ECE-305: Spring 2018
Exam 2 Review

Pierret, *Semiconductor Device Fundamentals* (SDF)
Chapter 3 (pp. 75-138)
Chapter 5 (pp. 195-226)

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Key topics to review

- Minority carrier diffusion equation
- Band structures
- PN junctions

Semiconductor equations: 2 key cases

- Diffusion problems ($\mathcal{E} = 0$): MCDE

$$\frac{\partial \Delta p}{\partial t} = D_p \frac{d^2 \Delta p}{dx^2} - \frac{\Delta p}{\tau_p} + G_L$$

- Drift problems ($\mathcal{E} \neq 0$): Drift current equations

$$\vec{J}_p = pq\mu_p \vec{\mathcal{E}} - qD_p \vec{\nabla} p = p\mu_p \vec{\nabla}\left(F_p / q\right)$$

$$\vec{J}_n = nq\mu_n \vec{\mathcal{E}} + qD_n \vec{\nabla} n = n\mu_n \vec{\nabla}\left(F_n / q\right)$$

How to solve (some) Exam 2 problems

Step 1: From material information (semiconductor, doping, etc.), calculate carrier densities, Fermi level, etc. Start with the majority carriers, $p = N_A - N_D, \quad or\ n = N_D - N_A$. Then get the other carrier from $np = n_i^2$

Step 2: Use band-diagram to calculate potential profile, electric field, $E = dE_c/dx$, or $E = -dV/dx$, and $\kappa_s \epsilon_0\, \rho = dE/dx\,,$ etc. For homogenous semiconductor with a battery attached, $E = V_B/L$.

Step 3: Decide if this is drift-related problem (resistivity, velocity, mobility, etc.), or a diffusion related problem (light turning on-off, etc.)

Step 4A: For a drift-problem use $\rho^{-1} = qn\mu_p E\ +\ qp\mu_p E$. For μ, you may be given a number, or table, or diffusion coefficient, etc. Learn how to read such a table.

Step 4B: For a diffusion problem, read carefully for clues to simplify the minority carrier equation.

How to solve equations

Step 4B: Two general types of minority diffusion problem.

i) Determine if electron or the hole is the minority carrier.

ii) **If** holes are the minority carriers, write the equation:

$$\frac{\partial \Delta p}{\partial t} = D_p \frac{d^2 \Delta p}{dx^2} - \frac{\Delta p}{\tau_p} + G_L$$

iii) If steady-state, drop the time-derivative. If transient, keep the time derivative. If spatially uniform, drop the diffusion term. Without light, drop the generation term. If the region is very short, drop the recombination term. Choose the solutions from the following table.

iv) Use the boundary conditions to complete solution.

How to solve equations

$$\frac{\partial \Delta p}{\partial t} = D_p \frac{d^2 \Delta p}{dx^2} - \frac{\Delta p}{\tau_p} + G_L$$

Transient

$$\frac{\partial \Delta p}{\partial t} = G_L - \frac{\Delta p}{\tau_p},$$

solution

$$\Delta p = G_L \tau_p + B\, e^{-\frac{t}{\tau_p}}$$

Boundary condition for B: Concentration before light was turned on?

Steady State

$$0 = D_p \frac{d^2 \Delta p}{dx^2} + G_L - \frac{\Delta p}{\tau_p}$$

solution

$$\Delta p = A e^{-\frac{x}{L_D}} + B e^{+\frac{x}{L_D}} + G_L \tau_p$$

If $L \ll L_D$,

$$\Delta p = A + Bx + G_L \tau_p$$

BC to determine A and B: Concentration at leftmost and rightmost points

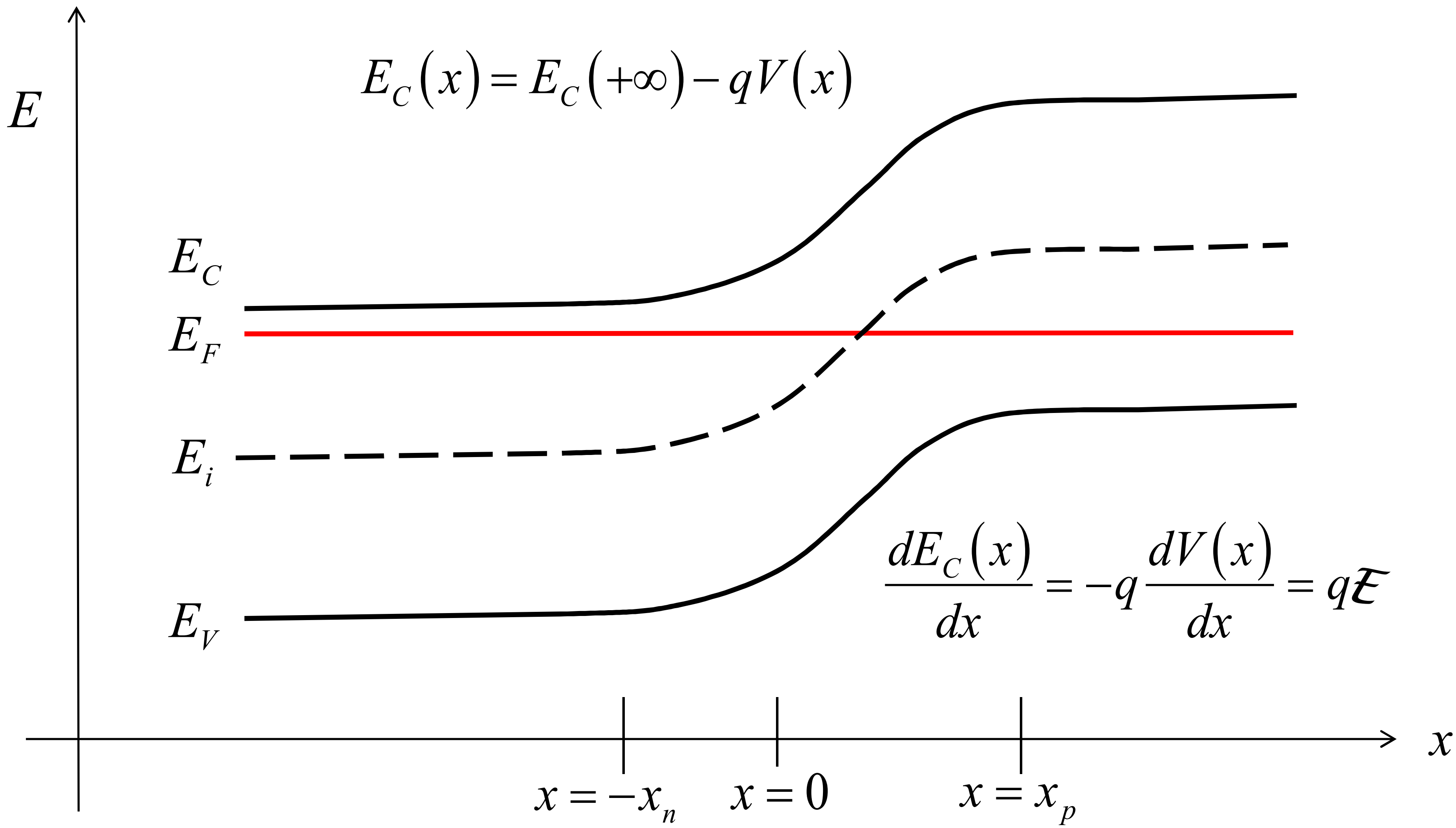
eq. energy band diagram

1) Begin with E_F
2) Draw the E-bands where you know the carrier density
3) Electrostatic potential by flipping E-band upside down.
4) E-field from slope
5) n(x), p(x) from the E-band diagram
6) rho(x) from n(x) and p(x)
7) diffusion current from (5) or from (6)

