NAME:____**FULL SOLUTION**_____   **PUID:**_____

ECE 305 – Spring 2018

Exam 1 – Thursday, January 25, 2018

This is a closed book exam. You may use a calculator and the formula sheet at the end of this exam. Following the ECE policy, the calculator **must** be a Texas Instruments TI-30X IIS scientific calculator.

To receive full credit, you must **show your work** (scratch paper is attached).
The exam is designed to be taken in 50 minutes (or less). However, you will be given a full hour.

Please be sure to fill in your name and Purdue student ID at the top of the page. DO NOT open the exam until told to do so, and stop working immediately when time is called. The last page is an equation sheet, which you may remove, if you want.

100 points possible,
- I)   40 points (8 points per question)
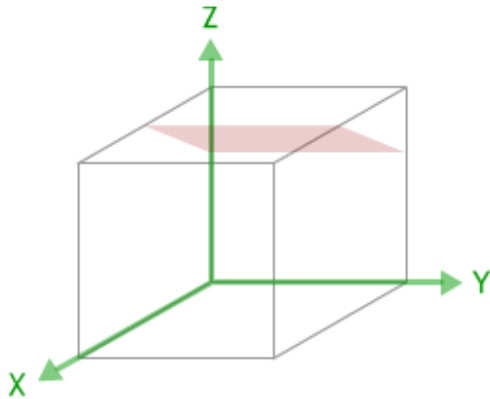- II) 30 points
- III) 30 points

1 (8 points). What is the Miller index for the plane shown below?

    a. $(\bar{2}03)$
    b. $(0\bar{2}3)$
    c. $(20\bar{3})$
    d. $(3\bar{2}0)$
    e. $(2\bar{3}0)$

2 (8 points). The concentration of silicon atoms in the lattice is 5×10^{22} atoms/cm^3. If germanium atoms are added, to the lattice, with a concentration of 1×10^{14} atoms/cm^3, the semiconductor silicon is

    a. lightly p-doped
    b. heavily p-doped
    c. lightly n-doped
    d. heavily n-doped
    e. still intrinsic

3 (8 points). GaSb has zincblende crystalline lattice. What is the Bravais lattice it is directly based upon?

    a. Simple (or primitive) cubic lattice
    b. Body centered cubic lattice
    c. Face centered cubic lattice
    d. Simple (or primitive) tetragonal lattice
    e. Body centered tetragonal lattice

4 (8 points). Sulfur introduced into GaSb can result in

       a. Crystallization
       b. N-type doping
       c. P-type doping
       d. Both b. and c.
       e. None of the above

5 (8 points). How does one calculate the effective density of states from the density of states?

       a.   Multiply the density of states with the effective carrier concentration
       b.   Integrate the product of the Fermi-Dirac distribution and the density of states
       c.   Divide the Boltzmann distribution by the Fermi energy
       d.   Apply space charge neutrality
       e.   They are identical

Part II (Free Response, 30 points)

