

ECE-305: Key Equations (for exam 2)
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Physical constants:

$$\hbar = 1.055 \times 10^{-34} \text{ [J-s]}$$

$$m_0 = 9.109 \times 10^{-31} \text{ [kg]}$$

$$k_B = 1.380 \times 10^{-23} \text{ [J/K]}$$

$$q = 1.602 \times 10^{-19} \text{ [C]}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ [F/m]}$$

Silicon parameters ($T = 300\text{K}$)

$$N_C = 3.23 \times 10^{19} \text{ cm}^{-3}$$

$$N_V = 1.83 \times 10^{19} \text{ cm}^{-3}$$

$$n_i = 1 \times 10^{10} \text{ cm}^{-3}$$

$$K_S = 11.8$$

$$\text{DOS: } g_C(E) = \frac{(m_n^*)^{3/2} \sqrt{2(E - E_C)}}{\pi^2 \hbar^3} \quad \text{FF: } f(E) = \frac{1}{1 + e^{(E - E_F)/k_B T}} \quad n_i = \sqrt{N_C N_V} e^{-E_G/2k_B T}$$

Equilibrium Carrier densities:

$$n_0 = N_C e^{(E_F - E_C)/k_B T} \text{ m}^{-3} \quad N_C = \frac{1}{4} \left(\frac{2m_n^* k_B T}{\pi \hbar^2} \right)^{3/2} \text{ m}^{-3} \quad n_0 = n_i e^{(E_F - E_i)/k_B T}$$

$$p_0 = N_V e^{(E_V - E_F)/k_B T} \text{ m}^{-3} \quad N_V = \frac{1}{4} \left(\frac{2m_p^* k_B T}{\pi \hbar^2} \right)^{3/2} \text{ m}^{-3} \quad p_0 = n_i e^{(E_i - E_F)/k_B T}$$

Space charge neutrality: $p - n + N_D^+ - N_A^- = 0$ **Law of Mass Action:** $n_0 p_0 = n_i^2$

Conductivity and resistivity: $\sigma = (\sigma_n + \sigma_p) = q(n\mu_n + p\mu_p) = 1/\rho$

Current equations: $J_n = n\mu_n \frac{dF_n}{dx}$ $J_n = nq\mu_n \mathcal{E}_x + qD_n \frac{dn}{dx}$ $D_n/\mu_n = k_B T/q$
 $J_p = p\mu_p \frac{dF_p}{dx}$ $J_p = pq\mu_p \mathcal{E}_x - qD_p \frac{dp}{dx}$ $D_p/\mu_p = k_B T/q$

Recombination: SRH: $R = \Delta n/\tau_n \text{ m}^{-3}\text{s}^{-1}$ or $R = \Delta p/\tau_p \text{ m}^{-3}\text{s}^{-1}$

Semiconductor Equations:

$$\frac{\partial n}{\partial t} = -\nabla \cdot \left(\frac{\vec{J}_n}{-q} \right) + G_n - R_n$$

$$\frac{\partial p}{\partial t} = -\nabla \cdot \left(\frac{\vec{J}_p}{q} \right) + G_p - R_p$$

$$0 = -\nabla \cdot (\epsilon \vec{E}) + \rho$$

Minority Carrier Diffusion Equation:

$$\frac{\partial \Delta p}{\partial t} = D_p \frac{\partial^2 \Delta p}{\partial x^2} - \frac{\Delta p}{\tau_p} + G_L$$

$$L_p = \sqrt{D_p \tau_p}$$

Non-equilibrium: $E_F \rightarrow F_n(x), F_p(x)$

$$n_0 p_0 = n_i^2 \rightarrow np = n_i^2 e^{(F_n - F_p)/k_B T}$$

$$n_0 = N_c e^{(E_F - E_C)/k_B T} \rightarrow n = N_c e^{(F_n - E_C)/k_B T}$$

$$p_0 = N_v e^{(E_V - E_F)/k_B T} \rightarrow p = N_v e^{(E_V - F_p)/k_B T}$$

PN junction electrostatics:

$$V_{bi} = \frac{k_B T}{q} \ln \left(\frac{N_D N_A}{n_i^2} \right) \quad \frac{d\mathcal{E}}{dx} = \frac{\rho(x)}{K_s \epsilon_0}$$

$$W = \left[\frac{2K_s \epsilon_0}{q} \left(\frac{N_A + N_D}{N_D N_A} \right) V_{bi} \right]^{1/2} \quad x_n = \frac{N_A}{N_A + N_D} W \quad x_p = \frac{N_D}{N_A + N_D} W \quad \mathcal{E}(0) = \sqrt{\frac{2qV_{bi}}{K_s \epsilon_0} \left(\frac{N_D N_A}{N_A + N_D} \right)}$$

Diode Current:

$$\Delta n(0) = \frac{n_i^2}{N_A} (e^{qV_A/k_B T} - 1) \quad \Delta p(0) = \frac{n_i^2}{N_D} (e^{qV_A/k_B T} - 1)$$

$$I_D = I_0 (e^{qV_A/k_B T} - 1) \quad I_0 = qA \left(\frac{D_n}{L_n} \frac{n_i^2}{N_A} + \frac{D_p}{L_p} \frac{n_i^2}{N_D} \right) \text{ (long)} \quad I_0 = qA \left(\frac{D_n}{W_p} \frac{n_i^2}{N_A} + \frac{D_p}{W_n} \frac{n_i^2}{N_D} \right) \text{ (short)}$$

$$\text{non-ideal } I_D = I_0 (e^{q(V - IR_s)/nk_B T} - 1) \quad I_{gen} = -qA \frac{n_i}{2\tau_0} W$$

small signal: $G_d = \frac{I_D + I_0}{k_B T/q}$

$$C_J(V_R) = \frac{K_s \epsilon_0 A}{\left[\frac{2K_s \epsilon_0}{qN_A} (V_{bi} - V_A) \right]^{1/2}} \quad C_D = G_d \tau_n$$