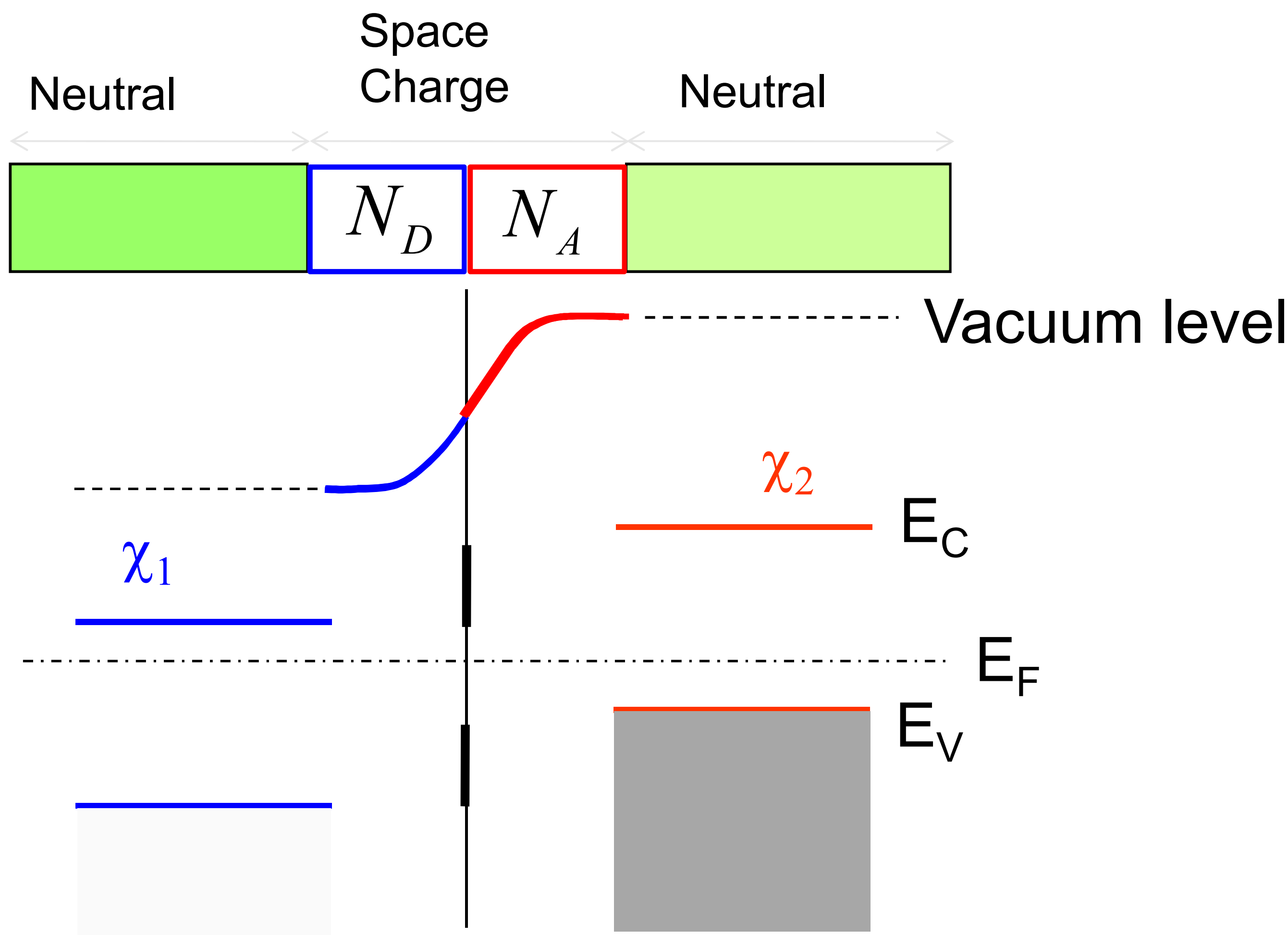
$$E_C(x) = E_{C-ref} - qV(x)$$

$$\mathcal{E}(x) = \frac{1}{q} dE_C(x)/dx$$

energy band diagram

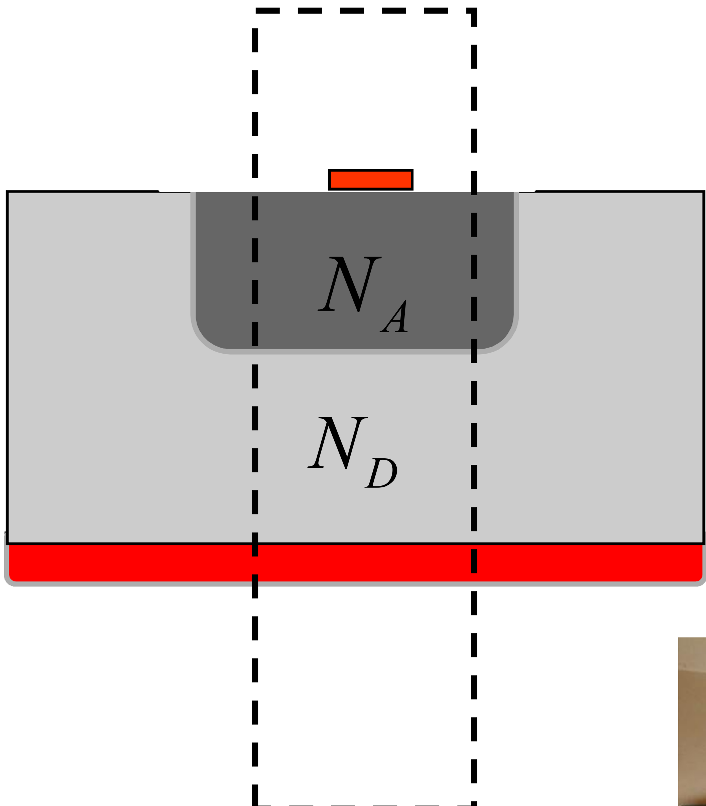


Short-cut to Band-diagram

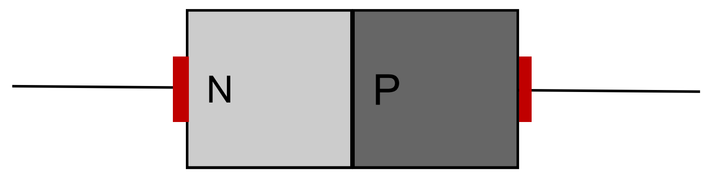


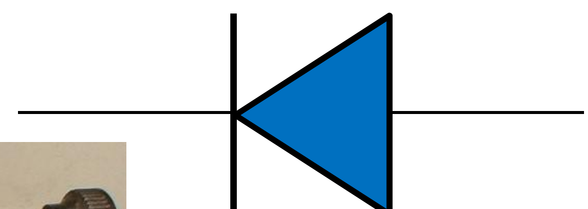
… is equivalent to solving the Poisson equation

p-n Junction Devices …



Symbols





Finding hotspot …

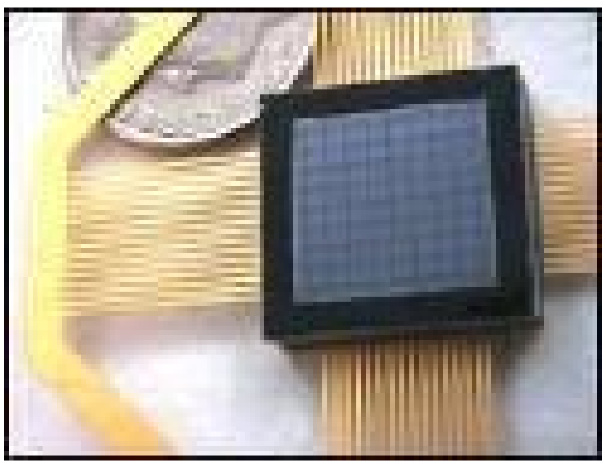
What is a Diode good for ....?

solar cells

GaAs lasers

Organic LED

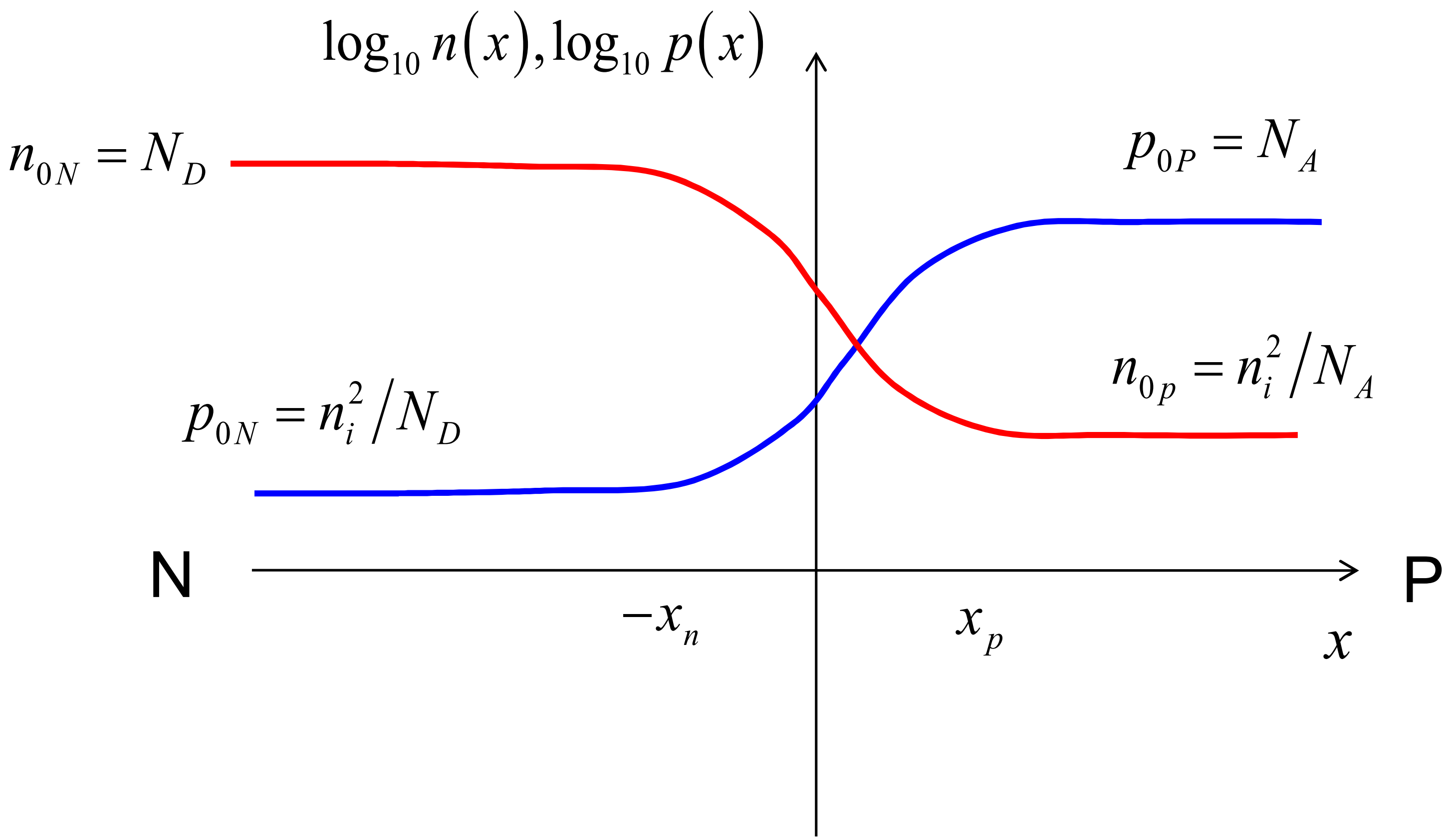
Avalanche Photodiode



GaN lasers

image.google.com

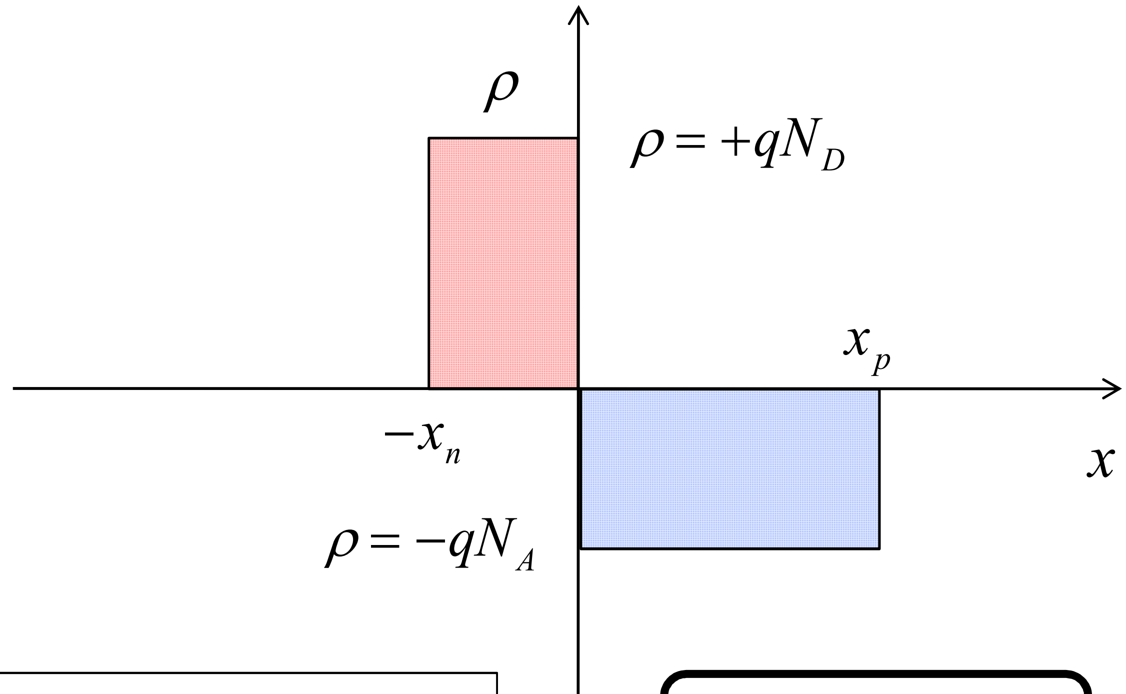
carrier densities vs. *x*



the "depletion approximation"

