NAME:_____FULL SOLUTION____________ **PUID:**______________________

ECE 305 – Fall 2017

Exam 1 – Thursday, September 14, 2017

This is a closed book exam. You may use a calculator and the formula sheet at the end of this exam. Following the ECE policy, the calculator **must** be a Texas Instruments TI-30X IIS scientific calculator.

To receive full credit, you must **show your work** (scratch paper is attached).
The exam is designed to be taken in 50 minutes (or less). Be sure to fill in your name and Purdue student ID at the top of the page. DO NOT open the exam until told to do so, and stop working immediately when time is called. The last page is an equation sheet, which you may remove, if you want.

100 points possible,

I) 40 points (8 points per question)
II) 30 points
III) 30 points

Course policy

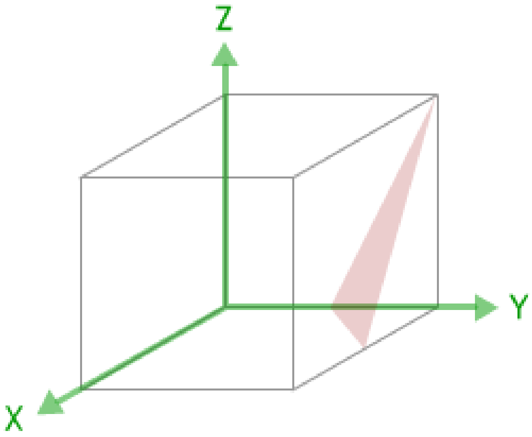
If I'm caught cheating, I'll earn an F in the course & be reported to the Dean of Students.

I repeat: ____

____

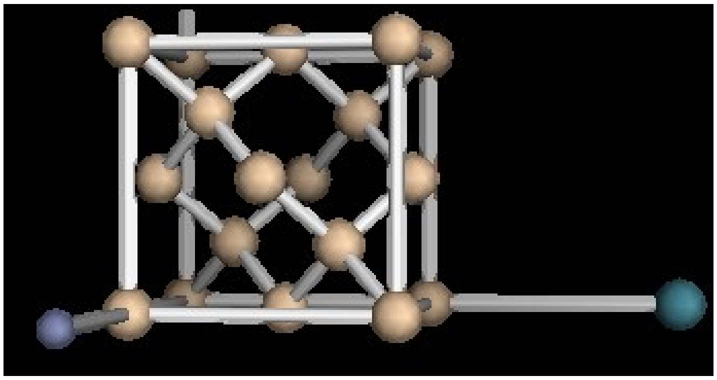
Signature: ____

1 (8 points). What is the Miller index for the plane shown below?



a. $(\bar{2}21)$
b. $(2\bar{2}1)$
c. $(22\bar{1})$
d. $(1\bar{2}2)$
e. $(2\bar{1}2)$

2 (8 points). The density of germanium atoms in a face-centered cubic diamond lattice (shown below) is 4.46×10^{22} atoms/cm^3. What is the lattice constant a for this structure? Note that 1 Å = 10^{-8} cm.



a. 0.53 Å
b. 2.82 Å
c. 3.89 Å
d. 5.64 Å
e. 11.3 Å

3 (8 points). The intrinsic carrier concentration of a semiconductor increases with temperature. If all the dopant atoms in a p-type semiconductor are already ionized at a certain temperature, further increases in temperature will:

a. shift E_F down, away from E_C and towards E_i
b. shift E_F up, toward E_C and away from E_i
c. shift E_F up, away from E_V and towards E_i
d. shift E_F down, towards E_V and away from E_i
e. not affect E_F

4 (8 points). What current is generated by <u>electrons</u> diffusing down a concentration gradient?

a. $+qE$
b. $-qE$
c. $+q\,D_n\,dp/dx$
d. $-q\,D_p\,dn/dx$
e. $+q\,D_n\,dn/dx$

5 (8 points). What is the approximate resistivity of intrinsic crystalline silicon at $T = 300$ K? Note that $\mu_n = 1400\ \mathrm{cm}^2/\mathrm{V{\cdot}s}$, and $\mu_p = 450\ \mathrm{cm}^2/\mathrm{V{\cdot}s}$.

