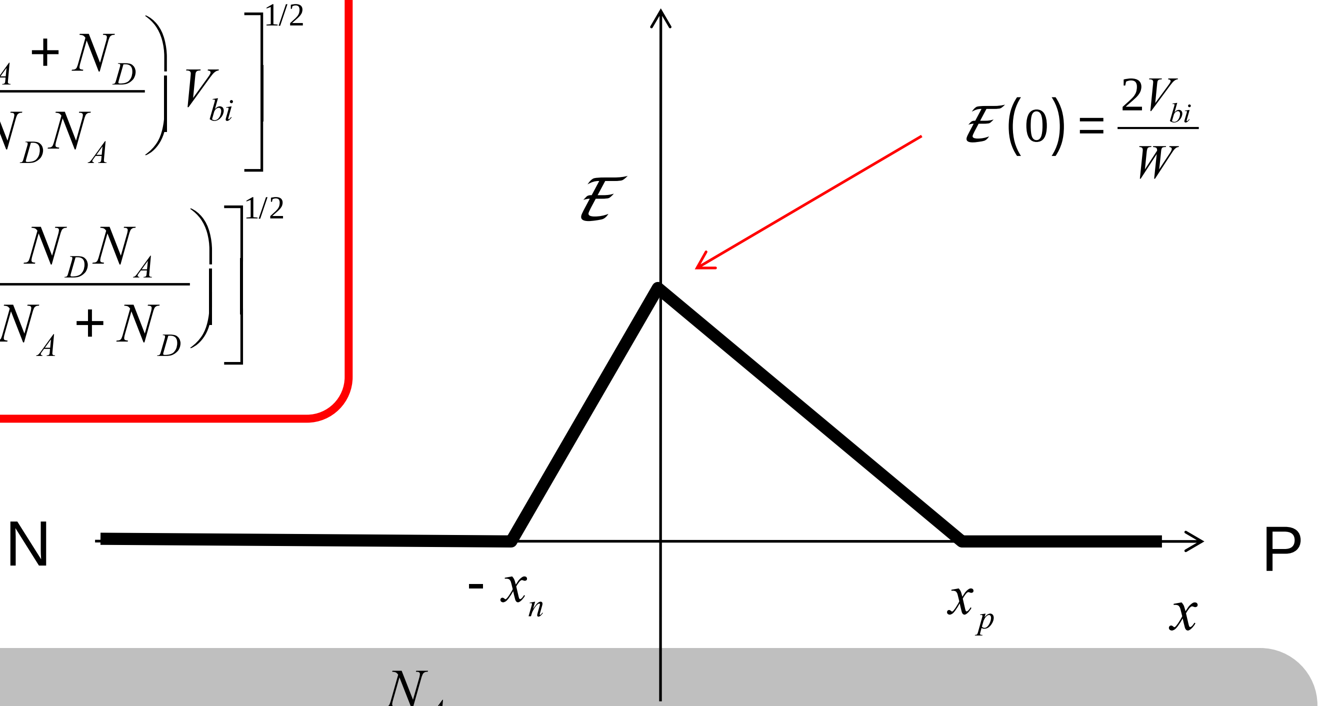


See Pierret, SDF, pp. 212-213

summary

$$W = \left[\frac{2K_S \varepsilon_0}{q} \left(\frac{N_A + N_D}{N_D N_A} \right) V_{bi} \right]^{1/2}$$

$$\mathcal{E}(0) = \left[\frac{2qV_{bi}}{K_s \varepsilon_0} \left(\frac{N_D N_A}{N_A + N_D} \right) \right]^{1/2}$$