N

P

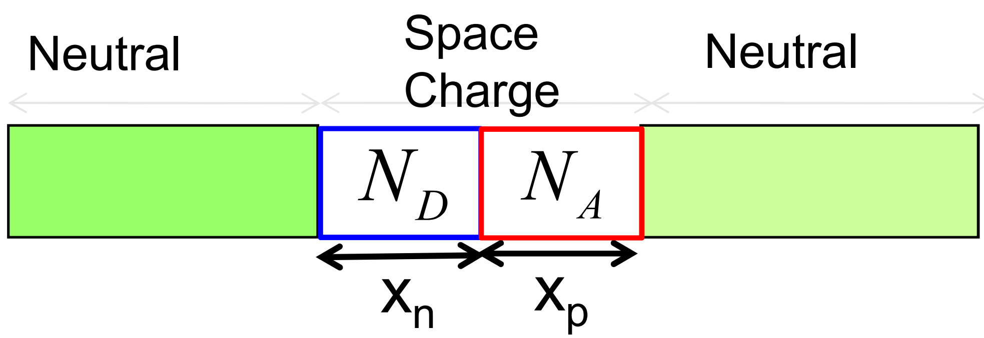


$$\frac{d\mathcal{E}}{dx} = \frac{\rho(x)}{K_S \varepsilon_0}$$

$$K_S \varepsilon_0 \frac{d^2V}{dx^2} = -q\left(p - n + N_D^+ - N_A^-\right)$$

$$N_D x_n = N_A x_p$$

Depletion Regions in Homojunctions



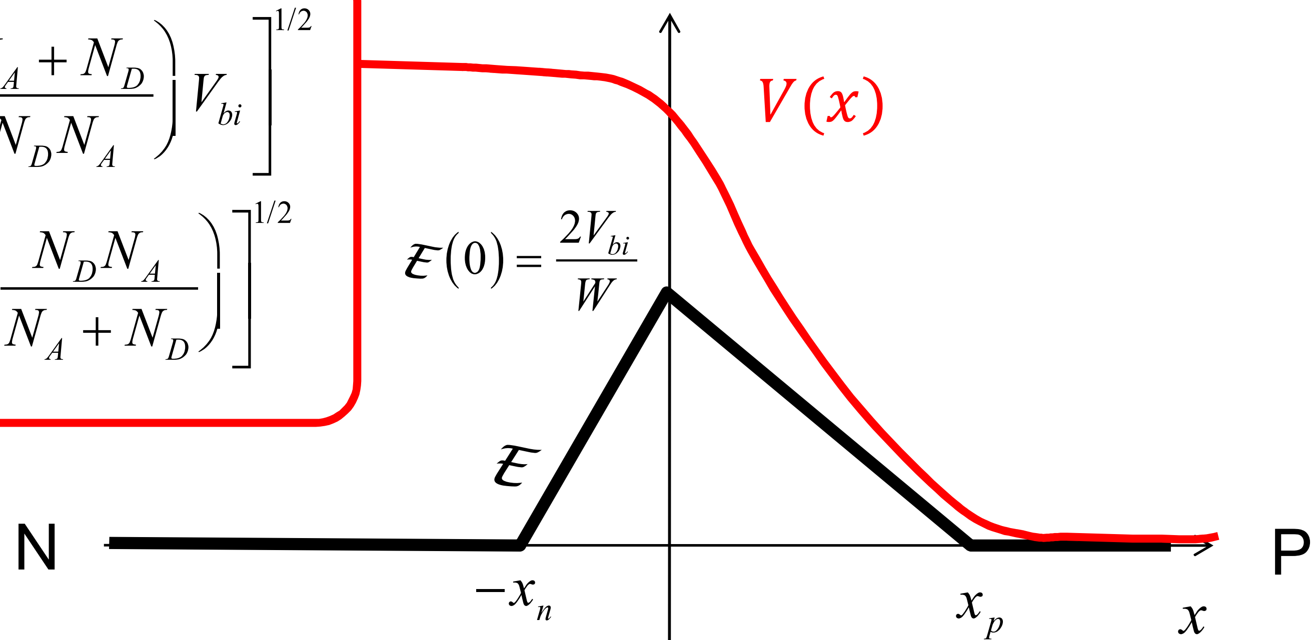
$$\left.\begin{aligned} N_D x_n &= N_A x_p \\ qV_{bi} &= \frac{qN_D x_n^2}{2k_s\varepsilon_0} + \frac{qN_A x_p^2}{2k_s\varepsilon_0} \end{aligned}\right\} \quad \begin{aligned} x_n &= \sqrt{\frac{2k_s\varepsilon_0}{q}\frac{N_A}{N_D\left(N_A+N_D\right)}V_{bi}} \\ x_p &= \sqrt{\frac{2k_s\varepsilon_0}{q}\frac{N_D}{N_A\left(N_A+N_D\right)}V_{bi}} \end{aligned}$$

Can you solve the same problem for a hetero-junction?

Key results for PN junctions

$$W=\left[\frac{2K_S\varepsilon_0}{q}\left(\frac{N_A+N_D}{N_DN_A}\right)V_{bi}\right]^{1/2}$$

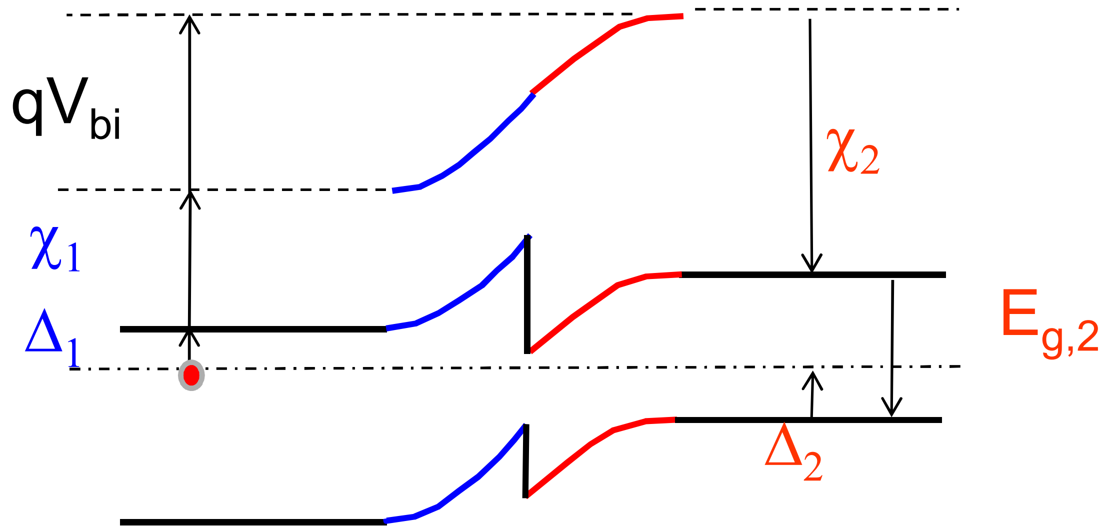
$$\mathcal{E}(0)=\left[\frac{2qV_{bi}}{K_s\varepsilon_0}\left(\frac{N_DN_A}{N_A+N_D}\right)\right]^{1/2}$$



$$W=x_n+x_P \qquad x_n=\frac{N_A}{N_A+N_D}W \qquad V_{bi}=\frac{k_BT}{q}\ln\left(\frac{N_DN_A}{n_i^2}\right)$$

$$N_Dx_n=N_Ax_P \qquad x_p=\frac{N_D}{N_A+N_D}W \qquad V(x)=-\int_x^{x_p}\mathcal{E}(x)\,dx$$

Built-in Potential: heterojunctions

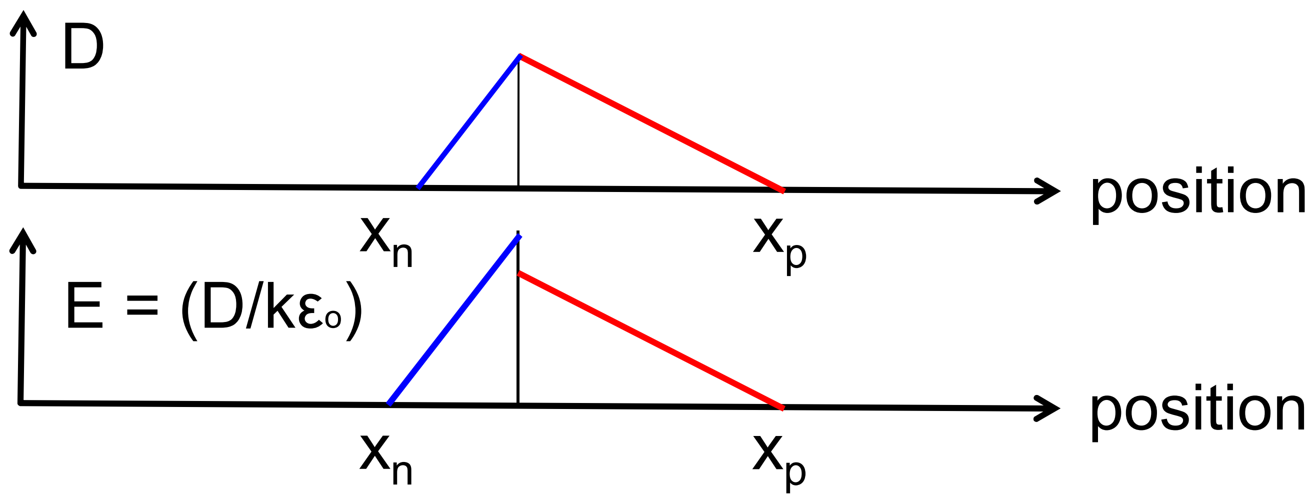


$$qV_{bi} = E_{g,2} - \Delta_2 - \Delta_1 + \chi_2 - \chi_1 \qquad \Delta_1 + \chi_1 + qV_{bi} = \chi_2 + E_{g,2} - \Delta_2$$

$$= \left(E_{g,2} + k_B T \ln \frac{N_A}{N_{V,2}} \right) + k_B T \ln \frac{N_D}{N_{C,1}} + \left(\chi_2 - \chi_1 \right)$$

$$= k_B T \ln \frac{N_A N_D}{N_{V,2} N_{C,1} e^{-E_{g,2}/k_B T}} + \left(\chi_2 - \chi_1 \right)$$