Assume that we have face-centered cubic lattice of copper with an atomic radius of 1.278 Å and a periodicity of 3.61 Å, as shown below (noting that 1 Å=10^{-8} cm).
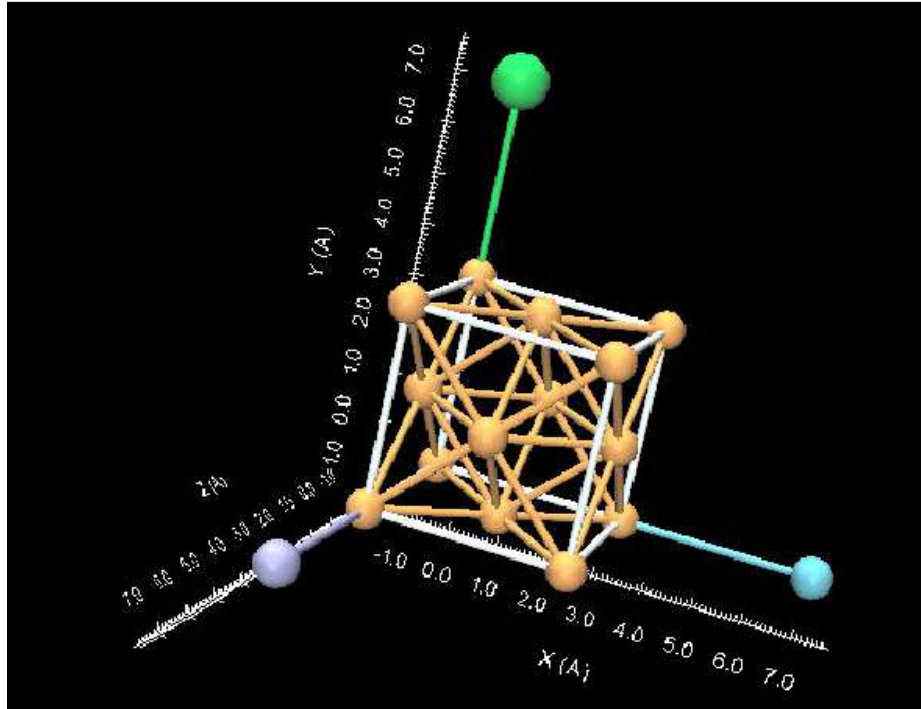
a. Calculate the atomic density of this material (in atoms/cm³)

We have 4 atoms per unit cell, and each cell has a volume $V_{cell} = (3.61 \text{ Å})^3 = 4.705 \times 10^{-23}$ cm³

Thus, $N_a = \frac{4}{4.705 \times 10^{-23} \text{ cm}^3} = 8.50 \times 10^{22}$ cm^{-3}.

b. Calculate the mass density of this material (g/cm³). The atomic mass of copper is 63.546 amu, while 1 amu $= 1.66 \times 10^{-24}$ g.

We obtain $\rho = mN_a = (63.546 \text{ amu}) \cdot \left(1.66 \times 10^{-24} \frac{\text{g}}{\text{amu}}\right) \cdot (8.50 \times 10^{22} \text{ cm}^{-3}) = 8.97$ g/cm³.
This value is very close to the literature value for copper at 20 °C.

c. Calculate the *packing fraction* of this material (i.e., the ratio of the total volume of the spheres to the volume of the cube enclosing them; a unitless value).

The volume of the atoms is given by $V_{at} = 4 * \frac{4\pi}{3} a^3 = 4 * \frac{4\pi}{3} (1.278 \text{ Å})^3 = 34.97$ Å³

Dividing by the cell volume V_{cell} yields $PF = \frac{V_{at}}{V_{cell}} = \frac{34.97 \text{ Å}^3}{47.05 \text{ Å}^3} = 0.74$.

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Part III (Free Response, 30 points)
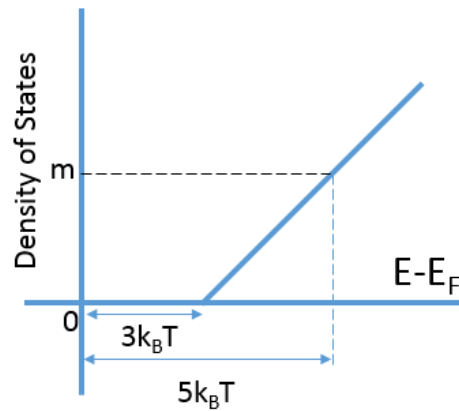
a. Using the graph, which depicts energy relative to the Fermi level E_F on the x-axis, and the density of states on the y-axis, write an expression for the density of states in terms of $E-E_F$.