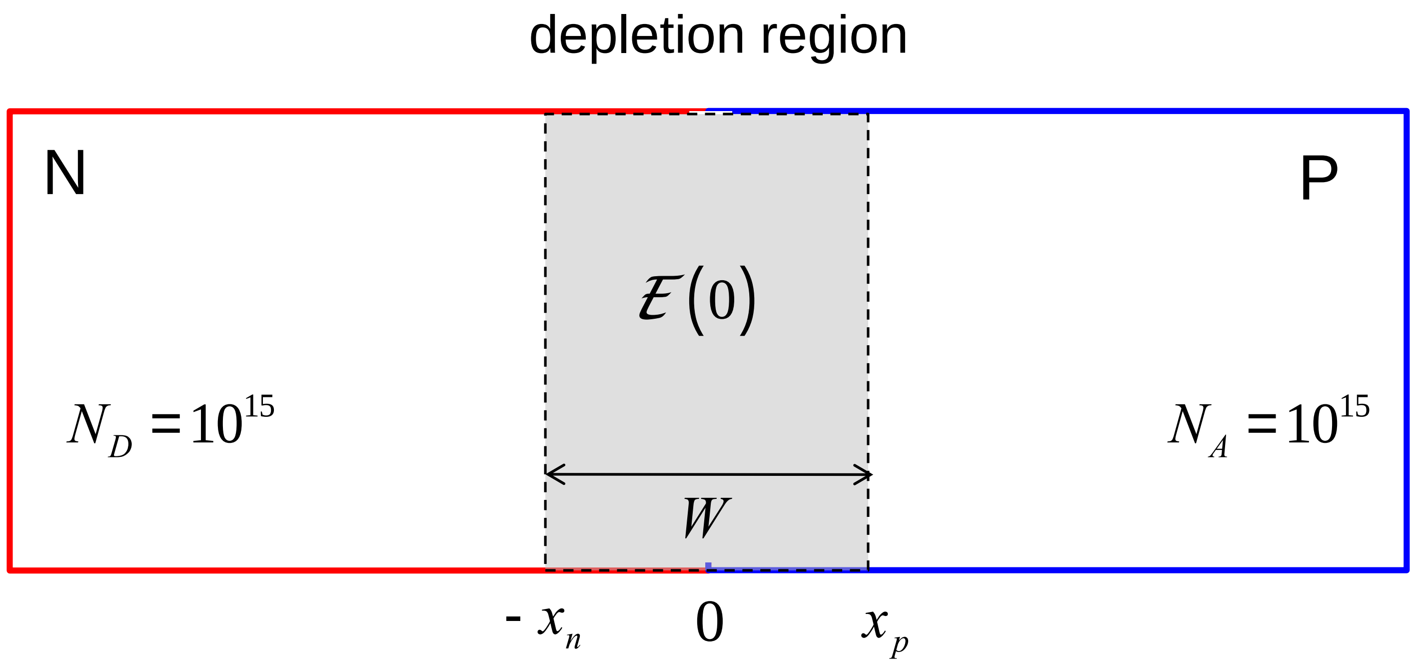


$$W = x_n + x_P \qquad x_n = \frac{N_A}{N_A + N_D} W$$

$$N_D x_n = N_A x_P \qquad x_p = \frac{N_D}{N_A + N_D} W$$

$$V_{bi} = \frac{k_B T}{q} \ln\left(\frac{N_D N_A}{n_i^2} \right)$$

example