Interface Boundary Conditions: heterojunctions

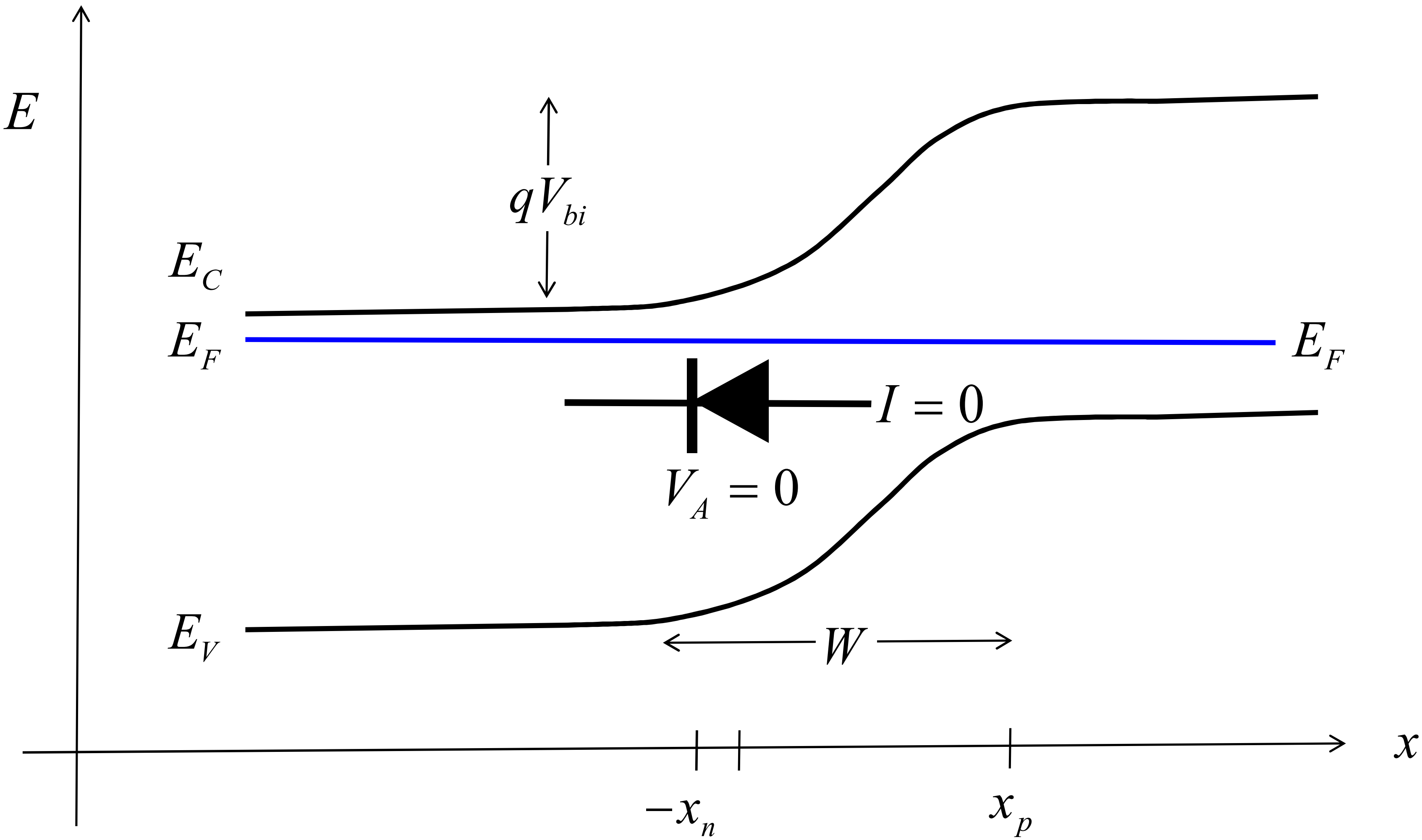


$$D_1 = K_1 \varepsilon_0 E(0^-) = K_2 \varepsilon_0 E(0^-) = D_2$$

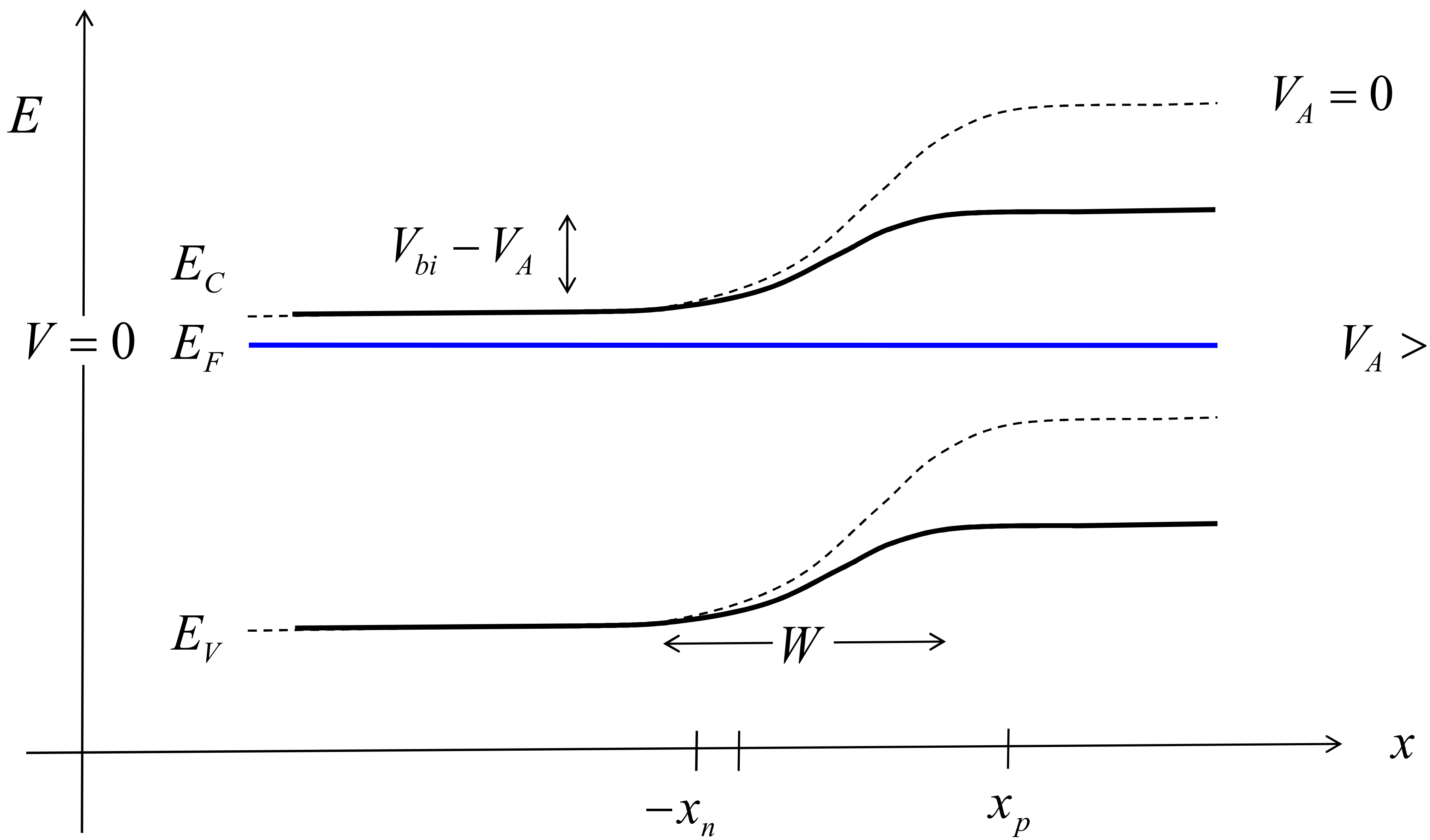
$$E(0^-) = \frac{K_2}{K_1} E(0^+)$$

Displacement is continuous across the interface, but field need not be ..

