ECE-305: Fall 2016

PN Junctions: Depletion Approximation

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Pierret, Semiconductor Device Fundamentals (SDF)
Chapter 5 (pp. 209-246)
Outline

1) **PN homo- and hetero-junctions**
2) PN Junctions under bias
3) Current flow in a p-n junction
4) Conclusions
What is a Diode good for ....?

- solar cells
- GaAs lasers
- Avalanche Photodiode
- GaN lasers
- Organic LED

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p-n Junction Devices …

Symbols

Finding hotspot …

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Step junction vs. graded junction

Metallurgical junction

$N_D, N_A$

$N_D - N_A$

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$N_D - N_A$

X

X
... is equivalent to solving the Poisson equation
Analytical Solution of Poisson Equation

Depleted Region

\[ Q = p \quad n + N_D^+ \quad N_A^- \]

\[ K_S \varepsilon_0 \frac{d^2V}{dx^2} = -q \left( p - n + N_D^+ - N_A^- \right) \]
Built-in voltage for **Homo-junctions**

\[ qV_{bi} = k_B T \ln \frac{N_A N_D}{N_V N_C e^{-E_g / k_B T}} + (\chi_2 - \chi_1) = k_B T \ln \frac{N_A N_D}{N_V N_C e^{-E_g / k_B T}} = k_B T \ln \frac{N_A N_D}{n_i^2} \]

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Analytical Solution for Homojunctions

\[ E(0^-) = \frac{qN_D x_n}{k_s \varepsilon_0} \]
\[ E(0^+) = \frac{qN_A x_p}{k_s \varepsilon_0} \]
\[ \Rightarrow N_D x_n = N_A x_p \]

\[ qV_{bi} = \frac{E(0^-) x_n}{2} + \frac{E(0^+) x_p}{2} \]
\[ = \frac{qN_D x_n^2}{2k_s \varepsilon_0} + \frac{qN_A x_p^2}{2k_s \varepsilon_0} \]

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Depletion Regions in Homojunctions

\[ N_D x_n = N_A x_p \]

\[ qV_{bi} = \frac{qN_D x_n^2}{2k_s \varepsilon_0} + \frac{qN_A x_p^2}{2k_s \varepsilon_0} \]

\[ x_n = \sqrt{\frac{2k_s \varepsilon_0}{q} \frac{N_A}{N_D (N_A + N_D)}} V_{bi} \]

\[ x_p = \sqrt{\frac{2k_s \varepsilon_0}{q} \frac{N_D}{N_A (N_A + N_D)}} V_{bi} \]

Can you solve the same problem for a hetero-junction?
Built-in Potential: heterojunctions

\[ qV_{bi} = E_{g,2} - \Delta_2 - \Delta_1 + \chi_2 - \chi_1 \]

\[ = \left( E_{g,2} + k_B T \ln \frac{N_A}{N_{V,2}} \right) + k_B T \ln \frac{N_D}{N_{C,1}} + (\chi_2 - \chi_1) \]

\[ = k_B T \ln \frac{N_A N_D}{N_{V,2} N_{C,1} e^{-E_{g,2}/k_B T}} + (\chi_2 - \chi_1) \]
Interface Boundary Conditions: heterojunctions

\[ E = \left( \frac{D}{k\varepsilon_0} \right) \]

\[ D_1 = K_1 \varepsilon_0 E(0^-) = K_2 \varepsilon_0 E(0^-) = D_2 \]

\[ E(0^-) = \frac{K_2}{K_1} E(0^+) \]

Displacement is continuous across the interface, but field need not be ..
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1) PN homo- and hetero-junctions
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Applying a Bias: Poisson Equation

\[ qV_{bi} + E_{C-E_F} + E_{F-E_V} \]

\[ qV_{bi} - V_A - E_{C-F_n} + F_p - E_V \]
equilibrium e-band diagram

\[ E_c \]
\[ E_F \]
\[ E_v \]

\[ qV_{bi} \]

\[ I = 0 \]
\[ V_A = 0 \]

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The applied voltage drops across the junction, but...
QFL’s split

\[ V = 0 \]

\[ E_C \]

\[ V_{bi} - V_A \uparrow \downarrow \]

\[ F_n \]

\[ F_n > F_p \]

\[ qV_A \]

\[ W \]

\[ V_A > 0 \]

\[ x_p \]

\[ x_n \]

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e-band diagram under reverse bias

\[ V_{bi} - V_A = V_{bi} + V_R \]

\[ F_n < F_p \]

\[ V_A < 0 \]
one-sided junction

\[ E(x) \]

\[ N \quad P \]

\[ V_A < 0 \]

\[ N_D >> N_A \]

\[ x_p >> x_n \]

\[ V_A > 0 \]
key points (one-sided NP junctions)

\[ V_{bi} \approx \frac{k_B T}{q} \ln \left( \frac{N_D N_A}{n_i^2} \right) \]

\[ W = \left[ \frac{2K_S \varepsilon_0}{qN_A} (V_{bi} - V_A) \right]^{1/2} \quad W \propto \sqrt{V_{bi} - V_A} \quad W \propto \frac{1}{\sqrt{N_A}} \]

\[ \mathcal{E}(0) = \frac{2(V_{bi} - V_A)}{W} \quad \mathcal{E}(0) \propto \sqrt{V_{bi} - V_A} \quad \mathcal{E}(0) \propto \sqrt{N_A} \]
Applying a Bias: Poisson Equation
Flat Quasi-Fermi Level up to Junction

