1. Minority carrier diffusion equation example
   #3: Steady-state, one-sided

2. Semiconductor fabrication
Example #3 summary: One-sided carrier diffusion

<table>
<thead>
<tr>
<th>Length scale</th>
<th>Solution type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Long ((L \gg L_D))</td>
<td>Decaying exponentials</td>
</tr>
<tr>
<td>Short ((L \ll L_D))</td>
<td>Linear</td>
</tr>
<tr>
<td>Intermediate ((L \sim L_D))</td>
<td>Hyperbolic functions</td>
</tr>
</tbody>
</table>

**Notes:**

- \(L\) is length of region where MCDE applies
- \(L_D = \sqrt{D_{n,p} \tau_{n,p}}\) is the diffusion length for carriers
Example 3C: One-sided carrier diffusion
Intermediate sample (steady state)

Steady-state, sample is **30 micrometers long**. No generation.

\[ \Delta n(x = 0) = 10^{12} \text{ cm}^{-3} \text{ fixed} \]

\[ \Delta n(x = 30 \mu m) = 0 \]

\[ \frac{\partial \Delta n}{\partial t} = D_p \frac{d^2 \Delta n}{dx^2} - \frac{\Delta}{\tau_n} + G_L \]

\[ 0 = D_p \frac{d^2 \Delta n}{dx^2} - \frac{\Delta}{\tau_n} + 0 \]

1) Simplify the MCDE  
2) Solve the MCDE  
3) Deduce \( F_p \) from \( \Delta p \)

\[ \frac{d^2 \Delta n}{dx^2} - \frac{\Delta n}{L_n} = 0 \quad L_n \equiv \sqrt{D_n \tau_n} \]

\[ L_n = 28 \mu m \quad L = 30 \mu m \]
Example 3C: One-sided carrier diffusion
Intermediate sample (steady state)

Steady-state, sample is 30 micrometers long. No generation.

\[ \Delta n(x = 0) = 10^{12} \text{ cm}^{-3} \quad \text{fixed} \]

\[ \Delta n(x = 30 \mu m) = 0 \]

1) Simplify the MCDE
2) Solve the MCDE
3) Deduce \( F_p \) from \( \Delta \rho \)

\[ \frac{d^2 \Delta n}{dx^2} - \frac{\Delta n}{L_n} = 0 \]

\[ \Delta n(x) = Ae^{-x/L_n} + Be^{+x/L_n} \]

\[ \Delta n(0) = A + B = 10^{12} \]

\[ \Delta n(L) = Ae^{-L/L_n} + Be^{+L/L_n} = 0 \]
Example 3C: One-sided carrier diffusion
Intermediate sample (steady state)

\[ L_n = \sqrt{D_n \tau_n} \]

\[ \Delta n(x) = \Delta n(0) \frac{\sinh\left(\frac{(L - x)}{L_n}\right)}{\sinh\left(\frac{L}{L_n}\right)} \]

\[ \Delta n(x = L) = 0 \]

Steady-state; sample neither long nor short compared to the diffusion length.

\[ L_n = \sqrt{D_n \tau_n} \]

\[ \Delta n(0) \]

\[ \Delta n(x) \]

\[ x = 0 \]

\[ x = L \]
1. Minority carrier diffusion equation example #3: Steady-state, one-sided
2. Semiconductor fabrication
current challenges in device fabrication

Smaller, Faster, Cheaper, Over: The Future of Computer Chips

By JOHN MARKOFF  SEPTE 26, 2015

Max Shulaker, a graduate student at Stanford, working in 2011 on a new kind of semiconductor circuit. As chips continue to shrink, computer scientists are seeking new technological breakthroughs.

integrated circuit resistors

\[ R = \rho_s \left( \frac{L}{W} \right) \]

\[ \rho_s \left( \frac{\Omega}{\square} \right) \]

P-type

N-type
Integrated circuit resistors

1) Oxidize
2) Coat with resist

resist
SiO$_2$
P-Si
Integrated circuit resistors

1) Oxidize

2) Coat with resist

3) Expose light
Integrated circuit resistors

1) Oxidize
2) Coat with resist
3) Expose
4) Develop
Integrated circuit resistors

1) Oxidize
2) Coat with resist
3) Expose
4) Develop
5) Etch
Integrated circuit resistors

1) Oxidize
2) Coat with resist
3) Expose
4) Develop
5) Etch
6) Strip resist

SiO\textsubscript{2}

P-Si
Integrated circuit resistors

1) Oxidize
2) Coat with resist
3) Expose
4) Develop
5) Etch
6) Strip resist
7) Dope
Integrated circuit resistors