equilibrium e-band diagram

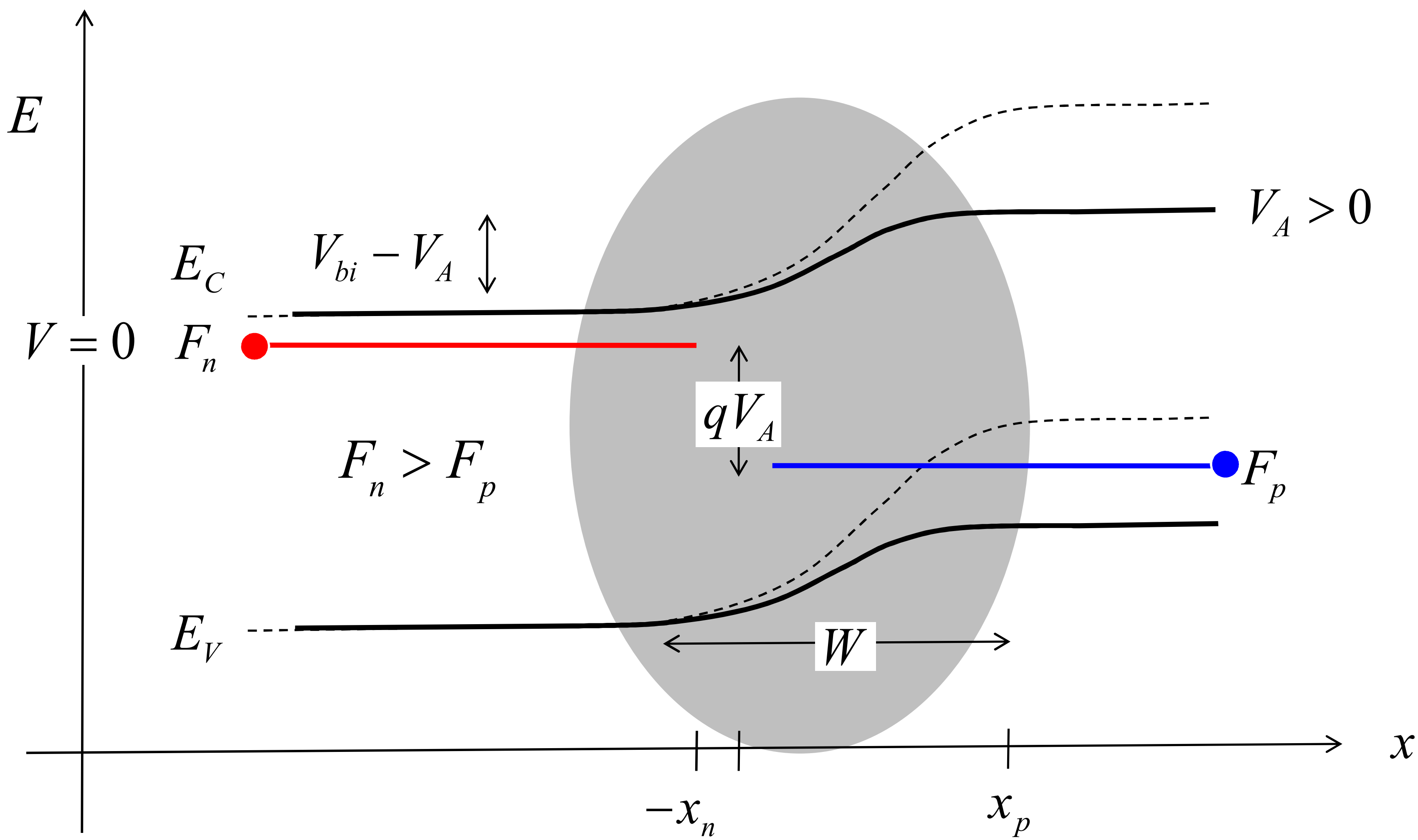


e-band diagram under forward bias

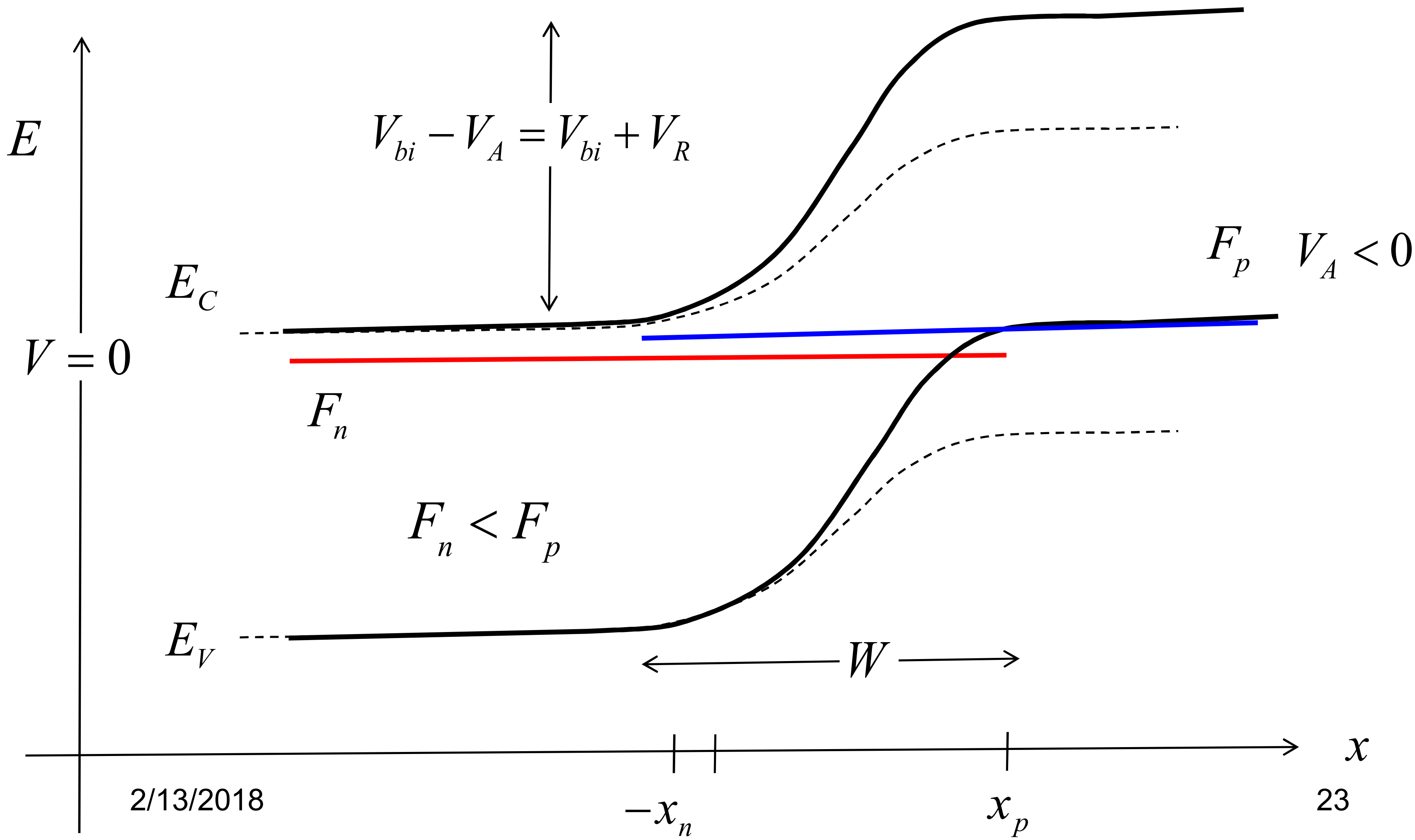


The applied voltage drops across the junction, but…

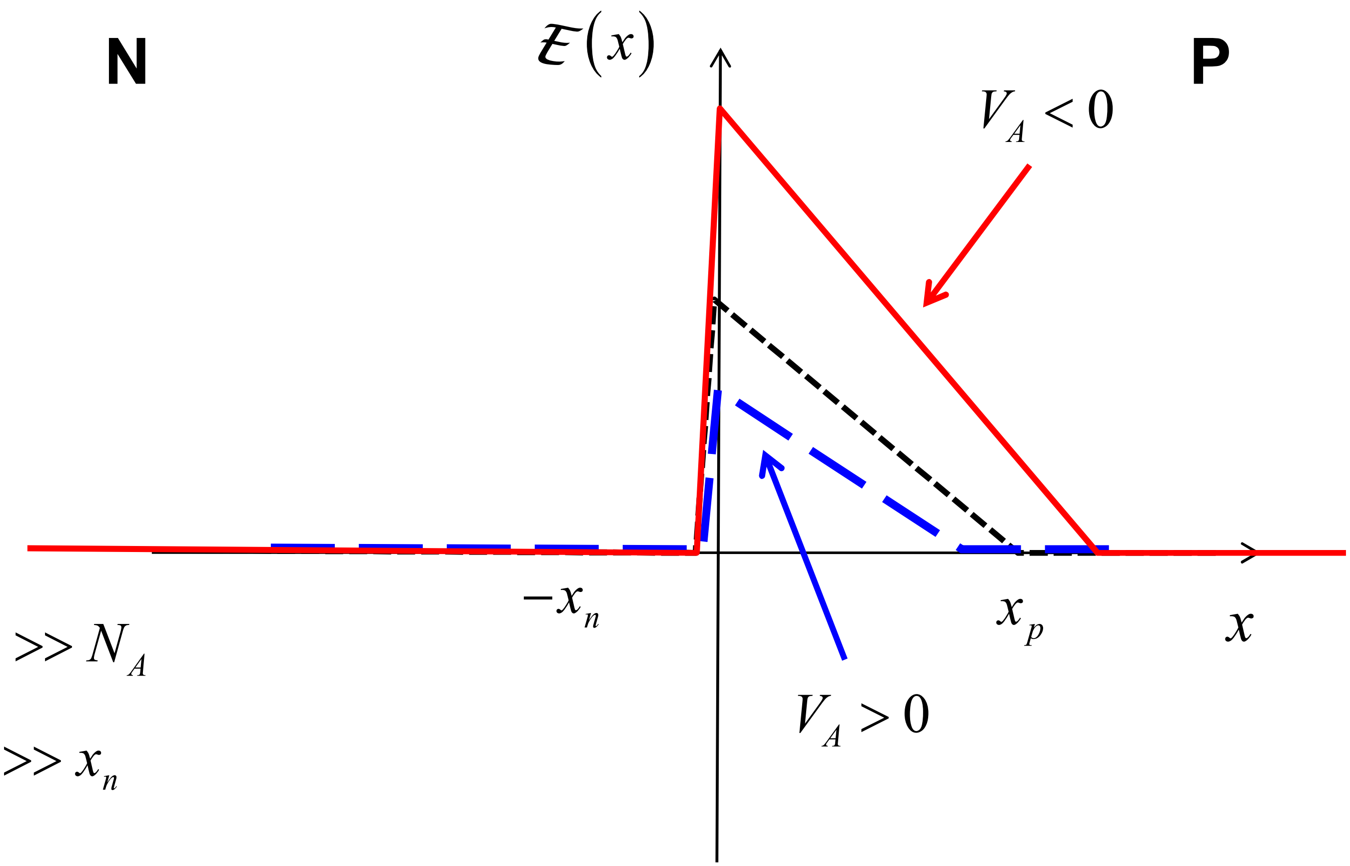
QFL's split



e-band diagram under reverse bias



one-sided junction



$N_D >> N_A$

$x_p >> x_n$

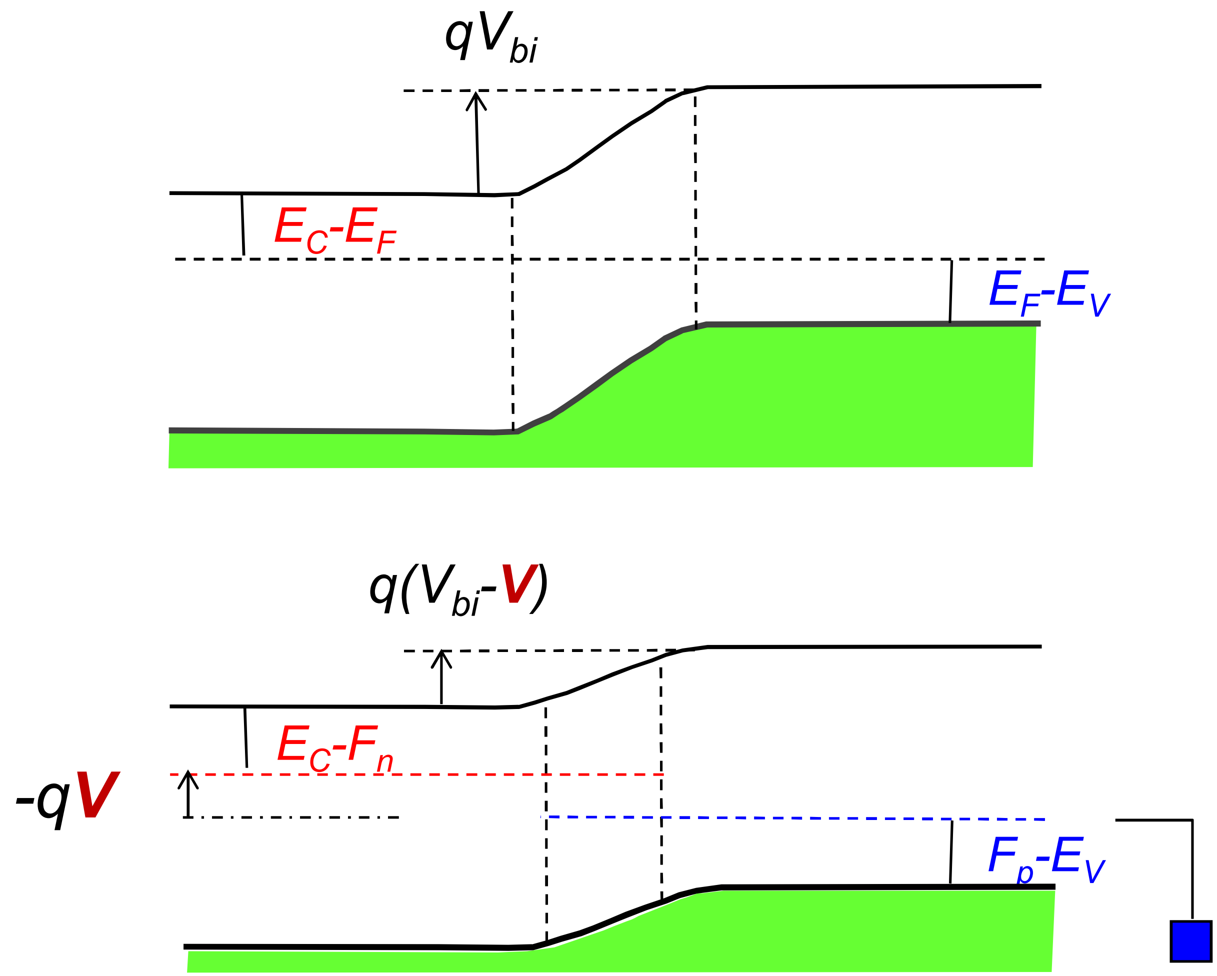
key points (one-sided NP junctions)

$$V_{bi} \approx \frac{k_B T}{q} \ln\left(\frac{N_D N_A}{n_i^2}\right)$$

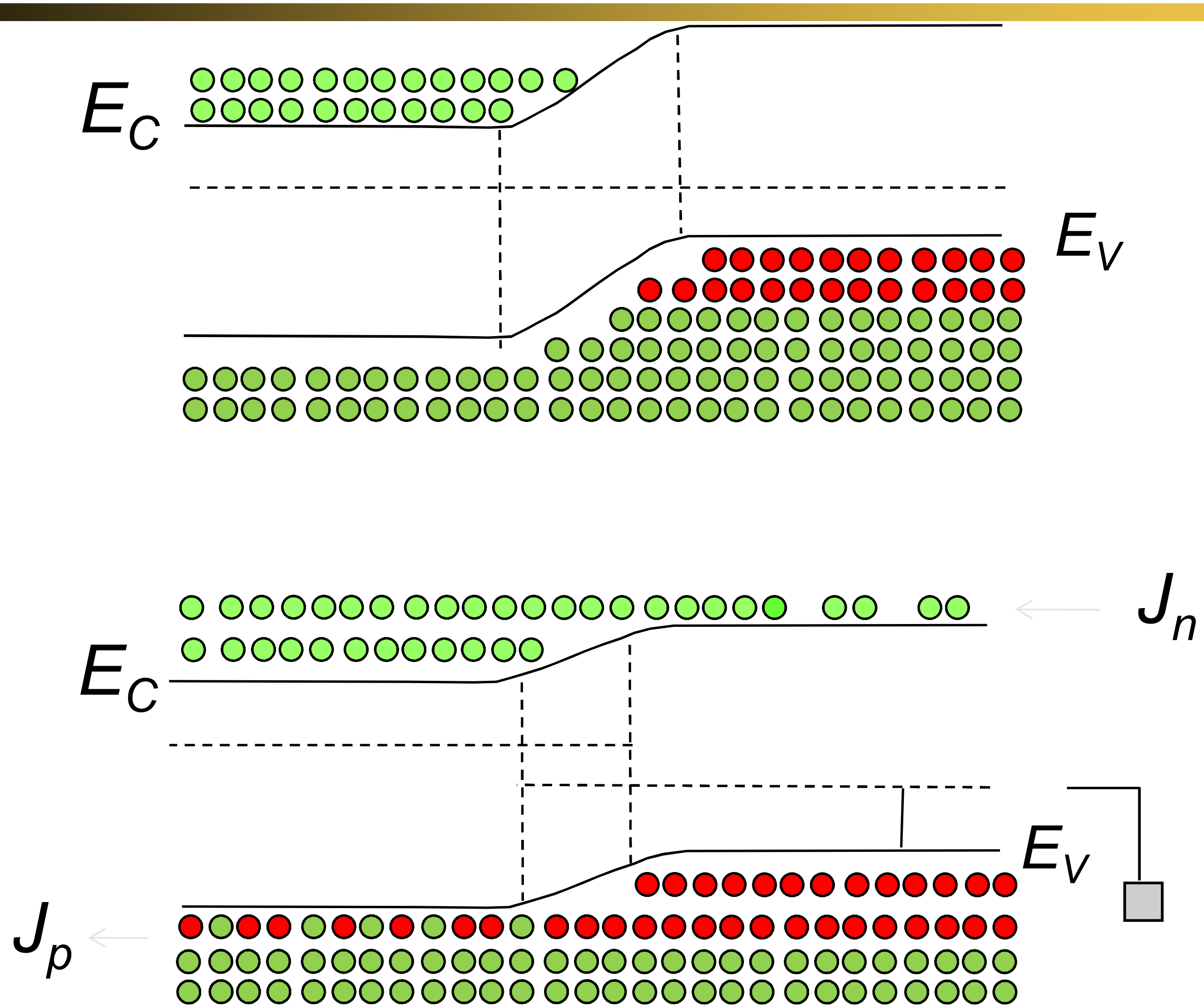