We can use two-point form: $g(E) = \frac{m-0}{5kT-3kT}(E - E_F - 3kT) = m\left(\frac{E-E_F}{2kT} - \frac{3}{2}\right)\Theta(E - E_F - 3kT)$

b. Formulate and sketch the carrier concentration per unit energy, $n(E)$. Assume that $E - E_F >> k_BT$ in your derivation.

$$n(E) = g(E) \times f(E) \approx \Theta(E - E_F - 3kT) \times m\left(\frac{E - E_F}{2kT} - \frac{3}{2}\right)\exp\left(-\frac{E - E_F}{kT}\right)$$

$(E - E_F)/kT$
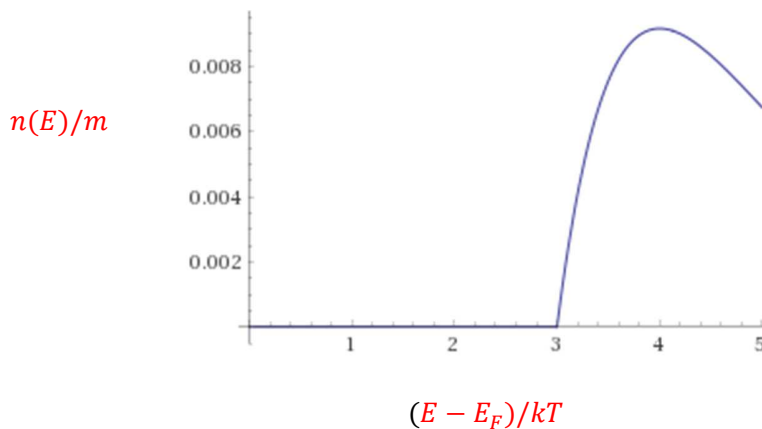
c. Assuming you only need to account for states up to $5k_BT$ above the Fermi level, find the total carrier concentration n.

$$n = \int\limits_{E_F+3kT}^{E_F+5kT} n(E)dE = mkT\int_3^5 \left(\frac{x}{2} - \frac{3}{2}\right)e^{-x}dx = \frac{mkT}{2}[(2 - x)e^{-x}]_3^5 = \frac{mkT}{2}(-3e^{-5} + e^{-3}).$$

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ECE 305 Exam 1 Formula Sheet (Spring 2018)

Physical Constants	Silicon parameters ($T = 300$ K)
$\hbar = 1.055 \times 10^{-34}$ J·s	$N_C = 3.23 \times 10^{19}$ cm^{-3}
$m_0 = 9.109 \times 10^{-31}$ kg	$N_V = 1.83 \times 10^{19}$ cm^{-3}
$k = 1.38 \times 10^{-23}$ J/K	$n_i = 1.1 \times 10^{10}$ cm^{-3}
$q = 1.602 \times 10^{-19}$ C	$K_s = 11.8$
$\epsilon_0 = 8.854 \times 10^{-12}$ F/m	

Miller Indices: (hkl) {hkl} [hkl] <hkl>   **Density of states** $g_C(E) = \frac{(m_n^*)^{3/2}\sqrt{2(E-E_C)}}{\pi^2\hbar^3}$

Fermi function $f(E) = \frac{1}{1+e^{(E-E_F)/kT}}$ **Intrinsic carrier concentration** $n_i = \sqrt{N_C N_V}\, e^{-E_g/2kT}$

Equilibrium carrier densities: $N_C = \frac{1}{4}\left(\frac{2m_n^* kT}{\pi\hbar^2}\right)^{3/2}$ $N_V = \frac{1}{4}\left(\frac{2m_p^* kT}{\pi\hbar^2}\right)^{3/2}$

$n = N_C e^{(E_F-E_C)/kT} = n_i e^{(E_F-E_i)/kT}$ $p = N_V e^{(E_V-E_F)/kT} = n_i e^{(E_i-E_F)/kT}$

Space charge neutrality: $p - n + N_D^+ - N_A^- = 0$ **Law of Mass Action**: $np = n_i^2$

Conductivity/resistivity: $\sigma = \sigma_n + \sigma_p = q(n\mu_n + p\mu_p) = 1/\rho$

Drift-diffusion current equations: $J_n = nq\mu_n \mathcal{E}_x + qD_n\frac{dn}{dx}$ $\frac{D_n}{\mu_n} = \frac{kT}{q}$

$J_p = pq\mu_p \mathcal{E}_x - qD_p\frac{dp}{dx}$ $\frac{D_p}{\mu_p} = \frac{kT}{q}$