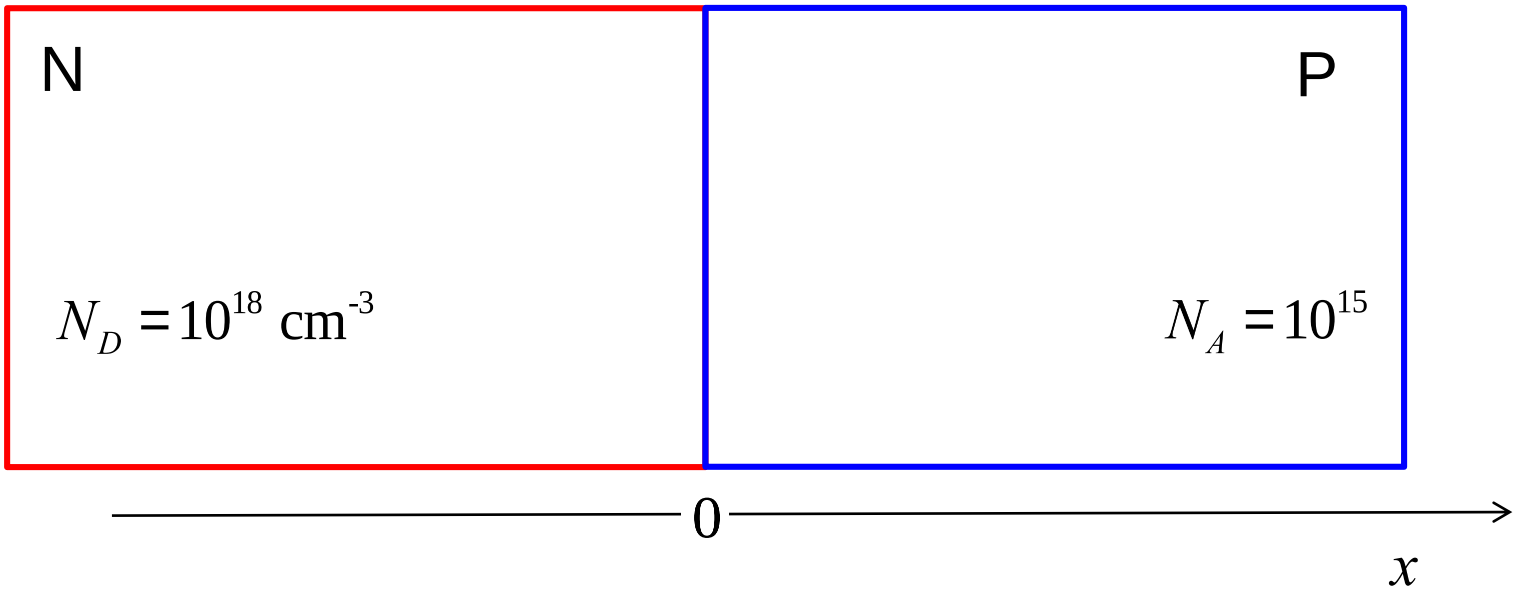


$$V_{bi} = \frac{k_B T}{q} \ln\left(\frac{N_D N_A}{n_i^2} \right) = 0.6 \text{ V}$$

$W = 1.25\ \mu\text{m}$ $\qquad x_n = x_p = 0.625\ \mu\text{m}$

$$\mathcal{E}(0) = 9.6 \times 10^3 \text{ V/cm}$$

"one-sided junction"