$$W = \left[\frac{2K_S\varepsilon_0}{qN_A}\left(V_{bi} - V_A\right)\right]^{1/2} \quad W \propto \sqrt{V_{bi} - V_A} \quad W \propto \frac{1}{\sqrt{N_A}}$$

$$\mathcal{E}(0) = \frac{2\left(V_{bi} - V_A\right)}{W} \quad \mathcal{E}(0) \propto \sqrt{V_{bi} - V_A} \quad \mathcal{E}(0) \propto \sqrt{N_A}$$

Applying a Bias: Poisson Equation



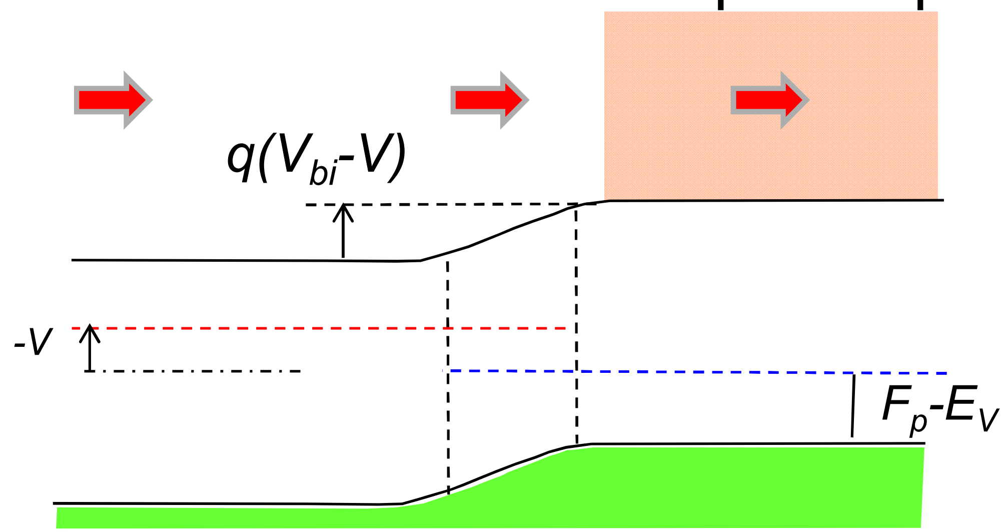
Flat Quasi-Fermi Level up to Junction



One Sided Minority Diffusion

Can calculate current anywhere, let us solve the problem where it is the easiest …

