ECE-305: Spring 2018
Bipolar Junction Transistor I-V

Pierret, *Semiconductor Device Fundamentals* (SDF)
Chapters 10 and 11 (pp. 371-385, 389-403)

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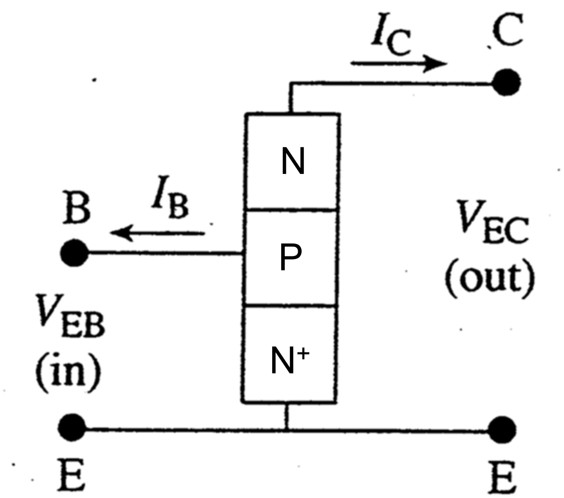
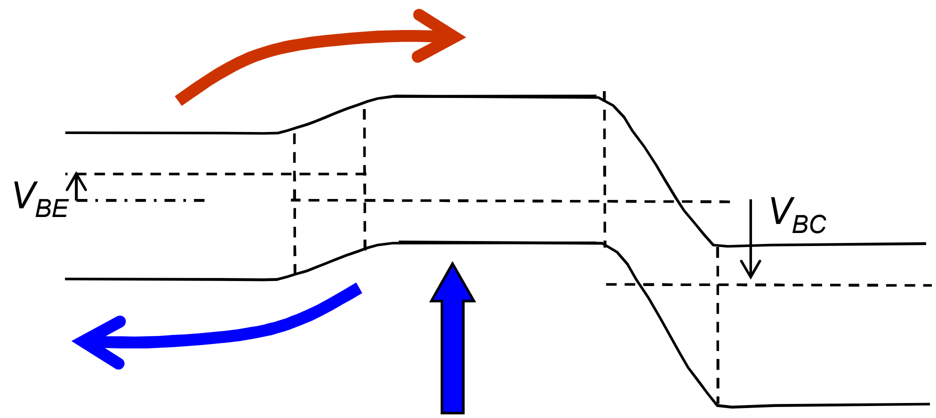
essence of current gain





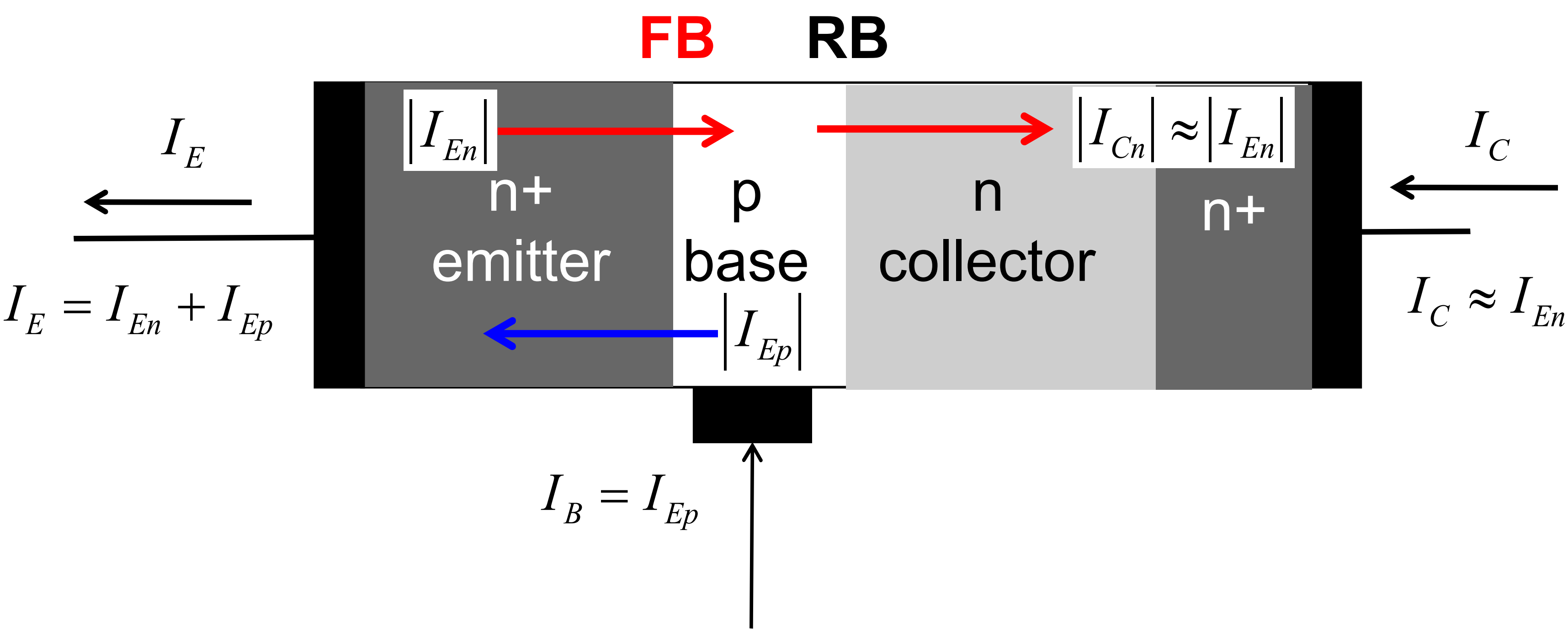
Input → Response → Input

$$I_B \approx \frac{qD_p}{W_E}\frac{n_{i,E}^2}{N_E}\left(e^{qV_{BE}\beta}-1\right)$$

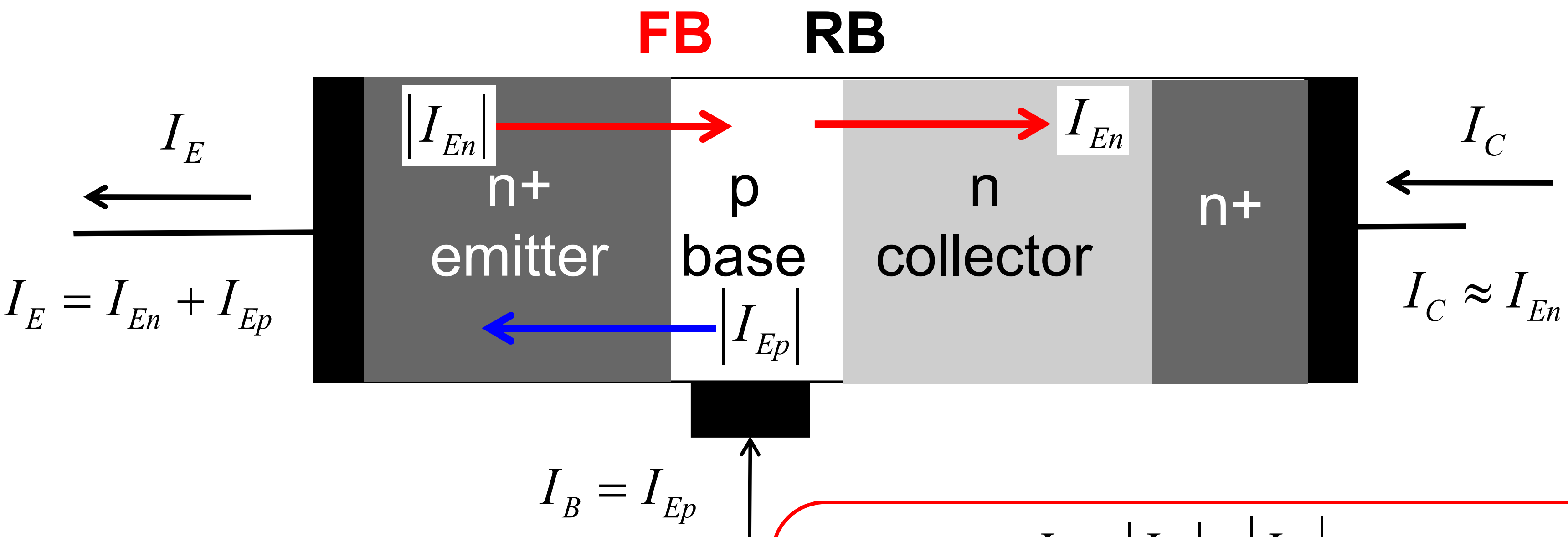
$$I_E \approx \frac{qD_n}{W_B}\frac{n_{i,B}^2}{N_B}\left(e^{qV_{BE}\beta}-1\right)$$

