NAME:____FULL SOLUTION__________ **PUID:________________________**

ECE 305 – Fall 2017

Exam 2 – Thursday, October 5, 2017

This is a closed book exam. You may use a calculator and the formula sheet at the end of this exam. Following the ECE policy, the calculator **must** be a Texas Instruments TI-30X IIS scientific calculator.

To receive full credit, you must **show your work** (scratch paper is attached).
The exam is designed to be taken in 50 minutes (or less). Be sure to fill in your name and Purdue student ID at the top of the page. DO NOT open the exam until told to do so, and stop working immediately when time is called. The last page is an equation sheet, which you may remove, if you want.

100 points possible,

I) 40 points (8 points per question)
II) 30 points
III) 30 points

Course policy

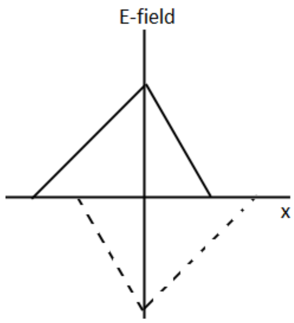
If I am caught cheating, I will earn an F in the course & be reported to the Dean of Students.

I repeat: ____

____

Signature: ____

Part I: Answer the 5 multiple choice questions below by entering them on your IDP-15 Scantron.



1 (8 points). The diagram to the left is drawn to scale. What is true about the the two E field profiles (solid and dashed)?

A. The built-in potential V_{bi} is negative for one of these.
B. Voltage is applied to transition from one to the other.
C. They are from identical pn junctions, reversed along the x-axis.
D. More negative charges exist in the depletion region than positive charges and vice versa for the separate profiles.
E. None of the above.

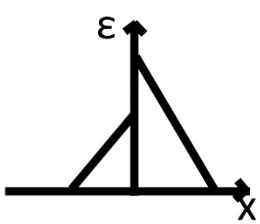
2 (8 points). What assumption(s) are made to derive the minority carrier diffusion equation (MCDE) given in the Formula Sheet?

A. Flat conduction and valence bands
B. Steady-state conditions
C. Low carrier injection
D. A and B
E. A and C

3 (8 points). What is the correct ordering of processes before doping in silicon?

A. Apply resist, expose, etch, develop, remove resist
B. Expose, apply resist, develop, etch, remove resist
C. Apply resist, expose, develop, remove resist, etch
D. Apply resist, expose, develop, etch, remove resist
E. Apply oxide, expose, develop, remove resist, etch

4 (8 points). What does the diagram below (E-field vs position x) represent?



A. Sheet charge embedded in a uniformly doped semiconductor
B. Trapped charges at the interface of a pn junction
C. E-field from voltage applied across pn junction
D. E-field from voltage applied across uniformly-doped semiconductor
E. Linear variation of dopant concentration across pn junction

5 (8 points). What assumption(s) are made in the equation pertaining to pn junctions: $N_D x_n = N_A x_p$?

A. All mobile charge carriers recombine in the depletion region
B. All acceptors and donors are fully ionized
C. The p and n-type materials are the same, forming a homojunction
D. A and B
E. A, B and C

Part II (Free Response, 30 points)

Assume that an n-type region of spatially uniform crystalline silicon with $\mu_p = 500 \text{ cm}^2/\text{V}\cdot\text{s}$ and lifetime $\tau_p = 10\ \mu\text{s}$ is uniformly illuminated by a photon flux $G_L = 10^{19}$ /cm³·s, which starts at $t = 0$ and is then switched off at $t = 50\ \mu\text{s}$.

a. Write down the minority carrier diffusion equations that describes its behavior from $t = 0$ to $t = 100\ \mu\text{s}$. You may write different equations for different time segments.

$$\frac{\partial \Delta p}{\partial t} = -\frac{\Delta p}{\tau_p} + G_L \quad 0 < t < 50\mu s$$

$$\frac{\partial \Delta p}{\partial t} = -\frac{\Delta p}{\tau_p} \qquad t \geq 50\mu s$$

Holes are our minority carriers. We can drop the diffusion term (with its two spatial derivatives) here, because of spatial uniformity. Because of the time-dependent behavior of the generation and recombination, we must keep all the other terms.

b. Sketch the time-dependent behavior of the excess minority carrier concentration from $t = 0$ to $t = 100\ \mu\text{s}$. Be sure to label both the x and y axes, and include at least 3 distinct numerical values on each axis (with justification).