Steady state
Acceptor doped



$$\cancel{\frac{\partial n}{\partial t}} = \frac{1}{q}\frac{dJ_n}{dx} - \cancel{r_n} + \cancel{g_n}$$

$$J_n = qn\cancel{\mu_n \mathcal{E}} + qD_n\frac{dn}{dx}$$

$$0 = D_n\frac{d^2 n}{dx^2}$$

Boundary Conditions

$$n(x=0^+) = n_i\, e^{(F_n - E_i)\beta}$$

$$p(x=0^+) = n_i\, e^{-(F_p - E_i)\beta}$$

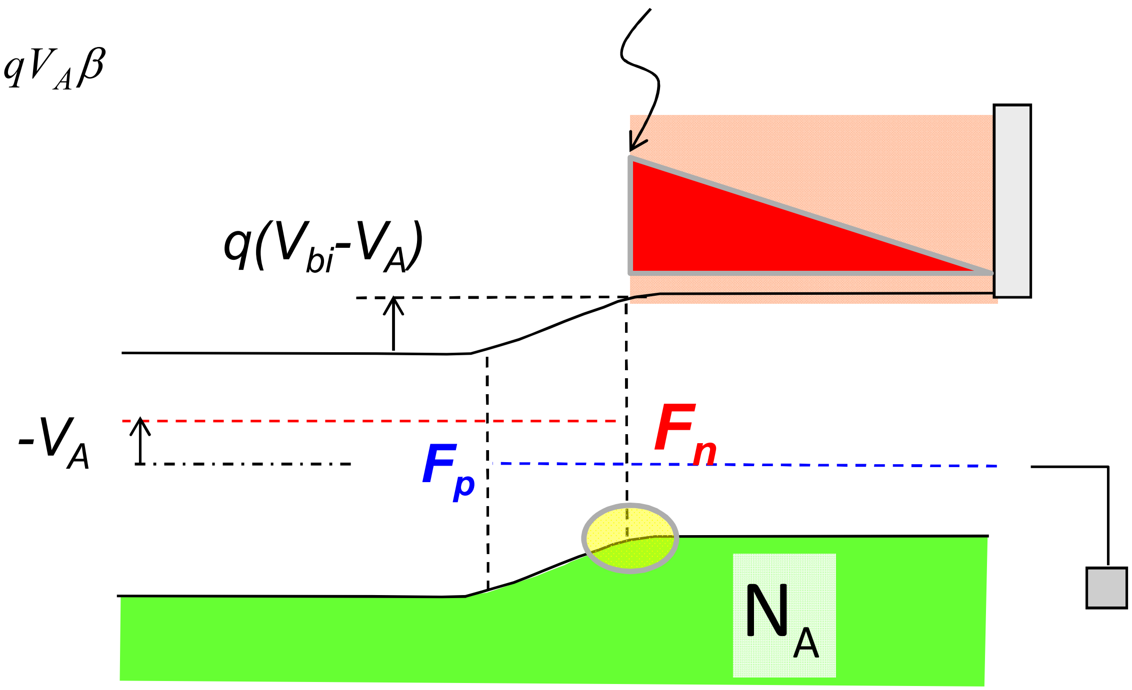
$$\Delta n(0^+) = n(0^+)_{V_G} - n(0^+)_{V_G=0}$$

$$= \frac{n_i^2}{N_A}\left(e^{qV_A\beta} - 1\right)$$

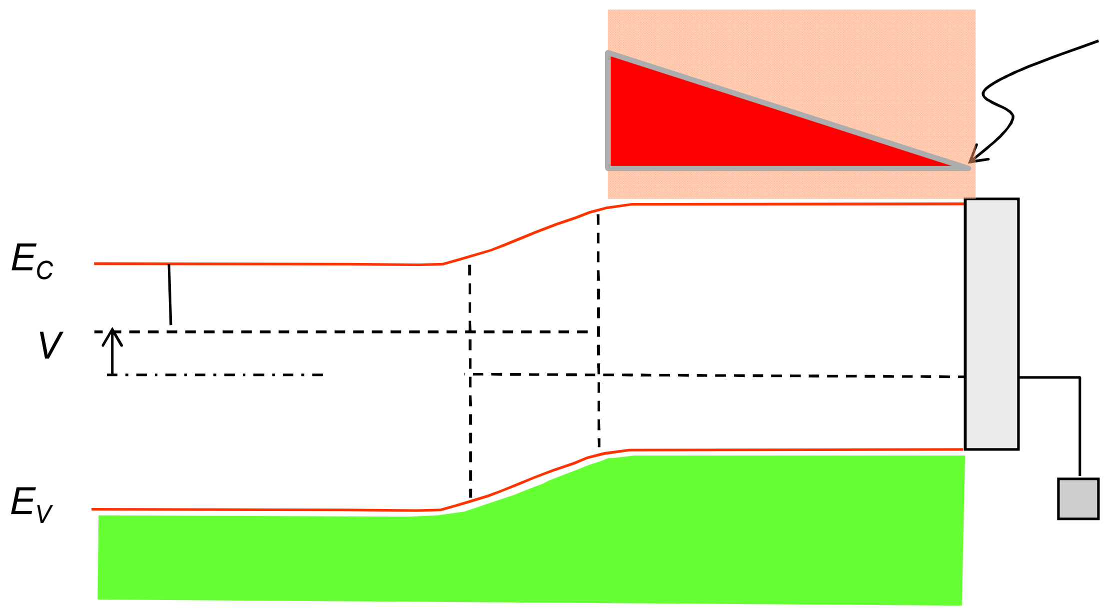
$$np = n_i^2 e^{(F_n - F_p)\beta} = n_i^2 e^{qV_A\beta}$$

$$p(0^+) = N_A$$

$$n(0^+) = \frac{n_i^2}{N_A} e^{qV_A\beta}$$



Right Boundary Condition



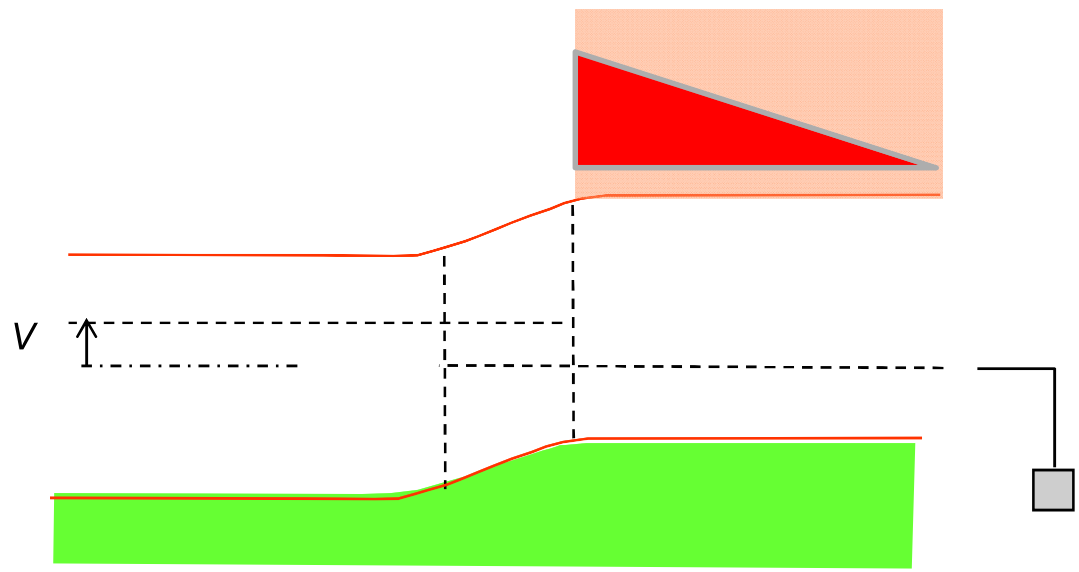
$$n(x = W_p) \approx \frac{n_i^2}{N_A}$$

$$\Delta n(x = W_p) = 0$$

Example: One Sided Minority Diffusion

$$D_N \frac{d^2 n}{dx^2} = 0$$

$$\Delta n(x,t) = C + Dx$$



$$x = W_p, \quad \Delta n(x = W_p) = 0 \Rightarrow C = -DW_p$$

$$x = 0', \quad \Delta n(x = 0) = \frac{n_i^2}{N_A}\left(e^{qV_A\beta} - 1\right) = C$$

$$\Delta n(x,t) = \frac{n_i^2}{N_A}\left(e^{qV_A\beta} - 1\right)\left(1 - \frac{x}{W_p}\right)$$

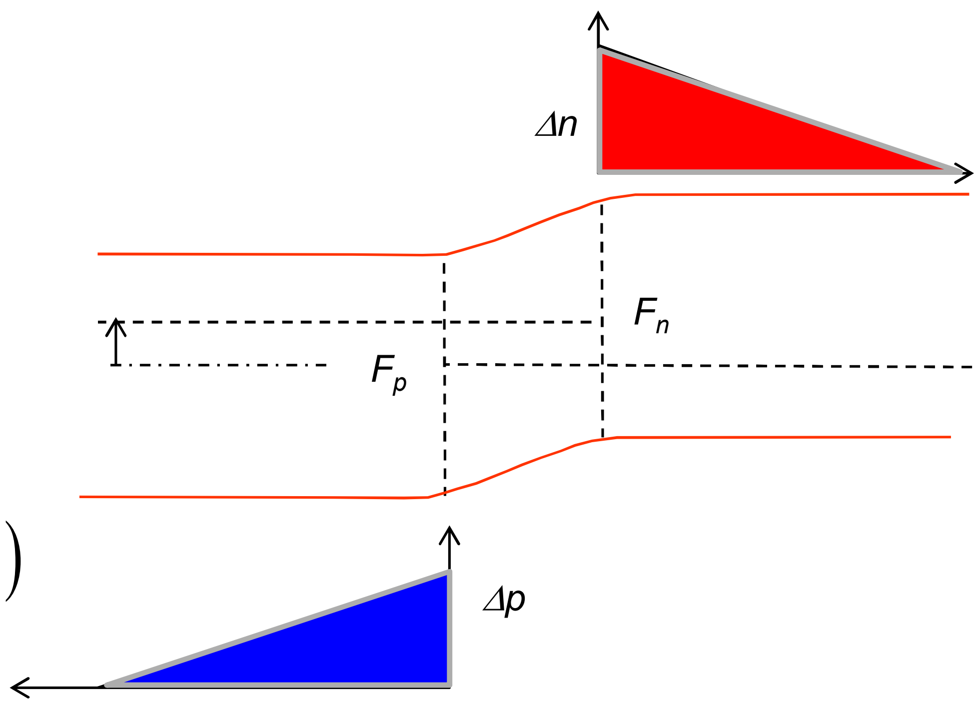
Electron & Hole Fluxes

$$\Delta n(x) = \frac{n_i^2}{N_A}\left(e^{qV_A\beta} - 1\right)\left(1 - \frac{x}{W_p}\right)$$

$$\mathbf{J}_N = qn\mu_N\mathcal{E} + qD_N\nabla n$$

$$J_n = qD_n \left.\frac{dn}{dx}\right|_{x=0} = -\frac{qD_n}{W_p}\frac{n_i^2}{N_A}\left(e^{qV_A\beta} - 1\right)$$

$$J_p = -qD_p \left.\frac{dp}{dx}\right|_{x=0'} = -\frac{qD_p}{W_n}\frac{n_i^2}{N_D}\left(e^{qV_A\beta} - 1\right)$$