Response

forward active region



emitter current: forward active region



$$|I_{En}| = qA_E\left(\frac{n_i^2}{N_{AB}}\right)\frac{D_n}{W_B}e^{qV_{BE}/k_BT}$$

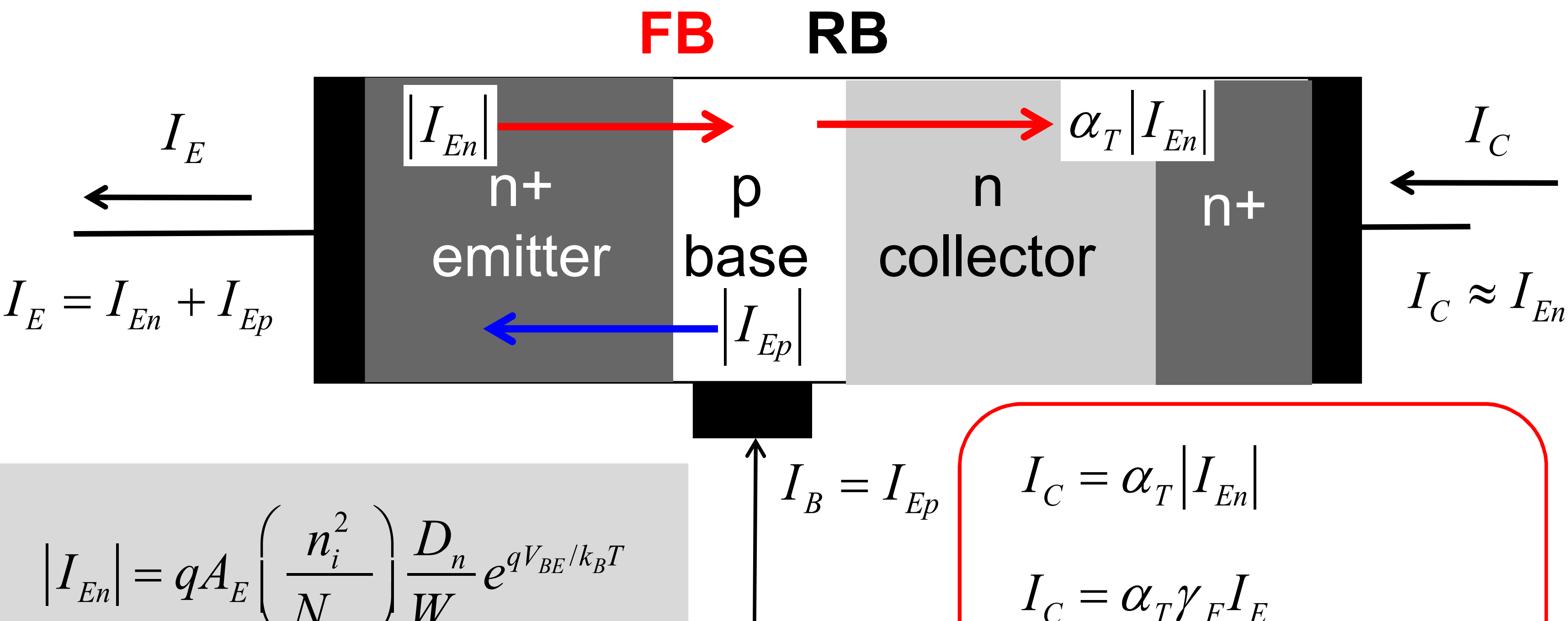
$$|I_{Ep}| = qA_E\left(\frac{n_i^2}{N_{DE}}\right)\frac{D_p}{W_E}e^{qV_{BE}/k_BT}$$

$$I_E = |I_{En}| + |I_{Ep}|$$

$$I_E = I_{F0}\left(e^{qV_{BE}/k_BT} - 1\right)$$

$$I_{F0} = qA_E\left(\frac{n_i^2}{N_{AB}}\right)\frac{D_n}{W_B} + qA_E\left(\frac{n_i^2}{N_{DE}}\right)\frac{D_p}{W_E}$$

collector current: forward active region



$$|I_{En}| = qA_E \left(\frac{n_i^2}{N_{AB}} \right) \frac{D_n}{W_B} e^{qV_{BE}/k_B T}$$

$$|I_{Ep}| = qA_E \left(\frac{n_i^2}{N_{DE}} \right) \frac{D_p}{W_E} e^{qV_{BE}/k_B T}$$

