ECE-305: Key Equations (for exam 6)
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Physical constants:

$\hbar = 1.055\times10^{-34}$ [J-s]

$m_0 = 9.109\times10^{-31}$ [kg]

$k_B = 1.380\times10^{-23}$ [J/K]

$q = 1.602\times10^{-19}$ [C]

$\varepsilon_0 = 8.854\times10^{-12}$ [F/m]

Miller Indices: (hkl) {hkl} [hkl] <hkl>

Silicon parameters (T = 300K)

$N_C = 3.23\times10^{19}$ cm^{-3}

$N_V = 1.83\times10^{19}$ cm^{-3}

$n_i = 1\times10^{10}$ cm^{-3}

$K_S = 11.8$

DOS: $g_C(E) = \dfrac{\left(m_n^*\right)^{3/2}\sqrt{2\left(E-E_C\right)}}{\pi^2\hbar^3}$ FF: $f(E) = \dfrac{1}{1+e^{(E-E_F)/k_BT}}$ $n_i = \sqrt{N_C N_V}\,e^{-E_G/2k_BT}$

Equilibrium Carrier densities:

$n_0 = N_C e^{\left(E_F-E_C\right)/k_BT}$ m^{-3} $N_C = \dfrac{1}{4}\left(\dfrac{2m_n^* k_BT}{\pi\hbar^2}\right)^{3/2}$ m^{-3} $n_0 = n_i\, e^{\left(E_F-E_i\right)/k_BT}$

$p_0 = N_V e^{\left(E_V-E_F\right)/k_BT}$ m^{-3} $N_V = \dfrac{1}{4}\left(\dfrac{2m_p^* k_BT}{\pi\hbar^2}\right)^{3/2}$ m^{-3} $p_0 = n_i e^{\left(E_i-E_F\right)/k_BT}$

Space charge neutrality: $p - n + N_D^+ - N_A^- = 0$ **Law of Mass Action:** $n_0 p_0 = n_i^2$

Conductivity and resistivity: $\sigma = \left(\sigma_n + \sigma_p\right) = q\left(n\mu_n + p\mu_p\right) = 1/\rho$

Current equations: $J_n = n\mu_n \dfrac{dF_n}{dx}$ $J_n = nq\mu_n\mathcal{E}_x + qD_n\dfrac{dn}{dx}$ $D_n/\mu_n = k_BT/q$

$J_p = p\mu_p\dfrac{dF_p}{dx}$ $J_p = pq\mu_p\mathcal{E}_x - qD_p\dfrac{dp}{dx}$ $D_p/\mu_p = k_BT/q$

Recombination: SRH: $R = \Delta n/\tau_n$ m^{-3}s^{-1} or $R = \Delta p/\tau_p$ m^{-3}s^{-1}

Semiconductor Equations:

$$\frac{\partial n}{\partial t} = -\nabla\bullet\left(\frac{\vec{J}_n}{-q}\right) + G_n - R_n$$

$$\frac{\partial p}{\partial t} = -\nabla\bullet\left(\frac{\vec{J}_p}{q}\right) + G_p - R_p$$

$$0 = -\nabla\bullet\left(\varepsilon\vec{\mathcal{E}}\right) + \rho$$

Minority Carrier Diffusion Equation:

$$\frac{\partial\Delta p}{\partial t} = D_p\frac{\partial^2\Delta p}{\partial x^2} - \frac{\Delta p}{\tau_p} + G_L$$

$$L_p = \sqrt{D_p\tau_p}$$

Non-equilibrium: $E_F \to F_n(x), F_p(x)$ $\quad n_0 p_0 = n_i^2 \to np = n_i^2 e^{(F_n - F_p)/k_B T}$

$$n_0 = N_C e^{(E_F - E_C)/k_B T} \to n = N_C e^{(F_n - E_C)/k_B T}$$

$$p_0 = N_V e^{(E_V - E_F)/k_B T} \to p = N_V e^{(E_V - F_p)/k_B T}$$

PN junction electrostatics:

$$V_{bi} = \frac{k_B T}{q} \ln\left(\frac{N_D N_A}{n_i^2}\right) \qquad \frac{d\mathcal{E}}{dx} = \frac{\rho(x)}{K_S \varepsilon_0}$$

$$W = \left[\frac{2K_S\varepsilon_0}{q}\left(\frac{N_A + N_D}{N_D N_A}\right)V_{bi}\right]^{1/2} \qquad x_n = \frac{N_A}{N_A + N_D}W \quad x_p = \frac{N_D}{N_A + N_D}W \qquad \mathcal{E}(0) = \sqrt{\frac{2qV_{bi}}{K_s\varepsilon_0}\left(\frac{N_D N_A}{N_A + N_D}\right)}$$

Diode Current:

$$\Delta n(0) = \frac{n_i^2}{N_A}\left(e^{qV_A/k_B T} - 1\right) \qquad \Delta p(0) = \frac{n_i^2}{N_D}\left(e^{qV_A/k_B T} - 1\right)$$