$E_C$  

$E_V$  

$J_n$  

$J_p$
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Continuity Equations for p-n junction Diode

\[ \nabla \cdot E = q \left( p - n + N^+_D - N^-_A \right) \]

\[ \frac{\partial n}{\partial t} = \frac{1}{q} \nabla \cdot J_N - r_N + g_N \]

\[ J_N = qn \mu_N E + qD_N \nabla n \]

\[ \frac{\partial p}{\partial t} = \frac{1}{q} \nabla \cdot J_P - r_P + g_P \]

\[ J_P = qp \mu_P E - qD_P \nabla p \]

Will focus on this part now ...
Recall: One Sided Minority Diffusion

Can calculate current anywhere, let us solve the problem where it is the easiest …

\[
\frac{\partial n}{\partial t} = \frac{1}{q} \frac{dJ_n}{dx} - r_n + g_n
\]

\[
J_n = qn \mu_n E + qD_n \frac{dn}{dx}
\]

\[
0 = D_n \frac{d^2 n}{dx^2}
\]

Steady state
Acceptor doped

\[q(V_{bi} - V)\]
Boundary Conditions

\[ n(x = 0^+) = n_i e^{(F_n - E_i)\beta} \]
\[ p(x = 0^+) = n_i e^{-(F_p - E_i)\beta} \]
\[ np = n_i^2 e^{(F_n - F_p)\beta} = n_i^2 e^{qV_A}\beta \]
\[ p(0^+) = N_A \]
\[ n(0^+) = \frac{n_i^2}{N_A} e^{qV_A}\beta \]
\[ \Delta n(0^+) = n(0^+)_V - n(0^+)_G = 0 \]
\[ = \frac{n_i^2}{N_A} (e^{qV_A}\beta - 1) \]
Right Boundary Condition

\[ n(x = W_p) \approx \frac{n_i^2}{N_A} \]

\[ \Delta n(x = W_p) = 0 \]
Example: One Sided Minority Diffusion

\[ D_N \frac{d^2 n}{dx^2} = 0 \]

\[ \Delta n(x,t) = C + Dx \]

\[ x = W_p, \quad \Delta n(x = W_p) = 0 \Rightarrow C = -DW_p \]

\[ x = 0', \quad \Delta n(x = 0) = \frac{n_i^2}{N_A} \left( e^{qV_A\beta} - 1 \right) = C \]

\[ \Delta n(x,t) = \frac{n_i^2}{N_A} \left( e^{qV_A\beta} - 1 \right) \left( 1 - \frac{x}{W_p} \right) \]
Electron & Hole Fluxes

\[ \Delta n(x) = \frac{n_i^2}{N_A} \left( e^{qV_A \beta} - 1 \right) \left( 1 - \frac{x}{W_p} \right) \]

\[ J_n = qD_n \frac{dn}{dx} \bigg|_{x=0} = -qD_n \frac{n_i^2}{W_p N_A} \left( e^{qV_A \beta} - 1 \right) \]

\[ J_p = -qD_p \frac{dp}{dx} \bigg|_{x=0'} = -qD_p \frac{n_i^2}{W_n N_D} \left( e^{qV_A \beta} - 1 \right) \]

\[ J_N = qn \mu_N \mathcal{E} + qD_N \nabla n \]
Total Current

Forward Bias

\[ \ln J_T \approx q V_A / k_B T + \ln(\text{const.}) \]

\[ J_T = -q \left[ \frac{D_n n_i^2}{W_p N_A} + \frac{D_p n_i^2}{W_n N_D} \right] \left( e^{qV_A/\beta} - 1 \right) \]

Reverse Bias

\[ J_T \approx \text{const.} \]
Various Regions of I-V Characteristics

1. **Diffusion limited**
2. Ambipolar transport
3. High injection
4. R-G in depletion
5. Breakdown
6. Trap-assisted R-G
7. Esaki Tunneling

\[ \ln(I) \sim \frac{q}{k_B T} V_A \]
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Conclusion

1) I-V characteristics of a p-n junction are defined by many interesting phenomena including diffusion, ambipolar transport, tunneling, etc.

2) The separate regions are identified by specific features. Once we learn to identify them, we can see if one or the other mechanism is dominated for a given technology.

3) In the next class, we will discuss a few more non-ideal effects.
Review Questions

1) Which terminal of a multi-terminal device should you ground?

2) If you apply negative bias to a terminal, which direction does the band move?

3) What is the difference between Fermi level and Imref level?

4) What is the difference between forward bias and reverse bias? Which end of a p-n junction should you attach to the positive terminal of the battery to forward bias it?

5) How can we get away with solving just the minority equation?