Exam 2 Equation Sheet

Physical Constants	Silicon parameters ($T = 300$ K)
$\hbar = 1.055\times10^{-34}$ J·s	$N_C = 3.23\times10^{19}$ cm^{-3}
$m_0 = 9.109\times10^{-31}$ kg	$N_V = 1.83\times10^{19}$ cm^{-3}
$k_B = k = 1.38\times10^{-23}$ J/K	$n_i = 1.1\times10^{10}$ cm^{-3}
$q = 1.602\times10^{-19}$ C	$K_s = 11.8$
$\epsilon_0 = 8.854\times10^{-12}$ F/m	

Miller Indices: (hkl) {hkl} [hkl] <hkl>

Density of states $g_C(E) = \frac{(m_n^*)^{3/2}\sqrt{2(E-E_C)}}{\pi^2\hbar^3}$

Fermi function $f(E) = \frac{1}{1+e^{(E-E_F)/kT}}$

Intrinsic carrier concentration $n_i = \sqrt{N_C N_V}e^{-E_g/2kT}$

Equilibrium carrier densities: $N_C = \frac{1}{4}\left(\frac{2m_n^* kT}{\pi\hbar^2}\right)^{3/2}$ $\quad N_V = \frac{1}{4}\left(\frac{2m_p^* kT}{\pi\hbar^2}\right)^{3/2}$

$n_0 = N_C e^{(E_F-E_C)/kT} = n_i e^{(E_F-E_i)/kT}$ $\quad p_0 = N_V e^{(E_V-E_F)/kT} = n_i e^{(E_F-E_i)/kT}$

Space charge neutrality: $p - n + N_D^+ - N_A^- = 0$

Law of Mass Action: $n_0 p_0 = n_i^2$

Non-equilibrium carriers: $n = N_C e^{(F_N-E_C)/kT}$ $\quad p = N_V e^{(E_V-F_P)/kT}$ $\quad np = n_i^2 e^{(F_N-F_P)/kT}$

Conductivity/resistivity: $\sigma = \sigma_n + \sigma_n = q(n\mu_n + p\mu_p) = 1/\rho$

Exam 2 Equation Sheet

Drift-diffusion current equations: $J_n = nq\mu_n \mathcal{E}_x + qD_n \frac{dn}{dx} = n\mu_n \frac{dF_n}{dx}$ $\frac{D_n}{\mu_n} = \frac{kT}{q}$

$J_p = pq\mu_p \mathcal{E}_x - qD_p \frac{dp}{dx} = p\mu_p \frac{dF_p}{dx}$ $\frac{D_p}{\mu_p} = \frac{kT}{q}$

Carrier conservation equations: $\frac{\partial n}{\partial t} = +\nabla \cdot \left(\frac{J_n}{q}\right) + G_n - R_n$

$\frac{\partial p}{\partial t} = -\nabla \cdot \left(\frac{J_p}{q}\right) + G_p - R_p$

Poisson's equation: $\nabla \cdot (\epsilon \mathcal{E}) = \rho$

SRH carrier recombination: $R = \Delta n / \tau_n$ or $R = \Delta p / \tau_p$

Minority carrier diffusion equation: $\frac{\partial \Delta n}{\partial t} = D_n \frac{\partial^2 \Delta n}{\partial x^2} - \frac{\Delta n}{\tau_n} + G_L$ $L_{D,n} = \sqrt{D_n \tau_n}$

PN homojunction electrostatics: $V_{bi} = \frac{kT}{q} \ln\left(\frac{N_D N_A}{n_i^2}\right)$ $\frac{d\mathcal{E}}{dx} = \frac{\rho(x)}{K_s \epsilon_o}$

$W = \sqrt{\frac{2K_s \epsilon_o V_{bi}}{q}\left(\frac{N_A + N_D}{N_A N_D}\right)}$ $x_n = \left(\frac{N_A}{N_A + N_D}\right) W$ $x_p = \left(\frac{N_D}{N_A + N_D}\right) W$ $\mathcal{E}(0) = \sqrt{\frac{2qV_{bi}}{K_s \epsilon_o}\left(\frac{N_A N_D}{N_A + N_D}\right)}$

Exam 2 Fall 2016

1 (8 points). If the applied voltage and thus the quasi-Fermi level splitting of a pn homojunction is $6kT$, then what is the np product? Assume that n_i is the intrinsic carrier concentration for this material.

A. n_i^2
B. $6n_i^2$
C. $e^6 n_i^2$
D. $\ln(6) \cdot n_i^2$
E. 6 cm^{-6}

2 (8 points). What assumption(s) are made to derive the minority carrier diffusion equation (MCDE) in its most general form?

A. Flat conduction and valence bands
B. Spatial uniformity
C. Steady-state conditions
D. A and C
E. All of the above

Exam 2 Fall 2016

3 (8 points). The built-in potential of an NP junction is roughly equal to what value?

A. The thermal voltage, kT/q
B. $3kT/2q$
C. 220 V
D. The electron affinity of the semiconductor
E. The bandgap of the semiconductor

4 (8 points). What is the purpose of photoresist in semiconductor manufacturing?

A. To deposit polycrystalline films
B. To dope semiconductors
C. To deposit metals
D. To help pattern semiconductor devices
E. To image the final semiconductor devices

Exam 2 Fall 2016

5 (8 points). What assumption(s) are made in the depletion approximation?

A. All mobile charge carriers recombine in the depletion region
B. All acceptors and donors are fully ionized
C. The dielectric constant is identical in the p and n regions
D. A and B
E. All of the above

Exam 2 Fall 2016

Assume that an n-type region of spatially uniform crystalline silicon with $\mu_p = 450\ \mathrm{cm}^2/\mathrm{V}\cdot\mathrm{s}$ and lifetime $\tau_p = 10\ \mu\mathrm{s}$ is uniformly illuminated by a photon flux $G_L = 10^{20}\ /\mathrm{cm}^3\cdot\mathrm{s}$, reaches a steady state, and is then switched off at $t = 0$.

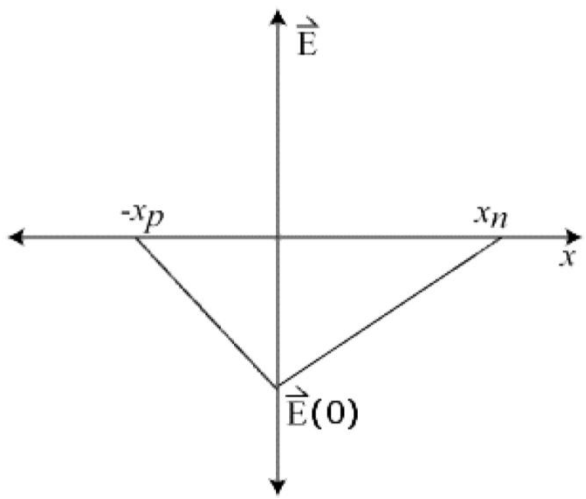
a. Write down the simplest form of the minority carrier diffusion equation that accurately describes its behavior for all times. Briefly justify your answer.

b. Sketch the time-dependent decay of the excess minority carrier concentration. Be sure to label both the x and y axes, and include at least 2 numerical values on each axis (with justification).

c. At what time does the excess minority carrier concentration reach 1% of its original steady-state value?

Exam 2 Fall 2016

The electric field versus position for a crystalline silicon ($K_s = 11.68$) pn homojunction is given by the following graph (may not be precisely to scale):



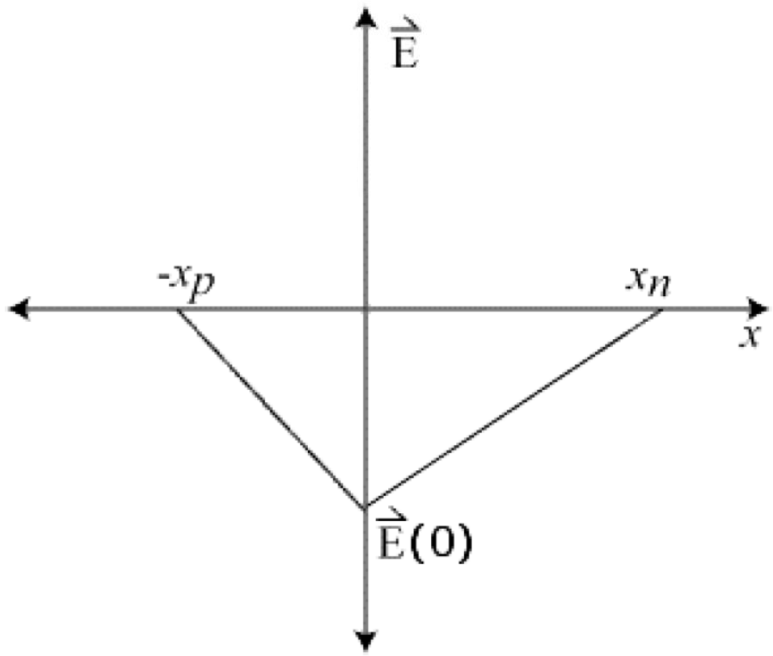
For this problem, assume that the junction is abrupt at $x = 0$ with a flat doping profile on each side, $\mathcal{E}(0) = -12$ kV/cm, the n-type depletion length $x_n = 620$ nm, and the p-type depletion length $x_p = 420$ nm. Please use the depletion approximation in your solutions.

a. What is the built-in voltage V_{bi}?

b. What is the value of N_A in the p-type region, and N_D in the n-type region?

Exam 2 Fall 2016

c. Assume that a forward bias of $V_{bi}/2$ is applied to the junction. How does this quantitatively change the electric field profile versus position? Please sketch the resulting graph on top of the diagram given previously in the space below. Your results need not be exactly to scale, but be sure to calculate the new values $\mathcal{E}'(0)$, x_p', and x_n', and indicate their position clearly relative to the previous values associated with zero bias.



Review Questions

1) If you apply negative bias to a terminal, which direction does the band move?
2) What is the difference between Fermi & Quasi-Fermi levels?
3) How can we get away with solving just the MCDE in certain cases?
4) What are the most basic parameters of a p-n junction, that can be used to calculate everything else?