1) Oxidize  
2) Coat with resist  
3) Expose  
4) Develop  
5) Etch  
6) Strip resist  
7) Dope  
8) Anneal and Oxidize
Integrated circuit resistors

1) Oxidize
2) Coat with resist
3) Expose
4) Develop
5) Etch
6) Strip resist
7) Dope
8) Anneal/Oxidize
9) Open contacts
Integrated circuit resistors

1) Oxidize  
2) Coat with resist  
3) Expose  
4) Develop  
5) Etch  
6) Strip resist  
7) Dope  
8) Anneal/Oxidize  
9) Open contacts  
10) Deposit metal pattern, etch

Our resistor is also PN junction!
Integrated circuit resistors

\[ R = \rho_s \left( \frac{L}{W} \right) \]
videos

- GLOBALFOUNDRIES Sand to Silicon
- Semiconductor Technology at TSMC, 2011
- Intel: The Making of a Chip with 22nm/3D Transistors
Diffusion from a gas, liquid or solid source

Pre-deposition (dose control)
- Silicon dioxide masks impurity diffusion in Si
- The mixture of dopant species, oxygen and inert gases pass over wafers at 900-1100°C in a diffusion furnace
- The dopant concentration reaches the solid solubility limit
- The impurities can be introduced into the carrier gas from solid (evaporated), liquid (vapor) or gas source.

Drive-in (profile control)

9/26/2017
Bermel ECE 305 F17

Adapted from Bo Cui, ECE, University of Waterloo; and Silicon VLSI Technology by Plummer, Deal and Griffin
Dopant solid solubility limits

**Solid solubility limit:** maximum concentration for an impurity before precipitation into a separate phase.
Fick’s first law of diffusion

\[ F(x, t) = -D \frac{\partial C(x, t)}{\partial x} \]

Cf. Ohm’s law

C is impurity concentration (number/cm\(^3\)), D is diffusivity (cm\(^2\)/sec).

D is related to atomic hops over an energy barrier (formation and migration of mobile species) and is exponentially activated.

Negative sign indicates that the flow is down the concentration gradient.
Intrinsic diffusivity $D_i$

Intrinsic: impurity concentration $N_A, N_D < n_i$

Note that $n_i$ is quite high at typical diffusion temperatures

$$D_i = D^0 \exp\left(-\frac{E_a}{kT}\right)$$

$E_a$: activation energy

<table>
<thead>
<tr>
<th>Element</th>
<th>$D^0$ (cm$^2$/s)</th>
<th>$E_a$ (eV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>1.0</td>
<td>3.46</td>
</tr>
<tr>
<td>In</td>
<td>1.2</td>
<td>3.50</td>
</tr>
<tr>
<td>P</td>
<td>4.70</td>
<td>3.68</td>
</tr>
<tr>
<td>As</td>
<td>9.17</td>
<td>3.99</td>
</tr>
<tr>
<td>Sb</td>
<td>4.58</td>
<td>3.88</td>
</tr>
</tbody>
</table>

Adapted from Bo Cui, ECE, University of Waterloo; and *Silicon VLSI Technology* by Plummer, Deal and Griffin
Fick’s second law

Since:

\[
\frac{\partial C(x,t)}{\partial t} = - \frac{\partial F(x,t)}{\partial x}
\]

\[F(x,t) = -D \frac{\partial C(x,t)}{\partial x}\]

We have:

\[
\frac{\partial C(x,t)}{\partial t} = - \frac{\partial F(x,t)}{\partial x} = \frac{\partial}{\partial x} \left[ D \frac{\partial C(x,t)}{\partial x} \right]
\]

If D is constant:

\[
\frac{\partial C(x,t)}{\partial t} = D \frac{\partial^2 C(x,t)}{\partial x^2}
\]
Solution to diffusion equation

\[ \frac{\partial C(x,t)}{\partial t} = D \frac{\partial^2 C(x,t)}{\partial x^2} \]

In equilibrium, C doesn’t change with time.

\[ \frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial x^2} = 0 \]

\[ C = a + bx \]

Diffusion of oxidant (O\textsubscript{2} or H\textsubscript{2}O) through SiO\textsubscript{2} during thermal oxidation.

Adapted from Bo Cui, ECE, University of Waterloo; and *Silicon VLSI Technology* by Plummer, Deal and Griffin
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2. Semiconductor fabrication
for more on IC manufacturing

ECE 612 Purdue University:

https://nanohub.org/resources/5788

https://nanohub.org/resources/5855

ECE 557 Purdue University:

http://www.purdue.edu/newsroom/releases/2015/Q3/purdue-uses-nanotechnology-cleanroom-to-expand-undergrad-class,-expose-students-to-high-end-research.html