a. 307 $\Omega \cdot \mathrm{cm}$
b. 0.307 $k\Omega \cdot \mathrm{cm}$
c. 0.307 $M\Omega \cdot \mathrm{cm}$
d. 0.307 $G\Omega \cdot \mathrm{cm}$
e. 0.307 $\Omega \cdot \mathrm{cm}$

Part II (Free Response, 30 points)

For this problem, consider crystalline silicon at room temperature ($T = 300$ K). Assume that $\tau_n = 1 \text{ ps}$ and $\tau_p = 0.26 \text{ ps}$.

a. Calculate the mobility and diffusion constants for both electrons and holes in this material. Recall that $\mu_n = q\tau_n/m_n^*$ and $\mu_p = q\tau_p/m_p^*$. Effective masses are provided on the formula sheet.

Inserting the given values, we obtain $\mu_n = 1600 \text{ cm}^2/(\text{V}\cdot\text{s})$ and $\mu_n = 400 \text{ cm}^2/(\text{V}\cdot\text{s})$.

Using Einstein's relation, we also obtain $D_n = 40 \text{ cm}^2/\text{s}$ and $D_p = 10 \text{ cm}^2/\text{s}$.

b. Assume that we use a 300 µm-thick n-type wafer ($N_D = 10^{15} \text{ cm}^{-3}$), uniformly contacted on the top and bottom. If we apply a voltage difference of 1 V to the contacts, what is the drift velocity and drift current density associated with the resulting motion of electrons?

$\sigma \approx \sigma_n = nq\mu_n = (10^{15} \text{ cm}^{-3})(1.602 \times 10^{-19} \text{ C})(1600 \text{ cm}^2/(\text{V}\cdot\text{s})) = 0.256 \text{ S/cm}$

$v_n = \mu_n \mathcal{E} = (1600 \text{ cm}^2/(\text{V}\cdot\text{s})) * (1 \text{ V})/(3 \times 10^{-2} \text{ cm}) = 53{,}333 \text{ cm/s}$

$J_n = \sigma\mathcal{E} = (0.256 \text{ S/cm}) * (1 \text{ V}) / (3 \times 10^{-2} \text{ cm}) = 8.544 \text{ A/cm}^2$

c. What concentration gradient would be needed to cancel out this drift current? Is this value reasonable? Justify your answer.

Now $J_n = 0$, so $qD_n \frac{dn}{dx} = -8.544 \text{ A/cm}^2$, which yields $\frac{dn}{dx} = -1.333 \times 10^{18} \text{cm}^{-3}/\text{cm}$. Integrated from the top to the bottom, this amounts to $\Delta n = 4 \times 10^{16} \text{cm}^{-3}$. This is not extremely reasonable, since it requires a change in carrier concentration significantly greater than the level of doping. Thus, it is unlikely that current could be entirely prevented from flowing.

Part III (Free Response, 30 points)

A p-type doped single-crystal wafer of silicon is in equilibrium at an elevated temperature ($T = 700$ K). The dopants are fully ionized, such that $N_A = 10^{16}\ \text{cm}^{-3}$, and $N_D = 0$. At this temperature, the bandgap of silicon is 0.99 eV.

a. For this sample, find the conduction and valence band effective densities of state.

$N_C(700\text{ K}) = N_C(300\text{ K}) \times (700)^{3/2}/(300)^{3/2} = 1.18 \times 10^{20}\ \text{cm}^{-3}$

$N_V(700\text{ K}) = N_V(300\text{ K}) \times (700)^{3/2}/(300)^{3/2} = 6.52 \times 10^{19}\ \text{cm}^{-3}$

or

$$N_C = \frac{1}{4}\left(\frac{2m_n^* kT}{\pi\hbar^2}\right)^{3/2};\ N_V = \frac{1}{4}\left(\frac{2m_p^* kT}{\pi\hbar^2}\right)^{3/2}$$

$N_C = 1.016 \times 10^{20}\ \text{cm}^{-3}$; $N_V = 1.10 \times 10^{20}\ \text{cm}^{-3}$

b. What is the electron concentration n (in units of cm^{-3})?