Boundary condition: $\Delta p(t = 0) = 0$, since illumination starts at $t = 0$;

General solution: $\Delta p = G_L \tau_p + Be^{-\frac{t}{\tau_p}}$

0=$G_L \tau_p + B$ implies B=$-G_L \tau_p$

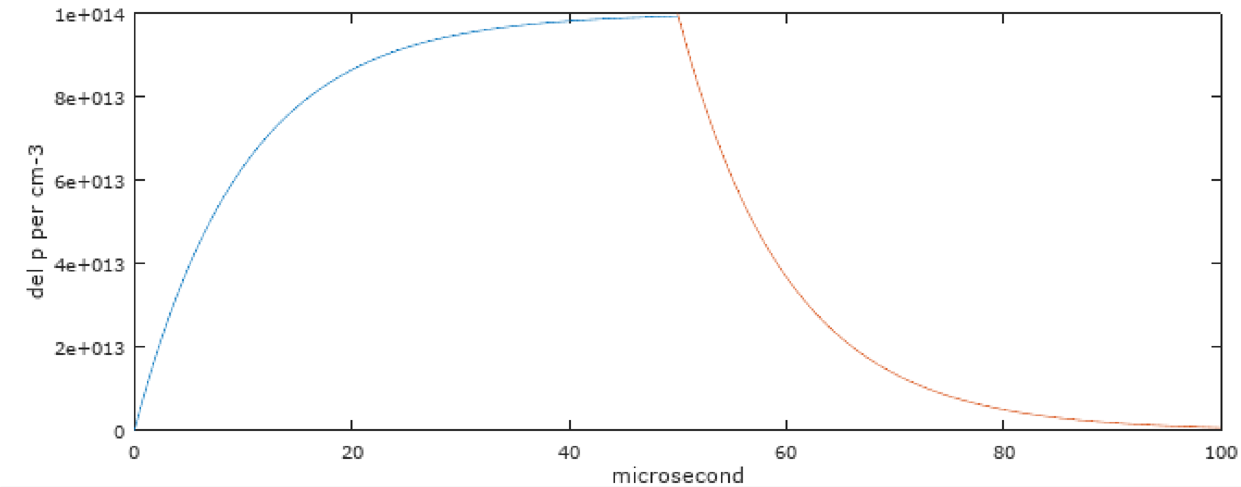
$$\Delta p = G_L \tau_p - G_L \tau_p e^{-\frac{t}{\tau_p}} \qquad 0 \ll t < 50\mu s$$

Boundary condition: $\Delta p(t = 50\mu s) \approx G_L \tau_p$; General solution: $\Delta p = Be^{-\frac{t}{\tau_p}}$

$G_L \tau_p = Be^{-\frac{50\mu s}{\tau_p}}$ implies B=$G_L \tau_p e^{\frac{50\mu s}{\tau_p}}$

$$\Delta p = G_L \tau_p e^{\frac{50\mu s}{\tau_p}} e^{-\frac{t}{\tau_p}} = G_L \tau_p e^{-\frac{t-50\mu s}{\tau_p}} \qquad t \geq 50\mu s$$

Thus the sketch should resemble:



c. Sketch the excess minority carrier concentration vs position at $t = 100$ μs if all conditions are same as the previous case, but the sample is infinitely long and the surface at $x = 0$ has a fixed excess minority carrier concentration $\Delta p(x = 0) = 10^{15}$ /cm³.