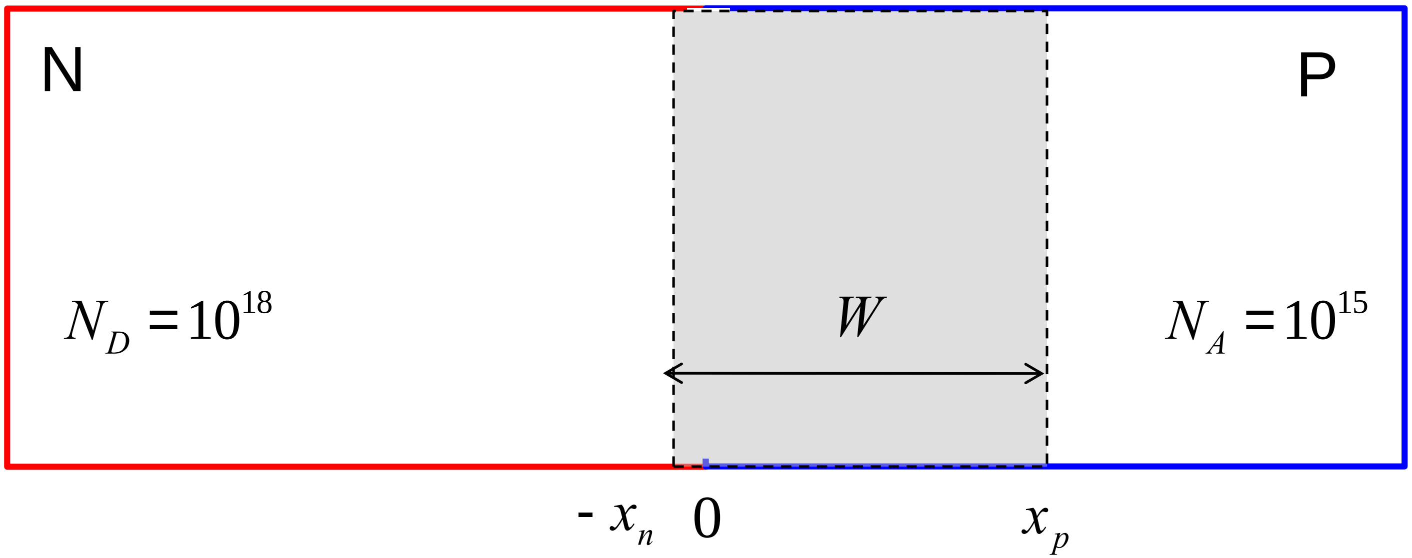


$$\mathcal{E}(0)$$

$$W$$

$$x_n, x_p$$

numbers: one-sided junction



$$V_{bi} = 0.78 \text{ V} \quad (0.60)$$

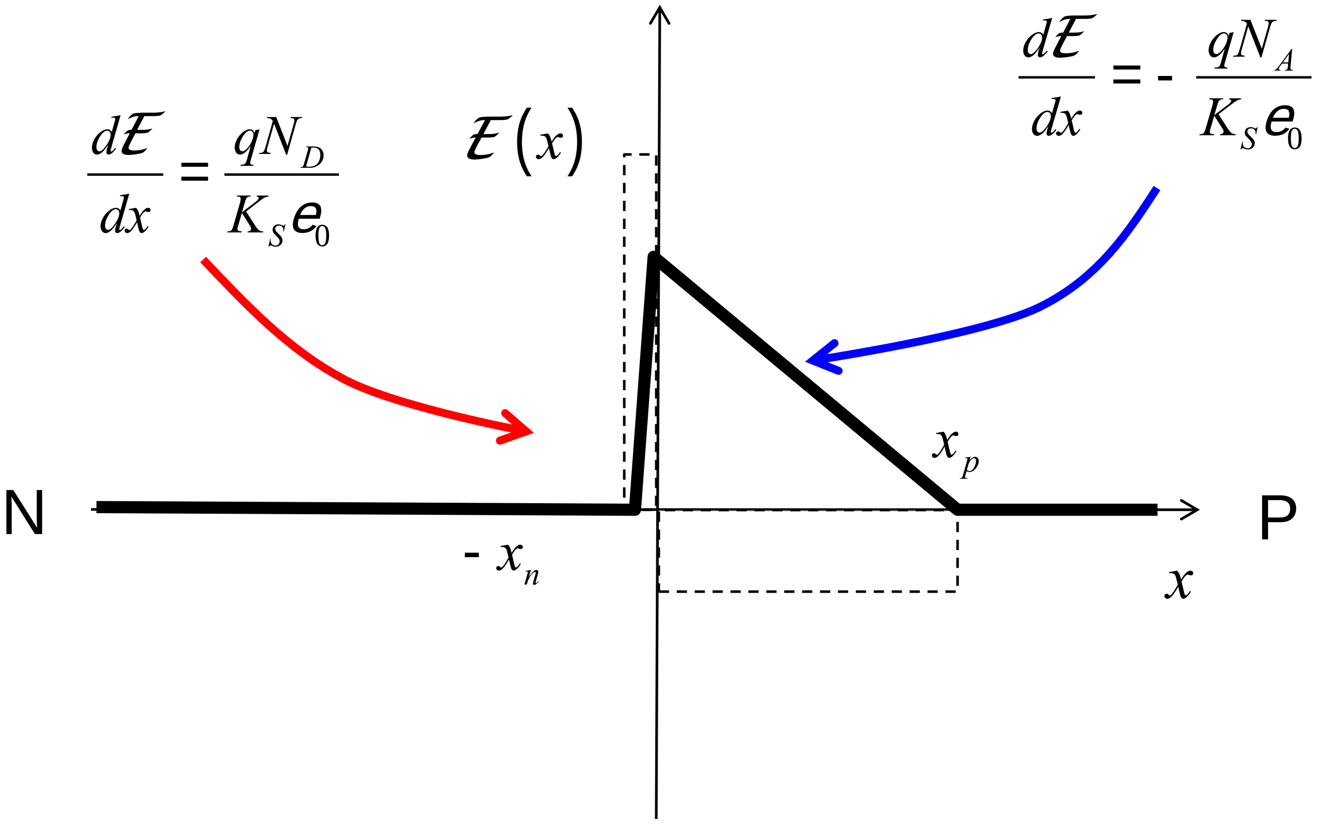
$$W = 1.010 \times 10^{-4} \text{ cm} \left(1.25 \times 10^{-4}\right)$$