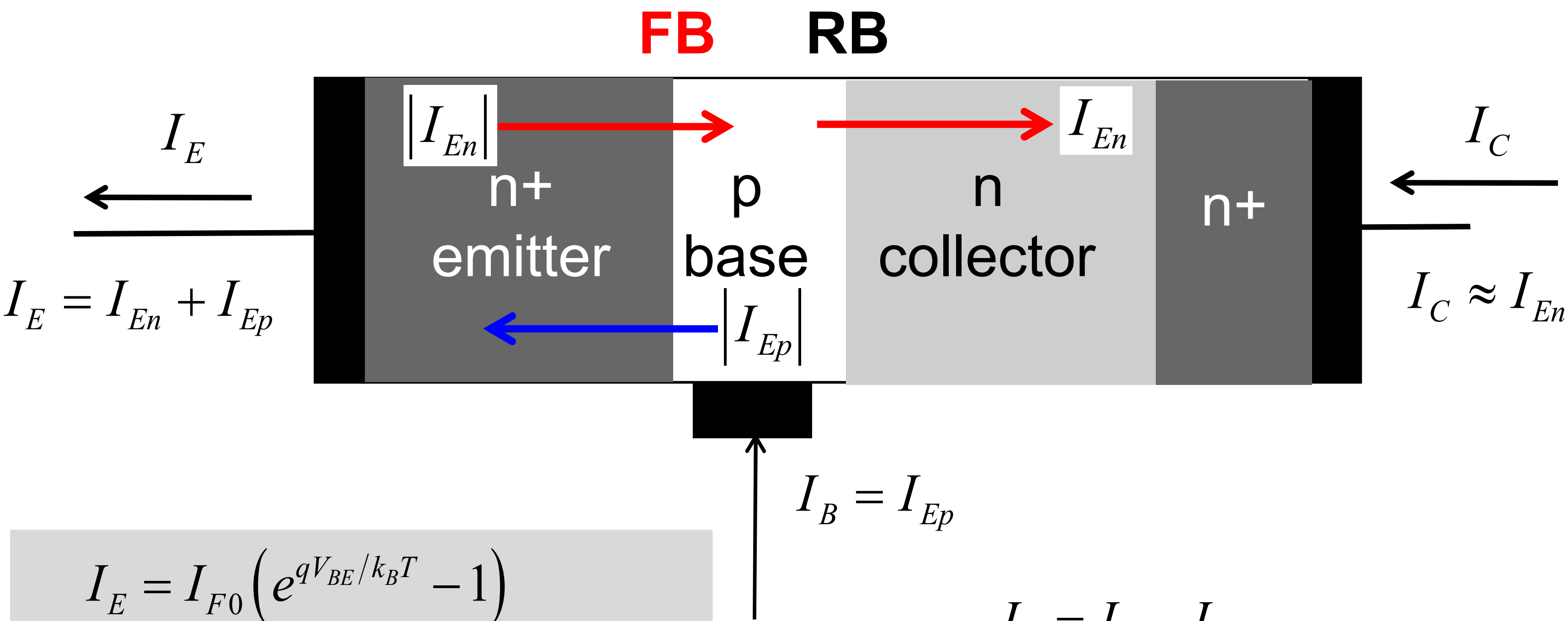
$$I_C = \alpha_T |I_{En}|$$

$$I_C = \alpha_T \gamma_F I_E$$

$$I_C = \alpha_F I_{F0} \left(e^{qV_{BE}/k_B T} - 1 \right)$$

$$\alpha_F = \alpha_T \gamma_F$$

base current: forward active region



$$I_E = I_{F0}\left(e^{qV_{BE}/k_BT} - 1\right)$$

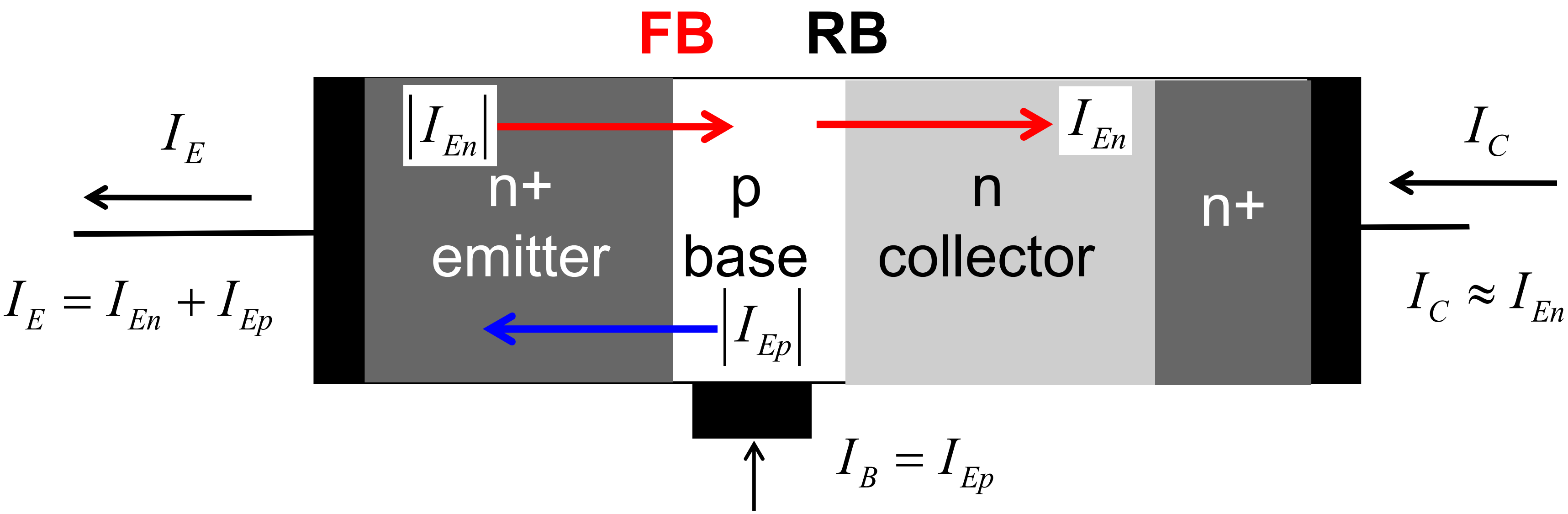
$$I_C = \alpha_F I_{F0}\left(e^{qV_{BE}/k_BT} - 1\right)$$

$$\alpha_F = \alpha_T \gamma_F$$

$$I_B = I_E - I_C$$

$$I_B = \left(1 - \alpha_F\right) I_{F0}\left(e^{qV_{BE}/k_BT} - 1\right)$$

summary: forward active region



$$I_E = I_{F0}\left(e^{qV_{BE}/k_BT} - 1\right)$$

$$I_C = \alpha_F I_{F0}\left(e^{qV_{BE}/k_BT} - 1\right)$$

$$I_B = \left(1 - \alpha_F\right) I_{F0}\left(e^{qV_{BE}/k_BT} - 1\right)$$

$$\alpha_F = \alpha_T \gamma_F$$

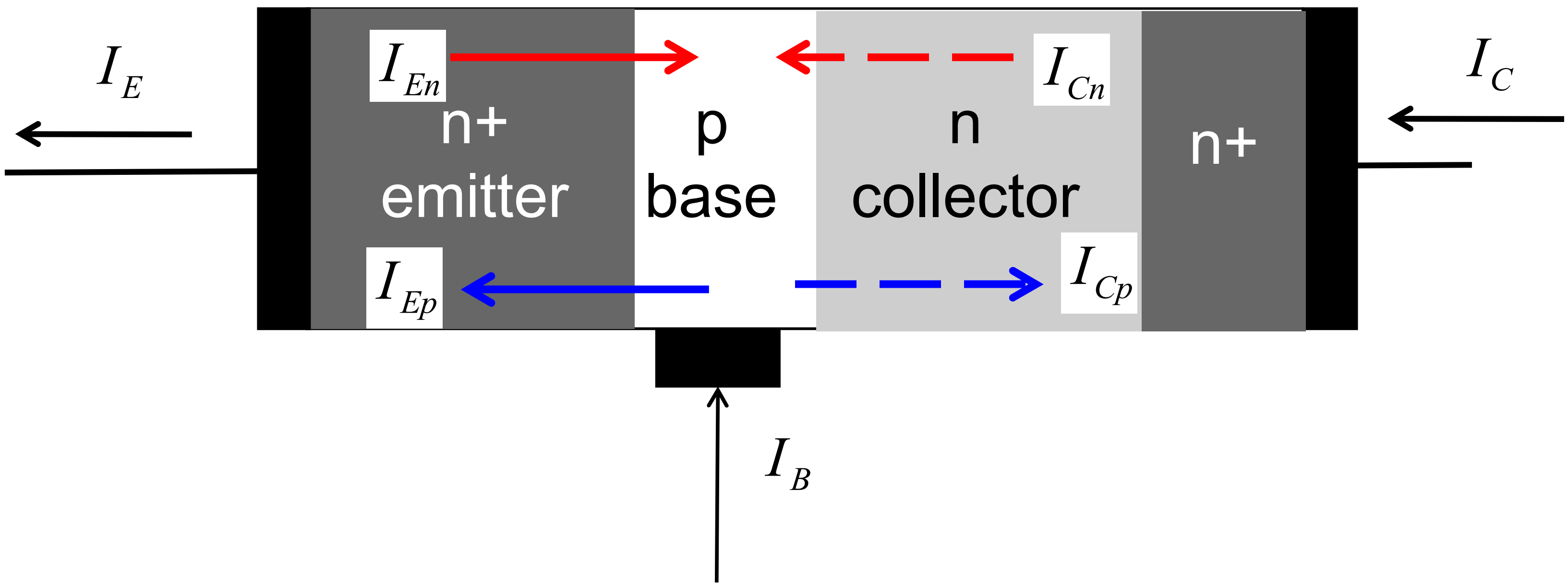
$$I_{F0} = qA_E\left(\frac{n_i^2}{N_{AB}}\right)\frac{D_n}{W_B} + qA_E\left(\frac{n_i^2}{N_{DE}}\right)\frac{D_p}{W_E}$$

Ebers-Moll model

Question:

How do we describe the BJT in **any** region of operation?

emitter-base junction (the **forward** diode)