Infinitely far away from surface there is no spatial dependence of Δp, so it is same as II. B, i.e.

$\Delta p(x=\infty, t=100\ \mu s) = G_L \tau_p e^{-\frac{(100-50)\mu s}{\tau_p}} \approx 0$

$\Delta p(x=0) = 10^{15}$ /cm³

At t=100 μs Δp has achieved steady state, so we can remove the partial differentiation with time.

We can also remove generation, since there is no illumination.

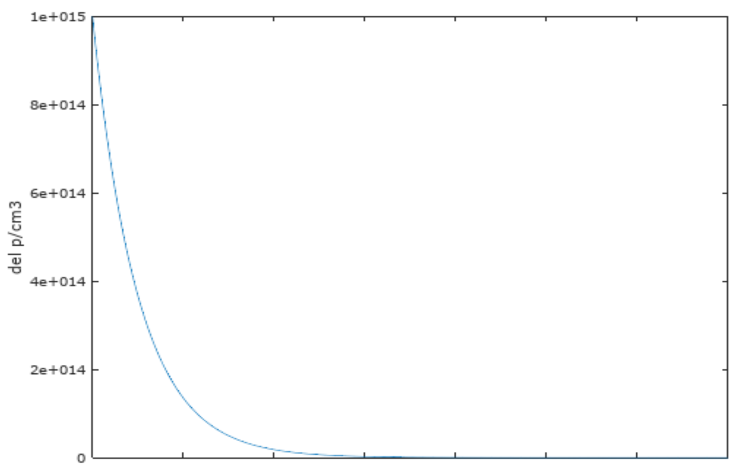
$0 = D_p \frac{\partial^2 \Delta p}{\partial x^2} - \frac{\Delta p}{\tau_n}$; General solution:

$\Delta p = Ae^{-\frac{x}{L_p}} + Be^{\frac{x}{L_p}}$

Applying the boundary conditions:

B=0, as otherwise Δp will exponentially blow up at x=∞ instead of equating to 0.

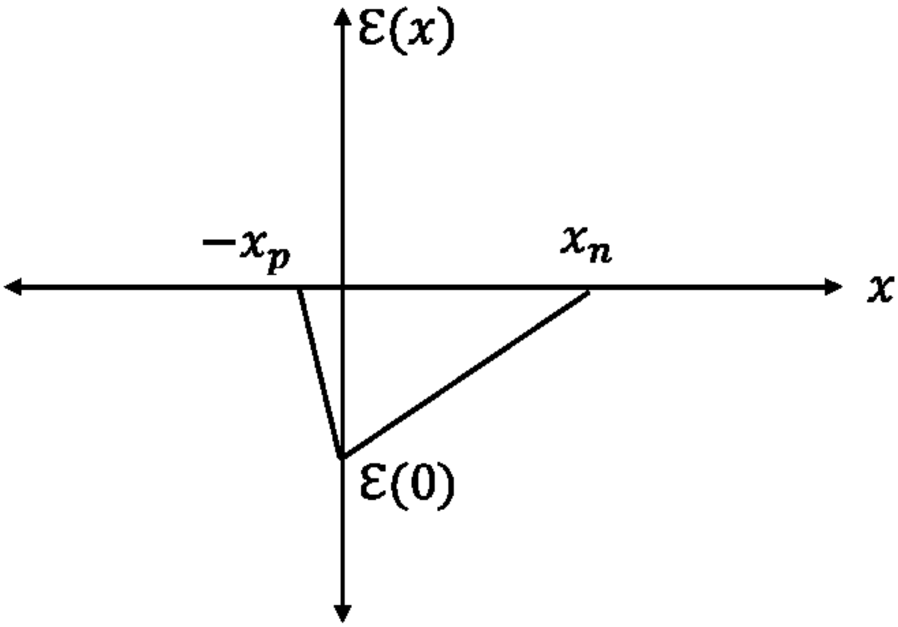
$\Delta p(x=0) = Ae^{-\frac{0}{L_p}}$ or $A = 10^{15}$



Therefore: $\Delta p = 10^{15} e^{-\frac{x}{L_p}}$.