$$x_n = 0.001 \times 10^{-4} \text{ cm}$$

$$x_p = 1.009 \times 10^{-4} \text{ cm}$$

$$\mathcal{E}(0) = \frac{2V_{bi}}{W} = 1.5 \times 10^{4} \text{ V/cm} \quad (0.96)$$

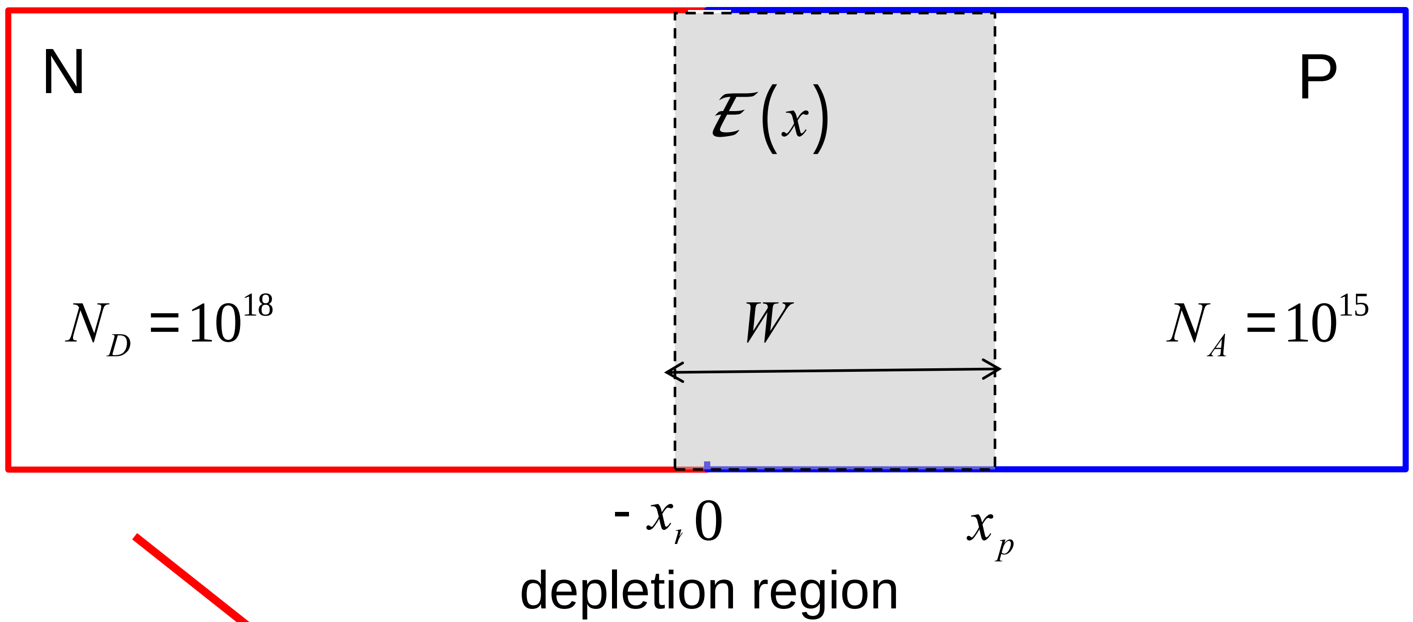
one-sided junction



$N_D >> N_A$

$x_p >> x_n$

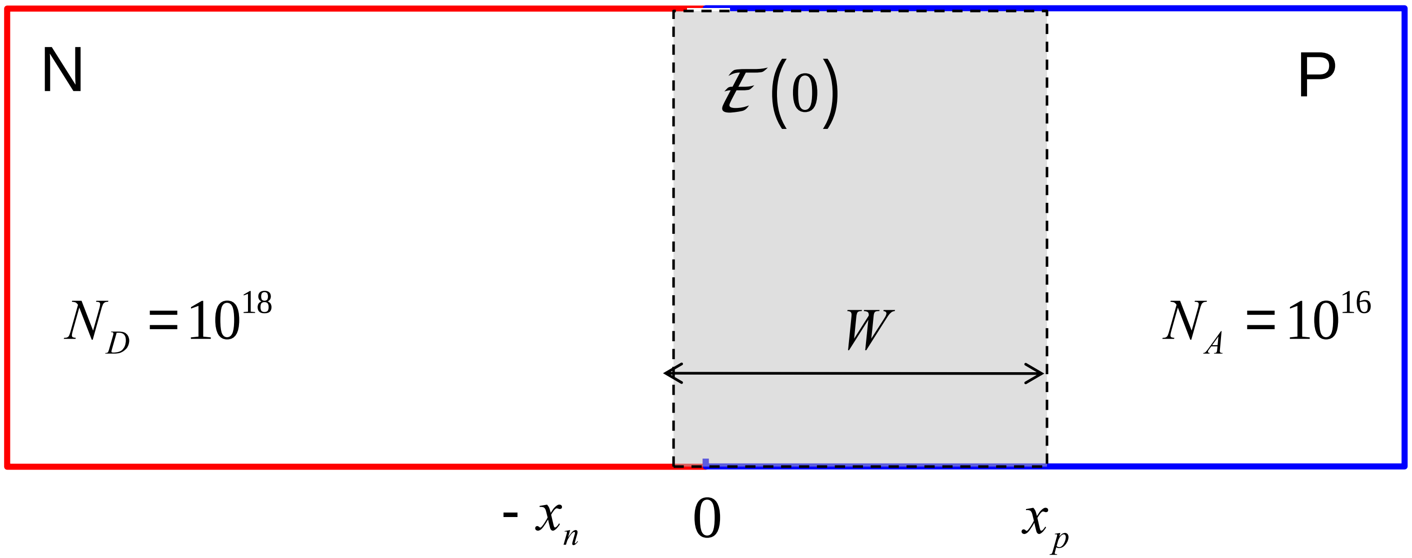
one-sided junction



$$V_{bi} \approx \frac{k_B T}{q} \ln\left(\frac{N_D N_A}{n_i^2}\right) \qquad x_n = \frac{N_A}{N_A + N_D} W \approx 0$$

$$W = \left[\frac{2K_S\varepsilon_0}{q}\left(\frac{N_A + N_D}{N_D N_A}\right)V_{bi}\right]^{1/2} \rightarrow \left[\frac{2K_S\varepsilon_0}{qN_A}V_{bi}\right]^{1/2} \qquad \mathcal{E}(0) = \frac{2V_{bi}}{W} \qquad x_p = \frac{N_D}{N_A + N_D} W \approx W$$

example: one-sided junction



$$V_{bi} \approx \frac{k_B T}{q} \ln\left(\frac{N_D N_A}{n_i^2}\right)$$

$$W = \left[\frac{2K_S e_0}{qN_A} V_{bi}\right]^{1/2} \qquad x_n » 0$$

$$\mathcal{E}(0) = \frac{2V_{bi}}{W} \qquad x_p » W$$

conclusions

- We will solve many problems in this class using the semiconductor equations:
  - In regions of zero field, we can use the minority carrier diffusion equation to understand the mechanics of carrier transport in electronic devices. Review the problem carefully to see if the assumption of minority carrier transport is satisfied.
  - In regions of non-zero field like NP junctions, we can use band diagrams to sketch $\mathcal{E}$, V, n, and p ; and Poisson's equation and 'depletion approximation' to quantify these values.
  - This approach also gives us the width of the 'depletion region' on both sides of the junction (x_n and x_p), plus the 'built-in' voltage V_{bi}