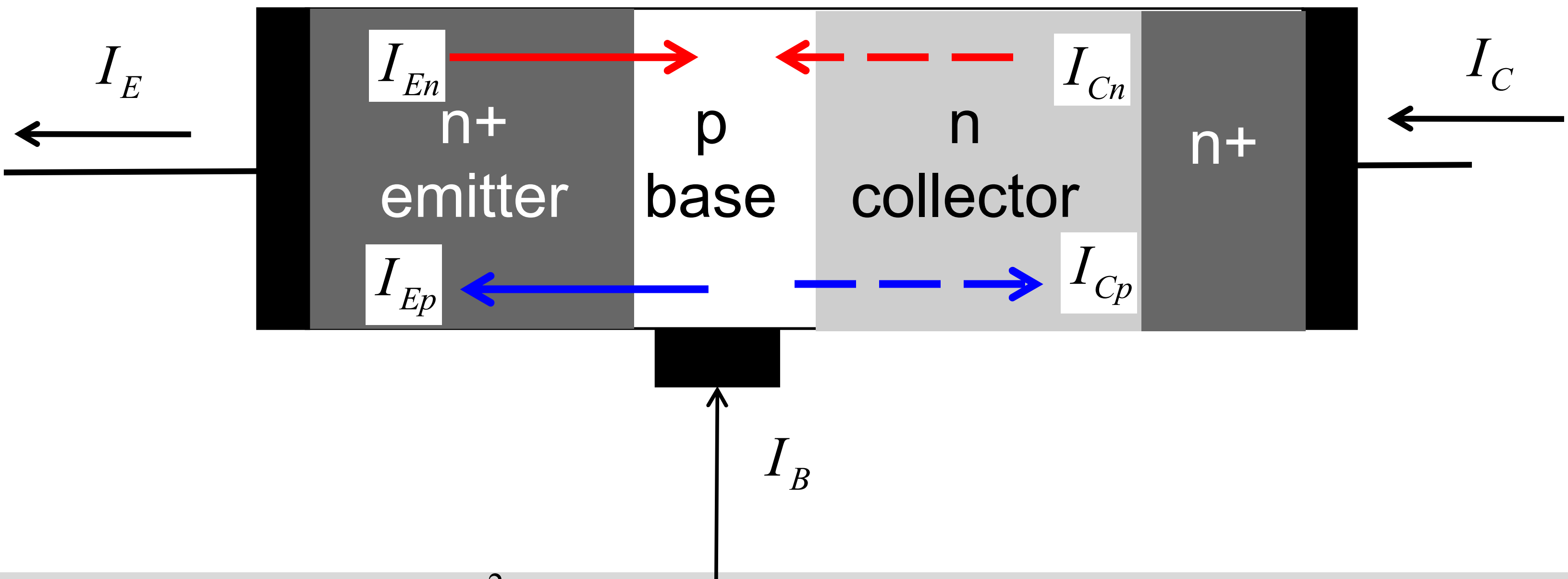
$$I_{En} = -qA\frac{D_n}{W_B}\frac{n_i^2}{N_{AB}}\left(e^{qV_{BE}/k_BT} - 1\right)$$

$$I_{Ep} = -qA\frac{D_p}{W_E}\frac{n_i^2}{N_{DE}}\left(e^{qV_{BE}/k_BT} - 1\right)$$

$$I_E\left(V_{BE}\right) = -I_{En}\left(V_{BE}\right) - I_{Ep}\left(V_{BE}\right)$$

$$I_E\left(V_{BE}\right) = I_{F0}\left(e^{qV_{BE}/k_BT} - 1\right)$$

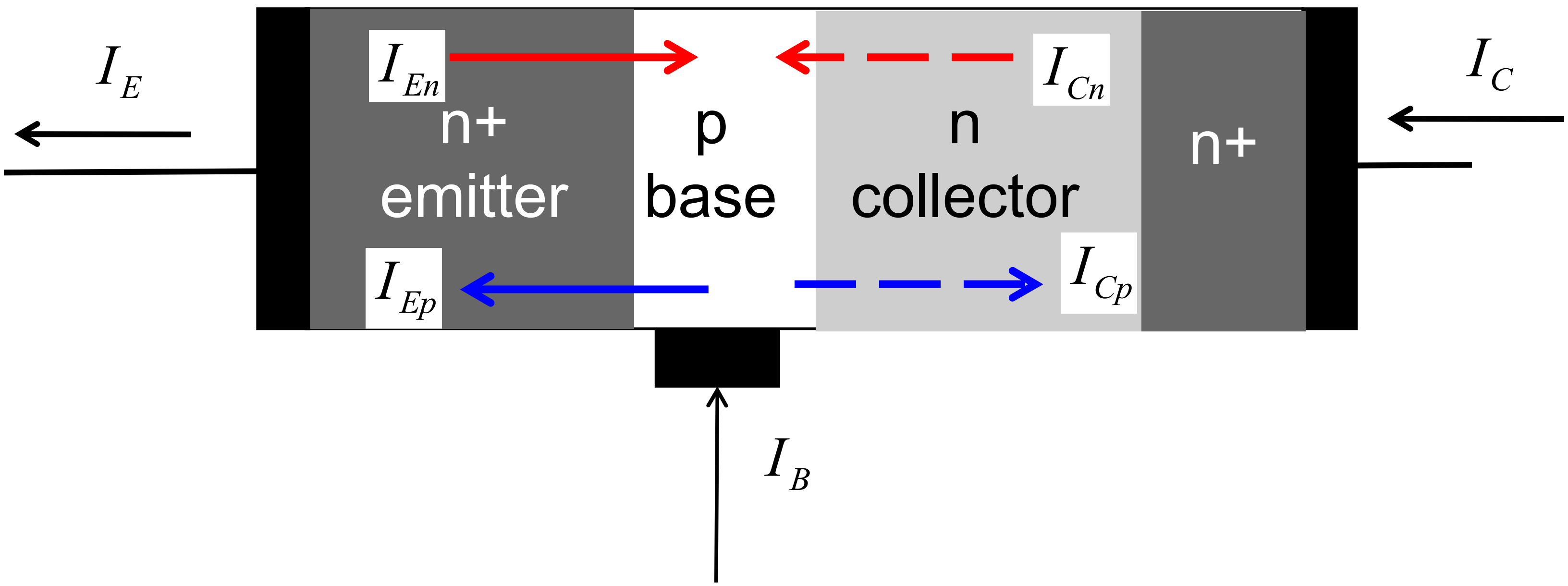
Base-collector junction (the **reverse** diode)



$$I_{Cn}(V_{BC}) = qA\frac{D_n}{W_B}\frac{n_i^2}{N_{AB}}\left(e^{qV_{BC}/k_BT} - 1\right) \qquad I_C(V_{BC}) = -\left[I_{Cn}(V_{BC}) + I_{Cp}(V_{BC})\right]$$

$$I_{Cp}(V_{BC}) = qA\frac{D_p}{W_C}\frac{n_i^2}{N_{DC}}\left(e^{qV_{BC}/k_BT} - 1\right) \qquad I_C(V_{BC}) = -I_{R0}\left(e^{qV_{BC}/k_BT} - 1\right)$$

Both junctions….



$$I_C(V_{BC}) = -I_{R0}\left(e^{qV_{BC}/k_BT} - 1\right)$$

$$I_E(V_{BE}) = I_{F0}\left(e^{qV_{BE}/k_BT} - 1\right)$$

But….
The two junctions are coupled!

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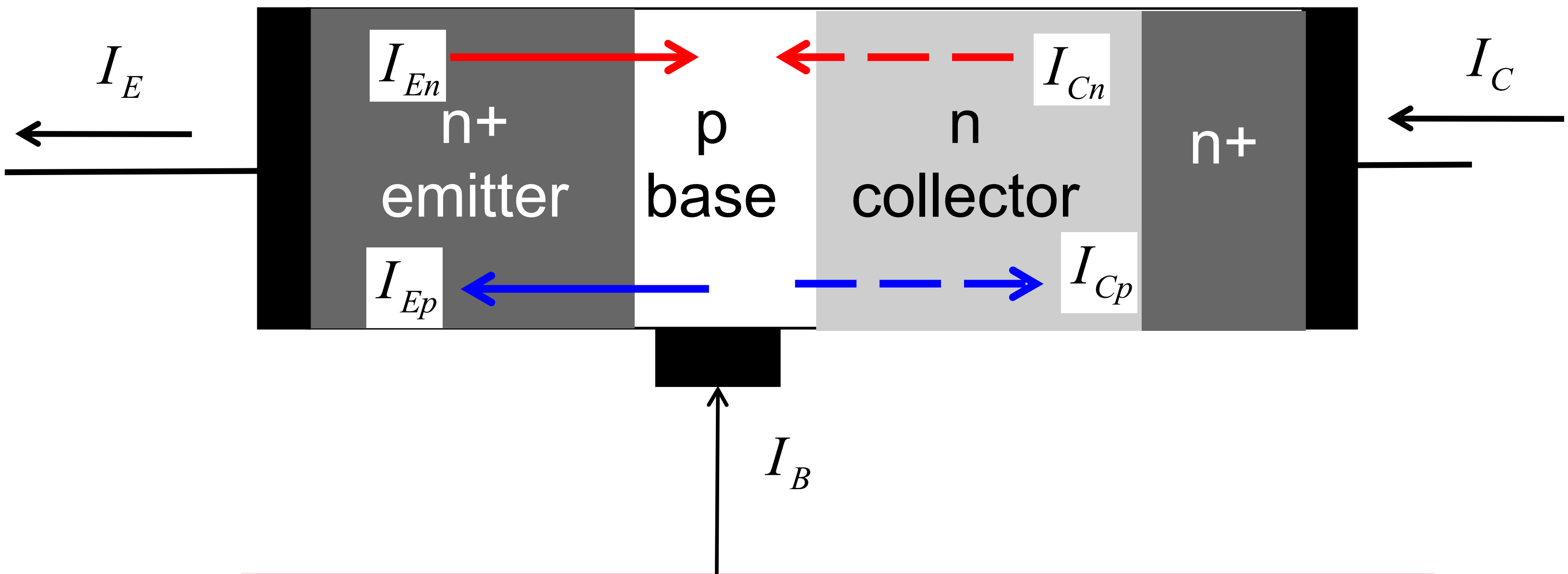
Ebers-Moll model