Part III (Free Response, 30 points)

The electric field versus position for a crystalline silicon ($K_s = 11.68$) pn homojunction at room temperature ($T = 300$ K) is given by the following graph (may not be precisely to scale):



For this problem, assume that the junction is abrupt at $x = 0$ with a flat doping profile on each side (p-doped on the left, and n-doped on the right), $\mathcal{E}(0) = -10$ kV/cm, n-type depletion length $x_n = 620$ nm, and the p-type depletion length $x_p = 100$ nm.

a. What is the built-in voltage V_{bi} obtained using depletion approximation?

$$V_{bi} = \frac{1}{2}\mathcal{E}(0)W = 0.5 \cdot \left(-10\,\frac{\text{kV}}{\text{cm}}\right) \cdot (720\text{ nm}) = 0.36\text{ V}$$

Continued on next page….

Part III (Continued)

b. What is the value of N_A in the p-type region, and N_D in the n-type region, if the depletion approximation is used?

From the formula sheet, we can apply $\frac{d\mathcal{E}}{dx} = \frac{\rho}{K_s\epsilon_o}$.

On the left-hand side, $\frac{d\mathcal{E}}{dx} = \frac{-qN_A}{K_s\epsilon_o}$, so $\mathcal{E}(0) = \frac{-qN_Ax_p}{K_s\epsilon_o}$, so $N_A = \frac{|\mathcal{E}(0)|K_s\epsilon_o}{qx_p} = 6.5 \cdot 10^{15}\ \mathrm{cm}^{-3}$.

On the right-hand side, $\frac{d\mathcal{E}}{dx} = \frac{qN_D}{K_s\epsilon_o}$, so $N_D = \frac{|\mathcal{E}(0)|K_s\epsilon_o}{qx_n} = 1.04 \cdot 10^{15}\ \mathrm{cm}^{-3}$.

Alternate solution (not recommended):

$V_{bi} = \frac{kT}{q}\ln\left(\frac{N_DN_A}{n_i^2}\right)$, so $N_DN_A = 10^{20}e^{0.36/.025}\mathrm{cm}^{-6} = 1.8 \cdot 10^{26}\ \mathrm{cm}^{-6}$

$N_Ax_p = N_Dx_n \rightarrow N_A/N_D = x_n/x_p = 6.2 \rightarrow 6.2N_D^2 = 1.8 \cdot 10^{26}$

$N_D = 5.4 \cdot 10^{12}\ \mathrm{cm}^{-3}$; $N_A = 3.3 \cdot 10^{13}\ \mathrm{cm}^{-3}$

c. Using the depletion approximation, calculate the np product just outside the right side of the junction (at $x = x_n^+$). How would this value change if you applied a forward bias of 0.28 V?

From the formula sheet, we can apply $np = n_i^2e^{(F_n-F_p)/kT}$.

In equilibrium, no quasi-Fermi level splitting, so $F_n = F_p = E_F$. Thus, $np = n_i^2 = 10^{20}\ \mathrm{cm}^{-6}$.

With forward bias of 0.28 V, $F_n - F_p = qV_A = 0.28$ eV. Thus, $np = n_i^2e^{\left(\frac{0.28}{0.0259}\right)} = 5 \cdot 10^{24}\ \mathrm{cm}^{-6}$.

ECE 305 Exam 2 Formula Sheet (Fall 2017)

You may remove this page from the exam packet, and take it with you.