$I_D = I_0\left(e^{qV_A/k_B T} - 1\right) \qquad I_0 = qA\left(\frac{D_n}{L_n}\frac{n_i^2}{N_A} + \frac{D_p}{L_p}\frac{n_i^2}{N_D}\right)$ (long) $\quad I_0 = qA\left(\frac{D_n}{W_P}\frac{n_i^2}{N_A} + \frac{D_p}{W_n}\frac{n_i^2}{N_D}\right)$ (short)

non-ideal $I_D = I_0\left(e^{q(V - IR_S)/nk_B T} - 1\right) \qquad I_{gen} = -qA\frac{n_i}{2\tau_0}W$

small signal: $G_d = \dfrac{I_D + I_0}{k_B T/q} \qquad C_J(V_R) = \dfrac{K_S\varepsilon_0 A}{\left[\dfrac{2K_S\varepsilon_0}{qN_A}\left(V_{bi} - V_A\right)\right]^{1/2}} \qquad C_D = G_d\,\tau_n$

MS Diodes: $qV_{bi} = \left|\Phi_M - \Phi_S\right| \qquad \Phi_{BP} = \chi + E_G - \Phi_M \qquad \Phi_{BN} = \Phi_M - \chi$

$$J = J_0\left(e^{qV_A/k_B T} - 1\right) \qquad J_0 = A^* T^2 e^{-\Phi_B/k_B T} \qquad A^* = \frac{4\pi q m^* k_B^2}{h^3}$$

MOS Capacitors:

$$W = \sqrt{\frac{2K_S\varepsilon_0\phi_S}{qN_A}}\ \text{cm} \qquad \mathcal{E}_S = \sqrt{\frac{2qN_A\phi_S}{K_S\varepsilon_0}}\ \text{V/cm} \qquad Q_B = -qN_A W(\phi_S) = -\sqrt{2q\,K_S\varepsilon_0 N_A\phi_S}\ \text{C/cm}^2$$

$$V_G = V_{FB} + \phi_S + \Delta\phi_{ox} = V_{FB} + \phi_S - \frac{Q_S(\phi_S)}{C_{ox}} \qquad C_{ox} = K_O\varepsilon_0/x_o \qquad V_{FB} = \Phi_{ms}/q - Q_F/C_{ox}$$

$$C = \frac{C_{ox}}{1 + \dfrac{K_O W(\phi_S)}{K_S x_o}} \qquad V_T = -\frac{Q_B(2\phi_F)}{C_{ox}} + 2\phi_F \qquad Q_n = -C_{ox}\left(V_G - V_T\right)$$

MOSFETs:

$$I_D = -WQ_n\left(y=0\right)\left\langle \upsilon_y\left(y=0\right)\right\rangle$$

$$I_D = \frac{W}{L}\mu_n C_{ox}\left(V_{GS}-V_T\right)V_{DS} \qquad I_D = W C_{ox}\,\upsilon_{sat}\left(V_{GS}-V_T\right)$$

Square Law theory:

$$I_D = \frac{W}{L}\mu_n C_{ox}\left[\left(V_{GS}-V_T\right)V_{DS} - V_{DS}^2/2\right] \quad \begin{pmatrix} 0 \le V_{DS} \le V_{GS}-V_T, \\ V_{GS} \ge V_T \end{pmatrix}$$

$$I_D = \frac{W}{2L}\mu_n C_{ox}\left(V_{GS}-V_T\right)^2 \quad \begin{pmatrix} V_{DS} > V_{GS}-V_T \\ V_{GS} \ge V_T \end{pmatrix}$$

Bipolar transistors: (NPN, short emitter, base, and collector)

Ebers-Moll equations**:**

$$I_C\left(V_{BE},V_{BC}\right) = \alpha_F I_{F0}\left(e^{qV_{BE}/k_BT}-1\right) - I_{R0}\left(e^{qV_{BC}/k_BT}-1\right)$$

$$I_E\left(V_{BE},V_{BC}\right) = I_{F0}\left(e^{qV_{BE}/k_BT}-1\right) - \alpha_R I_{R0}\left(e^{qV_{BC}/k_BT}-1\right)$$

$$I_{F0} = qA\left(\frac{D_{nB}}{W_B}\frac{n_i^2}{N_{AB}} + \frac{D_{pE}}{W_E}\frac{n_i^2}{N_{DE}}\right)$$

$$I_{R0} = qA\left(\frac{D_{nB}}{W_B}\frac{n_i^2}{N_{AB}} + \frac{D_{pC}}{W_C}\frac{n_i^2}{N_{DC}}\right)$$

$$\alpha_F = \gamma_F \alpha_T$$

$$\alpha_R = \gamma_R \alpha_T$$

$$\alpha_F I_{F0} = \alpha_R I_{R0}$$

$$\gamma_F = \frac{I_{En}}{I_{En}+I_{Ep}} = \frac{1}{1+\dfrac{D_{pE}}{D_{nB}}\dfrac{W_B}{W_E}\dfrac{N_{AB}}{N_{DE}}}$$

$$\alpha_T = \frac{I_{Cn}}{I_{En}} = \frac{1}{1+\dfrac{1}{2}\left(\dfrac{W_B}{L_{nB}}\right)^2}$$

$$\beta_F = \frac{\alpha_F}{1-\alpha_F}$$

$$\alpha_F = \frac{\beta_F}{1+\beta_F}$$