$$I_C(V_{BE},V_{BC}) = \alpha_F I_{F0}\left(e^{qV_{BE}/k_BT}-1\right) - I_{R0}\left(e^{qV_{BC}/k_BT}-1\right)$$

$$I_E(V_{BE},V_{BC}) = I_{F0}\left(e^{qV_{BE}/k_BT}-1\right) - \alpha_R I_{R0}\left(e^{qV_{BC}/k_BT}-1\right)$$

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Ebers-Moll model

$$I_C\left(V_{BE},V_{BC}\right)=\alpha_F I_{F0}\left(e^{qV_{BE}/k_BT}-1\right)-I_{R0}\left(e^{qV_{BC}/k_BT}-1\right)$$

$$I_E\left(V_{BE},V_{BC}\right)=I_{F0}\left(e^{qV_{BE}/k_BT}-1\right)-\alpha_R I_{R0}\left(e^{qV_{BC}/k_BT}-1\right)$$

$$I_B\left(V_{BE},V_{BC}\right)=I_E\left(V_{BE},V_{BC}\right)-I_C\left(V_{BE},V_{BC}\right)$$

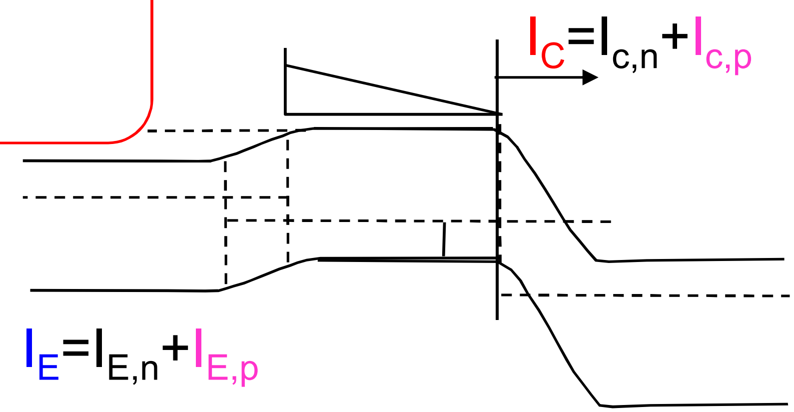
$$\alpha_F I_{F0}=\alpha_R I_{R0}$$

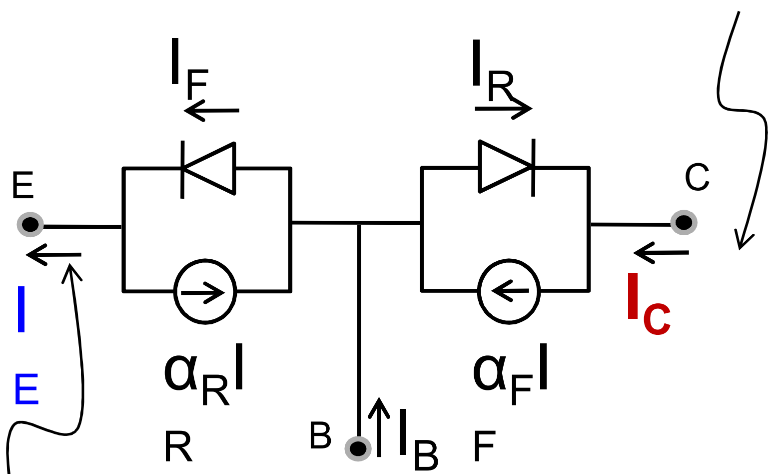
See Pierret SDF, Chapter 11, sec. 11.1.4

Ebers Moll Model

$$I_C = -A\frac{qD_n}{W_B}\frac{n_{i,B}^2}{N_B}\left(e^{qV_{BE}\beta}-1\right) + A\left[\frac{qD_n}{W_B}\frac{n_{i,B}^2}{N_B} + \frac{qD_p}{W_C}\frac{n_{i,C}^2}{N_C}\right]\left(e^{qV_{BC}\beta}-1\right)$$

$$\equiv \alpha_F I_{F0}\left(e^{qV_{BE}\beta}-1\right) - I_{R0}\left(e^{qV_{BC}\beta}-1\right)$$





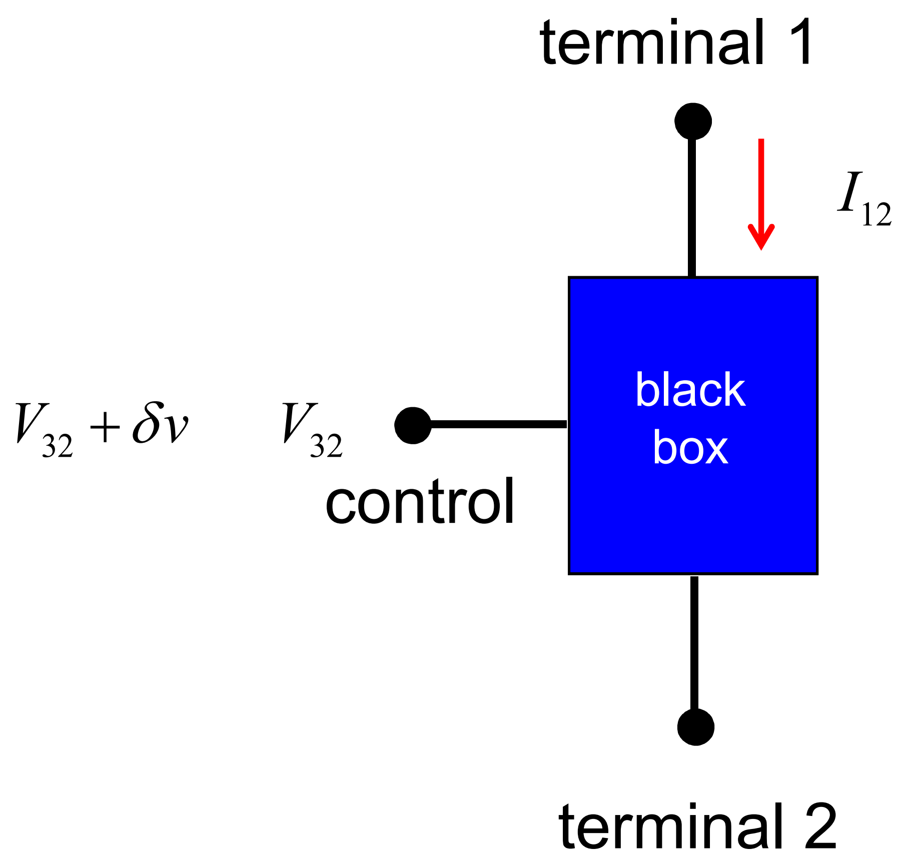
$$I_F = I_{F0}\left(e^{qV_{BE}\beta}-1\right)$$

$$I_R = I_{R0}\left(e^{qV_{BC}\beta}-1\right)$$

$$I_E = -A\left[\frac{qD_p}{W_E}\frac{n_{i,E}^2}{N_E} + \frac{qD_n}{W_B}\frac{n_{i,B}^2}{N_B}\right]\left(e^{qV_{BE}\beta}-1\right) + A\frac{qD_n}{W_E}\frac{n_{i,B}^2}{N_B}\left(e^{qV_{BC}\beta}-1\right)$$