Physical Constants	Silicon parameters ($T = 300$ K)
$\hbar = 1.055 \times 10^{-34}$ J·s	$N_C = 3.23 \times 10^{19}$ cm^{-3}
$m_0 = 9.109 \times 10^{-31}$ kg	$N_V = 1.83 \times 10^{19}$ cm^{-3}
$k_B = k = 1.38 \times 10^{-23}$ J/K	$n_i = 1.1 \times 10^{10}$ cm^{-3}
$q = 1.602 \times 10^{-19}$ C	$K_s = 11.8$
$\epsilon_0 = 8.854 \times 10^{-12}$ F/m	

Miller Indices: (hkl) {hkl} [hkl] <hkl>

Density of states $g_C(E) = \frac{(m_n^*)^{3/2}\sqrt{2(E-E_C)}}{\pi^2\hbar^3}$

Fermi function $f(E) = \frac{1}{1+e^{(E-E_F)/kT}}$

Intrinsic carrier concentration $n_i = \sqrt{N_C N_V} e^{-E_g/2kT}$

Equilibrium carrier densities: $N_C = \frac{1}{4}\left(\frac{2m_n^* kT}{\pi\hbar^2}\right)^{3/2}$ $\quad N_V = \frac{1}{4}\left(\frac{2m_p^* kT}{\pi\hbar^2}\right)^{3/2}$

$n = N_C e^{\frac{(E_F-E_C)}{kT}} = n_i e^{\frac{(E_F-E_i)}{kT}}$ $\quad p = N_V e^{\frac{(E_V-E_F)}{kT}} = n_i e^{\frac{(E_i-E_F)}{kT}}$

Space charge neutrality: $p - n + N_D^+ - N_A^- = 0$

Law of Mass Action: $n_0 p_0 = n_i^2$

Non-equilibrium carriers: $n = N_C e^{(F_N-E_C)/kT}$ $\quad p = N_V e^{(E_V-F_P)/kT}$ $\quad np = n_i^2 e^{(F_N-F_P)/kT}$

Conductivity/resistivity: $\sigma = \sigma_n + \sigma_n = q\left(n\mu_n + p\mu_p\right) = 1/\rho$

Drift-diffusion current equations: $J_n = nq\mu_n \mathcal{E}_x + qD_n \frac{dn}{dx} = n\mu_n \frac{dF_n}{dx}$ $\quad \frac{D_n}{\mu_n} = \frac{kT}{q}$

$J_p = pq\mu_p \mathcal{E}_x - qD_p \frac{dp}{dx} = p\mu_p \frac{dF_p}{dx}$ $\quad \frac{D_p}{\mu_p} = \frac{kT}{q}$

Carrier conservation equations: $\frac{\partial n}{\partial t} = +\nabla \cdot \left(\frac{J_n}{q}\right) + G_n - R_n$

$\frac{\partial p}{\partial t} = -\nabla \cdot \left(\frac{J_p}{q}\right) + G_p - R_p$

Poisson's equation: $\nabla \cdot (\epsilon\mathcal{E}) = \rho$

SRH carrier recombination: $R = \Delta n/\tau_n$ or $R = \Delta p/\tau_p$

Minority carrier diffusion equation: $\frac{\partial \Delta n}{\partial t} = D_n \frac{\partial^2 \Delta n}{\partial x^2} - \frac{\Delta n}{\tau_n} + G_L$ $\quad L_{D,n} = \sqrt{D_n \tau_n}$

PN homojunction electrostatics: $V_{bi} = \frac{kT}{q}\ln\left(\frac{N_D N_A}{n_i^2}\right)$ $\quad \frac{d\mathcal{E}}{dx} = \frac{\rho(x)}{K_s\epsilon_o}$

$W = \sqrt{\frac{2K_s\epsilon_o V_{bi}}{q}\left(\frac{N_A+N_D}{N_A N_D}\right)}$ $\quad x_n = \left(\frac{N_A}{N_A+N_D}\right)W$ $\quad x_p = \left(\frac{N_D}{N_A+N_D}\right)W$ $\quad \mathcal{E}(0) = \sqrt{\frac{2qV_{bi}}{K_s\epsilon_o}\left(\frac{N_A N_D}{N_A+N_D}\right)}$