Intrinsic carrier concentration $n_i = \sqrt{N_C N_V}\, e^{-E_g/2kT}$

$n_i = \sqrt{1.18 \times 10^{20} \times 6.52 \times 10^{19}}\ \exp\left[-\frac{0.99}{2\times 8.625\times 10^{-5}\times 700}\right]\ \text{cm}^{-3} = 1.8 \times 10^{16}\ \text{cm}^{-3}$ or $2.41\times 10^{16}\ \text{cm}^{-3}$ or $2.88\times 10^{16}\ \text{cm}^{-3}$

Space charge neutrality: $p - n + N_D^+ - N_A^- = 0$

$$\frac{n_i^2}{n} - n - N_A^- = 0$$

$$n_i^2 - n^2 - N_A^- n = 0$$

Solving the quadratic equation yields $n = 1.36 \times 10^{16}\ \text{cm}^{-3}$ or $1.96\times 10^{16}\ \text{cm}^{-3}$ or $2.42\times 10^{16}\ \text{cm}^{-3}$

c. Calculate the Fermi level position with respect to the intrinsic level, defined as $E_F - E_i$.

$$n = n_i e^{(E_F - E_i)/kT}$$

$$E_F - E_i = kT \ln\frac{n}{n_i} = 8.625 \times 10^{-5} \times 700 \times \ln\frac{1.36 \times 10^{16}}{1.8 \times 10^{16}}$$

$= -0.016$ eV or -0.01 eVs

ECE 305 Exam 1 Formula Sheet (Fall 2017)

Physical Constants	**Silicon parameters ($T = 300$ K)**
$\hbar = 1.055 \times 10^{-34}$ J·s	$N_C = 3.23 \times 10^{19}\ \mathrm{cm}^{-3}$
$m_0 = 9.109 \times 10^{-31}$ kg	$N_V = 1.83 \times 10^{19}\ \mathrm{cm}^{-3}$
$k = 1.38 \times 10^{-23}$ J/K	$n_i = 1.1 \times 10^{10}\ \mathrm{cm}^{-3}$
$q = 1.602 \times 10^{-19}$ C	$K_s = 11.8$
$\epsilon_0 = 8.854 \times 10^{-12}$ F/m	$m_n^* = 1.09 m_o$
$c = 2.998 \times 10^{10}$ cm/s	$m_p^* = 1.15 m_o$

Miller Indices: (hkl) {hkl} [hkl] <hkl>

Density of states $g_C(E) = \frac{(m_n^*)^{3/2}\sqrt{2(E-E_C)}}{\pi^2\hbar^3}$

Fermi function $f(E) = \frac{1}{1+e^{(E-E_F)/kT}}$

Intrinsic carrier concentration $n_i = \sqrt{N_C N_V} e^{-E_g/2kT}$

Equilibrium carrier densities: $N_C = \frac{1}{4}\left(\frac{2m_n^* kT}{\pi\hbar^2}\right)^{3/2}$ $\quad N_V = \frac{1}{4}\left(\frac{2m_p^* kT}{\pi\hbar^2}\right)^{3/2}$

$n = N_C e^{(E_F-E_C)/kT} = n_i e^{(E_F-E_i)/kT}$ $\quad p = N_V e^{(E_V-E_F)/kT} = n_i e^{(E_i-E_F)/kT}$

Space charge neutrality: $p - n + N_D^+ - N_A^- = 0$

Law of Mass Action: $np = n_i^2$

Conductivity/resistivity: $\sigma = \sigma_n + \sigma_p = q(n\mu_n + p\mu_p) = 1/\rho$

Drift-diffusion current equations:

$$J_n = nq\mu_n \mathcal{E}_x + qD_n \frac{dn}{dx} \qquad \frac{D_n}{\mu_n} = \frac{kT}{q}$$

$$J_p = pq\mu_p \mathcal{E}_x - qD_p \frac{dp}{dx} \qquad \frac{D_p}{\mu_p} = \frac{kT}{q}$$

Carrier conservation equations:

$$\frac{\partial n}{\partial t} = +\nabla \cdot \left(\frac{J_n}{q}\right) + G_n - R_n$$

$$\frac{\partial p}{\partial t} = -\nabla \cdot \left(\frac{J_p}{q}\right) + G_p - R_p$$

Poisson's equation:

$$\nabla \cdot (\epsilon \mathcal{E}) = \rho$$