$$\equiv I_{F0}\left(e^{qV_{BE}\beta}-1\right) - \alpha_R I_{R0}\left(e^{qV_{BC}\beta}-1\right)$$

transconductance



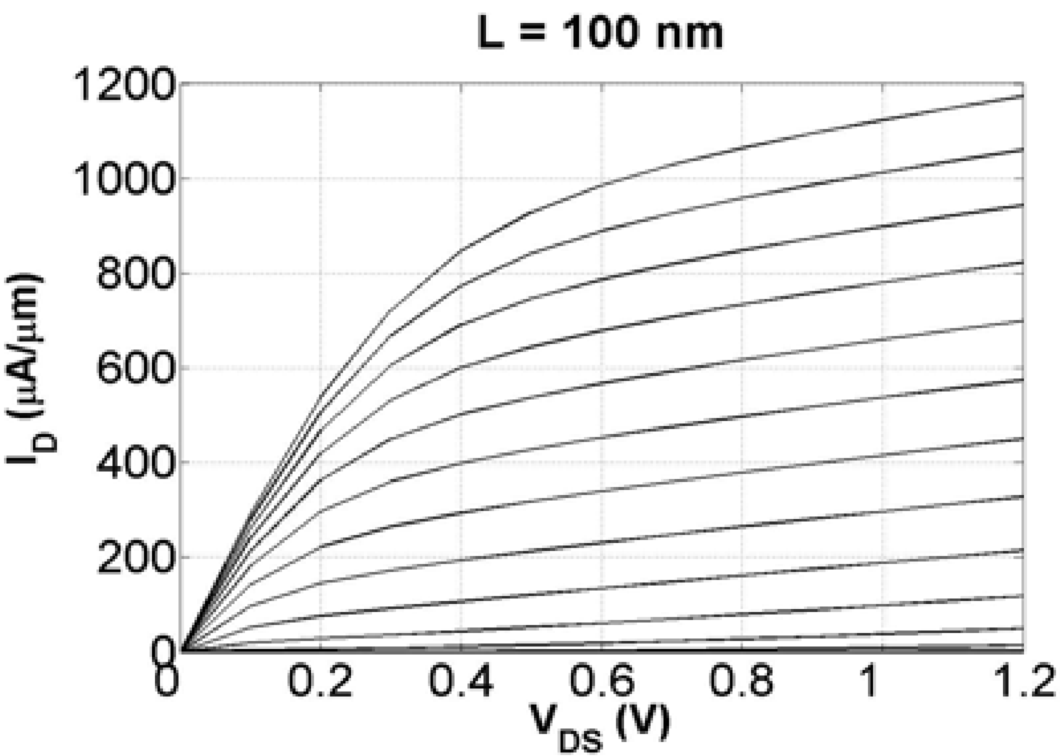
$$I_{12} + \delta i$$

$$I_{12} + g_m \delta v$$

$$g_m = \frac{\delta i}{\delta v}$$

$$g_m \equiv \frac{\partial I_{12}}{\partial V_{12}} \, \frac{\text{A}}{\text{V}} (\text{S})$$

transconductance



$V_{GS} = 1.2 \text{ V}$

$V_{GS} = 1.1 \text{ V}$

$$g_m = \left.\frac{\Delta I_D}{\Delta V_{GS}}\right|_{V_{DS}} \quad \frac{\text{A}}{\text{V}} = \text{S}$$

$$g_m = 1000 \text{ mS/mm}$$

$$g_m = \frac{(1180 - 1080) \times 10^{-6}}{0.1} = 0.001 \text{ S}/\mu\text{m} \qquad g_m = 1000 \ \mu\text{S}/\mu\text{m}$$

MOSFETs vs. BJTs

MOS (saturated)

$$g_m = \partial I_D / \partial V_{GS}\big|_{V_{DS}}$$

$$I_D = WC_{ox}\langle \upsilon(0)\rangle \left(V_{GS} - V_T\right)$$

$$g_m = WC_{ox}\langle \upsilon(0)\rangle$$

$$g_m / I_D = 1/\left(V_{GS} - V_T\right)$$

$$g_m / I_D = 1/\left(1.0 - 0.2\right) \approx 1.25 \text{ V}^{-1}$$

Bipolar (active)

$$g_m = \partial I_C / \partial V_{BE}\big|_{V_{CE}}$$

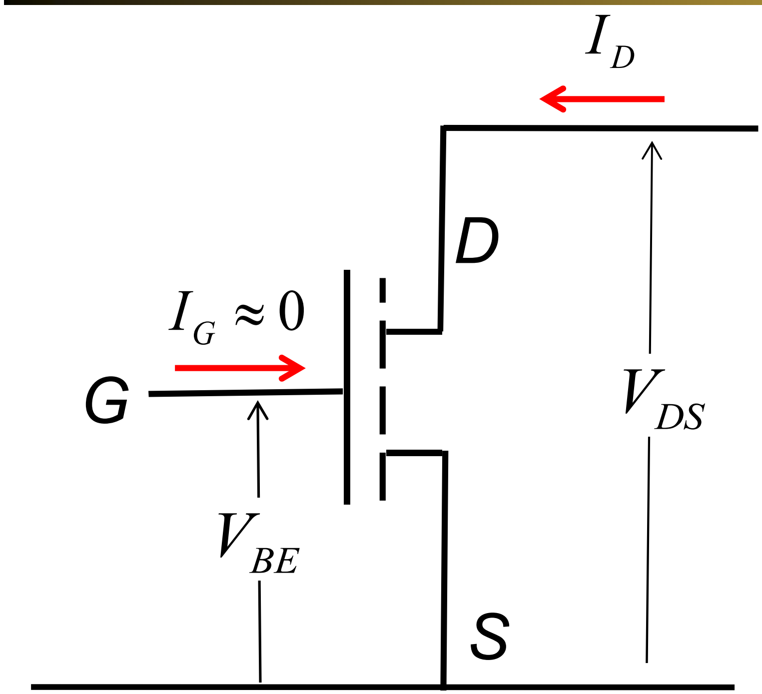
$$I_C = \alpha_F I_{F0} e^{qV_{BE}/k_B T}$$

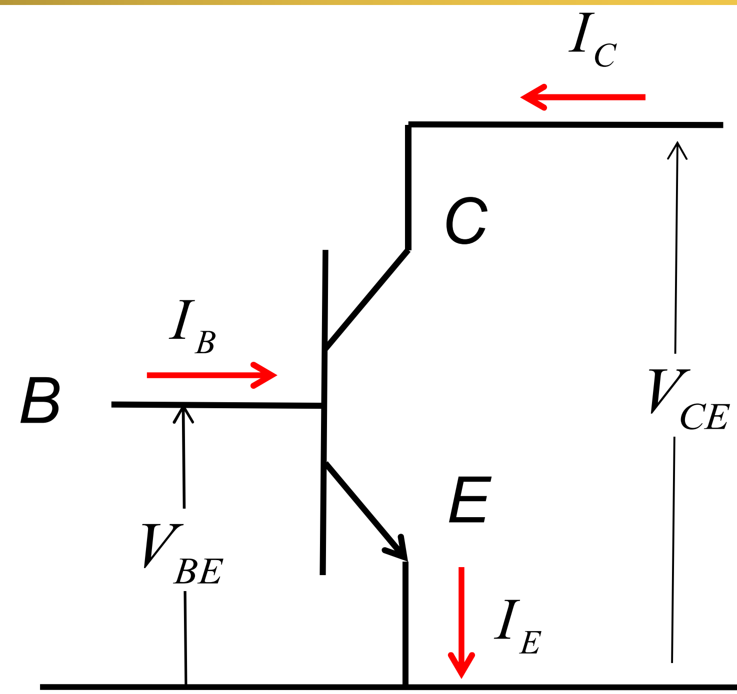
$$g_m = I_C / \left(k_B T / q\right)$$

$$g_m / I_C = 1/\left(k_B T / q\right)$$

$$g_m / I_C = 1/\left(0.026\right) \approx 40 \text{ V}^{-1}$$

MOSFETs vs. BJTs





MOSFET characteristics:

- simple to make
- no gate current
- moderate gm
- low capacitance

BJT characteristics:

- more complex to make
- base current
- large gm
- high capacitance

active region example 1 (NPN)

$$A_E = 10\ \mu\text{m} \times \mu\text{m}$$

$$I_C = 50\ \mu\text{A}$$

$N_{DE} = 1.5 \times 10^{19}\ \text{cm}^{-3}$	$W_B = 0.50\ \mu\text{m}$	$\tau_{pE} = 0.1\ \text{ns}$
$N_{AB} = 1.0 \times 10^{17}\ \text{cm}^{-3}$	$W_B = 0.25\ \mu\text{m}$	$\tau_{nB} = 75\ \text{ns}$
$N_{DC} = 2.0 \times 10^{16}\ \text{cm}^{-3}$	$W_C = 1.50\ \mu\text{m}$	$\tau_{pC} = 150\ \text{ns}$

Step 1: diffusion coefficients and lengths

$$N_{DE} = 1.5 \times 10^{19} \text{ cm}^{-3} \qquad W_E = 0.50\ \mu\text{m} \qquad \tau_{pE} = 0.1 \text{ ns}$$

$$N_{AB} = 1.0 \times 10^{17} \text{ cm}^{-3} \qquad W_B = 0.25\ \mu\text{m} \qquad \tau_{nB} = 75 \text{ ns}$$

$$N_{DC} = 2.0 \times 10^{16} \text{ cm}^{-3} \qquad W_C = 1.50\ \mu\text{m} \qquad \tau_{pC} = 150 \text{ ns}$$

Pierret, SDF, Fig. 3.5a, p. 80

$$\mu_{pE} \approx 70 \text{ cm}^2/\text{V-s} \qquad D_{pE} = \frac{k_B T}{q}\mu_{pE} \approx 1.8 \text{ cm}^2/\text{s} \qquad L_{pE} = \sqrt{D_{pE}\tau_{pE}} \approx 0.13\ \mu\text{m}$$

$$\mu_{nB} \approx 800 \text{ cm}^2/\text{V-s} \qquad D_{nB} \approx 20.8 \text{ cm}^2/\text{s} \qquad L_{nB} \approx 12.5\ \mu\text{m}$$

$$\mu_{pC} \approx 420 \text{ cm}^2/\text{V-s} \qquad D_{pC} \approx 10.9 \text{ cm}^2/\text{s} \qquad L_{pC} \approx 12.8\ \mu\text{m}$$

find the emitter injection efficiency

$$\gamma_F = \frac{1}{1 + \frac{D_{pE}}{D_{nB}} \frac{W_B}{W_E} \frac{N_{AB}}{N_{DE}}}$$

$$\gamma_F = \frac{1}{1 + 1.1 \times 10^{-3}} = 0.9989$$

find the base transport factor

$$\alpha_T = \frac{1}{1 + \frac{1}{2}\left(\frac{W_B}{L_{nB}}\right)^2}$$

$$\alpha_T = \frac{1}{1 + 2.0 \times 10^{-4}} = 0.9998$$

find beta

$$\beta_F = \frac{\gamma_F \alpha_T}{1-\gamma_F \alpha_T} = \frac{\alpha_F}{1-\alpha_F}$$

$$\beta_F = \frac{0.9989 \times 0.9998}{1-0.9989 \times 0.9998} = \frac{0.9987}{1-0.9987} = 768$$

find the base current

$$I_C = 50\ \mu\text{A}$$

$$\beta_F = 768$$

$$I_B = I_C / \beta_F$$

$$I_B = I_C / \beta_F = 65\ \text{nA}$$

$$\alpha_F = \frac{\beta_F}{\beta_F + 1} = 0.9987$$

Forward saturation current density

$$I_C\left(V_{BE},V_{BC}\right)=\alpha_F I_{F0}\left(e^{qV_{BE}/k_BT}-1\right)-I_{R0}\left(e^{qV_{BC}/k_BT}-1\right)$$

$$I_E\left(V_{BE},V_{BC}\right)=I_{F0}\left(e^{qV_{BE}/k_BT}-1\right)-\alpha_R I_{R0}\left(e^{qV_{BC}/k_BT}-1\right)$$

$$I_{F0}=qA\left(\frac{D_{nB}}{W_B}\frac{n_i^2}{N_{AB}}+\frac{D_{pE}}{W_E}\frac{n_i^2}{N_{DE}}\right)\rightarrow qA\left(\frac{D_{nB}}{W_B}\frac{n_i^2}{N_{AB}}+\frac{D_{pE}}{L_{pE}}\frac{n_i^2}{N_{DE}}\right)$$

$$I_{F0}=1.33\times10^{-16}\text{ A}$$

Reverse saturation current density

$$I_C\left(V_{BE},V_{BC}\right)=\alpha_F I_{F0}\left(e^{qV_{BE}/k_BT}-1\right)-I_{R0}\left(e^{qV_{BC}/k_BT}-1\right)$$

$$I_E\left(V_{BE},V_{BC}\right)=I_{F0}\left(e^{qV_{BE}/k_BT}-1\right)-\alpha_R I_{R0}\left(e^{qV_{BC}/k_BT}-1\right)$$

$$I_{R0}=qA\left(\frac{D_{nB}}{W_B}\frac{n_i^2}{N_{AB}}+\frac{D_{pC}}{W_C}\frac{n_i^2}{N_{DC}}\right)$$

$$I_{R0}=1.91\times10^{-16}\,\mathrm{A}$$

Ebers-Moll parameters summary

$$I_C\left(V_{BE},V_{BC}\right)=\alpha_F I_{F0}\left(e^{qV_{BE}/k_BT}-1\right)-I_{R0}\left(e^{qV_{BC}/k_BT}-1\right)$$

$$I_E\left(V_{BE},V_{BC}\right)=I_{F0}\left(e^{qV_{BE}/k_BT}-1\right)-\alpha_R I_{R0}\left(e^{qV_{BC}/k_BT}-1\right)$$

$$\alpha_R=\alpha_F\frac{I_{F0}}{I_{R0}}$$

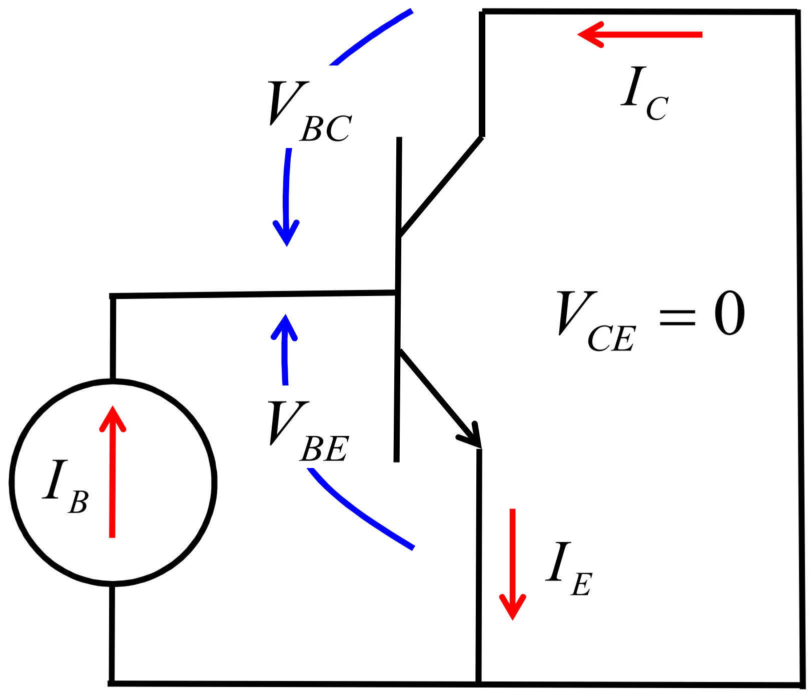
$$\alpha_F=0.9987$$

$$I_{F0}=1.33\times10^{-16}\,\mathrm{A}$$

$$\alpha_R=0.70 \qquad \beta_R=\frac{\alpha_R}{1-\alpha_R}=2.3$$

$$I_{R0}=1.91\times10^{-16}\,\mathrm{A}$$

What is I_C?



$$I_C = \alpha_F I_{F0}\left(e^{qV_{BE}/k_BT} - 1\right) - I_{R0}\left(e^{qV_{BC}/k_BT} - 1\right)$$

$$I_E = I_{F0}\left(e^{qV_{BE}/k_BT} - 1\right) - \alpha_R I_{R0}\left(e^{qV_{BC}/k_BT} - 1\right)$$

$$I_B = 50 \text{ nA}$$

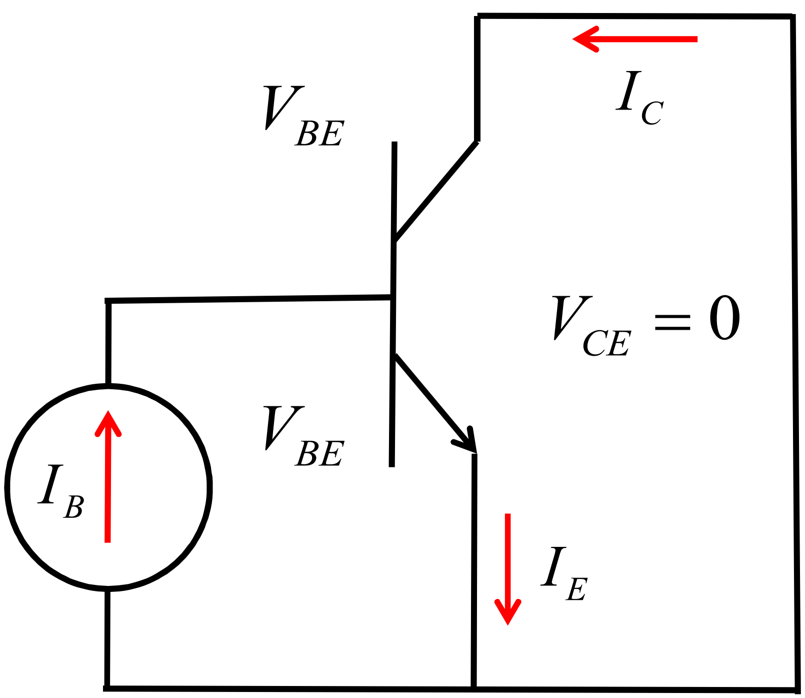
$$V_{CE} = V_{BE} - V_{BC} = 0$$

$$V_{BC} = V_{BE}$$

$$I_C = \left(\alpha_F I_{F0} - I_{R0}\right)\left(e^{qV_{BE}/k_BT} - 1\right)$$

$$I_E = \left(I_{F0} - \alpha_R I_{R0}\right)\left(e^{qV_{BE}/k_BT} - 1\right)$$

What is I_C?



$I_B = 50\text{ nA}$

$$I_C = \left(\alpha_F I_{F0} - I_{R0}\right)\left(e^{qV_{BE}/k_BT} - 1\right)$$

$$I_E = \left(I_{F0} - \alpha_R I_{R0}\right)\left(e^{qV_{BE}/k_BT} - 1\right)$$

$$I_B = I_E - I_C$$

$$I_B = \left[\left(1-\alpha_F\right)I_{F0} + \left(1-\alpha_F\right)I_{R0}\right]\left(e^{qV_{BE}/k_BT} - 1\right)$$

$$\left(e^{qV_{BE}/k_BT} - 1\right) = \frac{I_B}{\left[\left(1-\alpha_F\right)I_{F0} + \left(1-\alpha_F\right)I_{R0}\right]}$$

What is I_C?

$$I_C = \left\{ \frac{\left(\alpha_F I_{F0} - I_{R0}\right)}{\left(1-\alpha_F\right) I_{F0} + \left(1-\alpha_F\right) I_{R0}} \right\} I_B$$

$I_B = 50 \text{ nA}$

$I_{F0} = 1.33 \times 10^{-16} \text{ A}$

$I_C = -1.01 \times I_B$

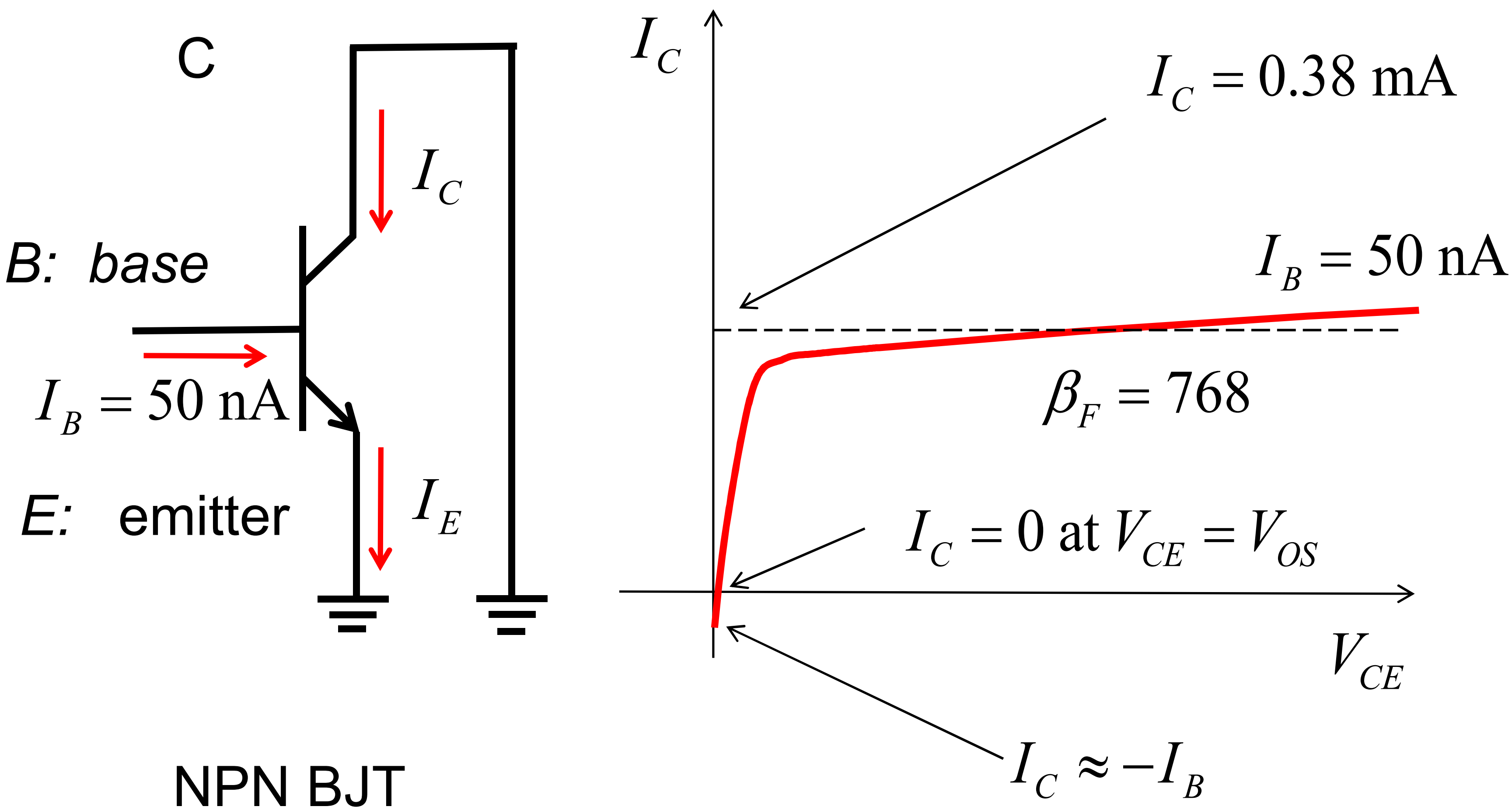
$I_{R0} = 1.91 \times 10^{-16} \text{ A}$

$I_B = -50 \text{ nA}$

$\alpha_F = 0.9987$

$\alpha_R = 0.70$

result


C
I_C
B: base
$I_B = 50$ nA
E: emitter
I_E
NPN BJT
I_C
$I_C = 0.38$ mA
$I_B = 50$ nA
$\beta_F = 768$
$I_C = 0$ at $V_{CE} = V_{OS}$
V_{CE}
$I_C \approx -I_B$

Why?

Conclusions

- BJT current is described through a pair of coupled diodes and current sources facing back-to-back: the Ebers-Moll model
- The Ebers-Moll model can be solved for key performance parameters quickly and self-consistently
- Current gain can be very substantial, under the correct circumstances