NAME:_____   **PUID: :**_____

SOLUTIONS: ECE 305 Exam 2: Spring 2015
February 13, 2015
Mark Lundstrom
Purdue University

This is a closed book exam. You may use a calculator and the formula sheet at the end of this exam. Following the ECE policy, the calculator **must** be a Texas Instruments TI-30X IIS scientific calculator.

There are three equally weighted questions. To receive full credit, you must **show your work** (scratch paper is attached).

The exam is designed to be taken in 50 minutes.

Be sure to fill in your name and Purdue student ID at the top of the page.

DO NOT open the exam until told to do so, and stop working immediately when time is called.

The last page is an equation sheet, which you may remove, if you want.

75 points possible, 10 per question

  1)   25 points (5 point per part)

  2)   25 points (5 points per part)

  3)   25 points (5 points per part)

------------------------ Course policy ------------------------

I understand that if I am caught cheating in this course, I will earn an F for the course and be reported to the Dean of Students.


Read and understood: _____
                                                    signature

Answer the **five multiple choice questions** below by **drawing a circle** around the **one, best answer**.

1a)   Consider holes diffusing down a concentration gradient. What force pushes them down?
a) $+q\mathcal{E}$
b) $-q\mathcal{E}$
c) $+D_p\, dp/dx$
d) $-D_p\, dp/dx$
e) none of the above

1b)   For a semiconductor in equilibrium at very low temperatures, what is the most important scattering mechanism?
a) Lattice scattering.
b) Ionized impurity scattering.
c) Auger scattering.
d) Impact ionization scattering.
e) Polar optical phonon scattering.

1c)   Which of the following is a statement of "low-level injection" in a p-type semiconductor?
a) $N_D \ll N_A$.
b) $N_A \ll N_D$.
c) $p_0 \ll N_A$.
d) $\Delta n \ll N_A$.
e) $\Delta p \ll n_i$.

1d)   To write the steady-state, minority carrier diffusion equation for a p-type semiconductor of length, W, as $d^2\Delta n_p/dx^2 = 0$, which one of the following must be true of the diffusion length, L_n?
a) $L_n \ll W$.
b) $L_n \gg W$.
c) $L_n = W$.
d) $L_n \ll L_p$.
e) There is no restriction on L_n.

1e)   If the quasi-Fermi level splitting is $F_n - F_p = 10k_BT$, then what is np?

   a) $10n_i^2$

   b) $\ln(10)n_i^2$

   c) $e^{10}n_i^2$

   d) n_i^2

   e) $\sqrt{10}n_i^2$

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2)   This problem concerns the energy band diagram shown below. Answer each of the questions that follow.
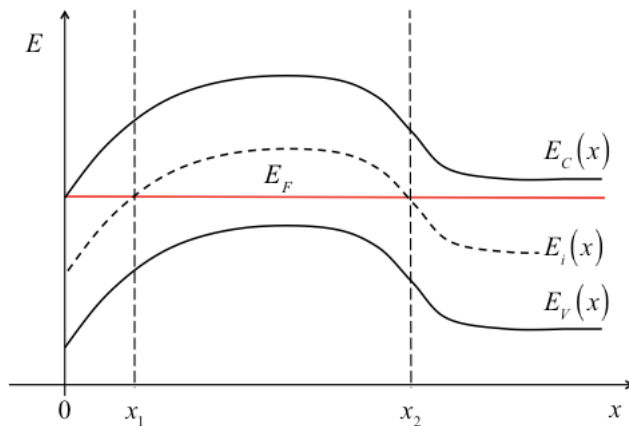
2a)   Assume that the semiconductor is Si at room temperature. Determine the electron density at $x = 0$ (a numerical answer is required).

Solution:

At x = 0, $E_F = E_C$. We know that $n_0 = N_C e^{(E_F - E_C)/k_BT}$, so $n(x = 0) = N_C$

From the formula sheet, $N_C(300\text{ K}) = 3.23 \times 10^{19}$ cm^{-3}, so

$$\boxed{n(x = 0) = N_C = 3.23 \times 10^{19} \text{ cm}^{-3}}$$

2b) Assume that the semiconductor is Si at room temperature. Determine the electron density at $x = x_1$ (a numerical answer is required).

Solution:

At $x = x_1$, $E_F = E_i$. We know that $n_0 = n_i e^{(E_F - E_i)/k_B T}$, so $n(x = x_1) = n_i$

From the formula sheet, $n_i(300 \text{ K}) = 1.00 \times 10^{10}$ cm^{-3}, so

$$\boxed{n(x = x_1) = n_i = 1.00 \times 10^{10}} \text{ cm}^{-3}$$

2c) Sketch the electrostatic potential vs. position. Assume that $V = 0$ for $x \gg x_2$. Make your sketch as accurate as possible without putting numbers on the vertical axis.
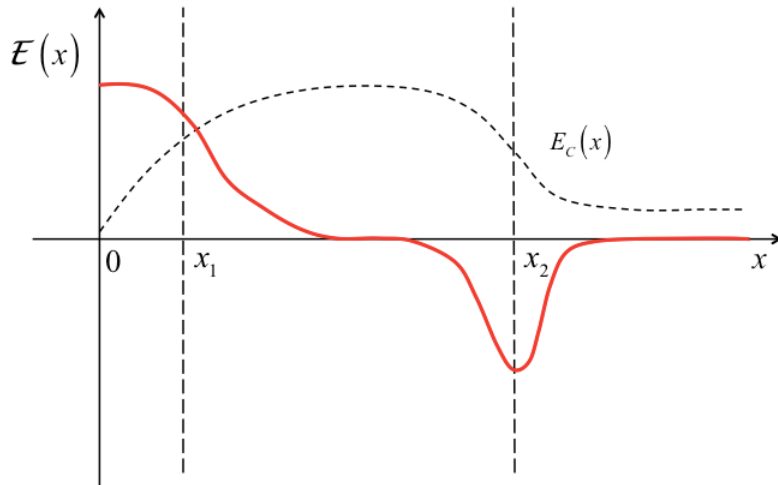
Solution:

The electron potential energy is a constant minus q times the electrostatic potential, so we flip $E_C(x)$ $E_V(x)$, or $E_i(x)$ upside down. We also need to define the reference potential so that that $V = 0$ for $x \gg x_2$.
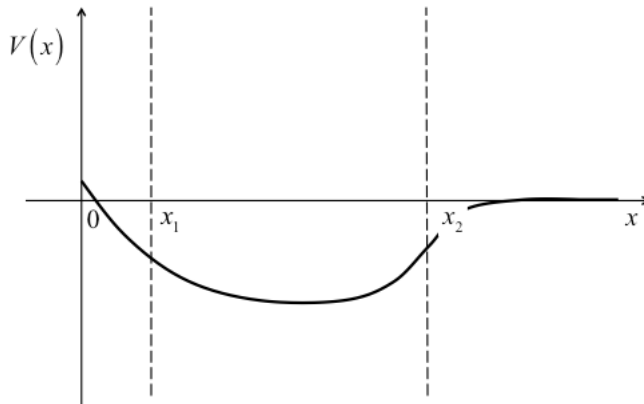
2d) Sketch the electric field vs. position. Make your sketch as accurate as possible without putting numbers on the vertical axis.

Solution:

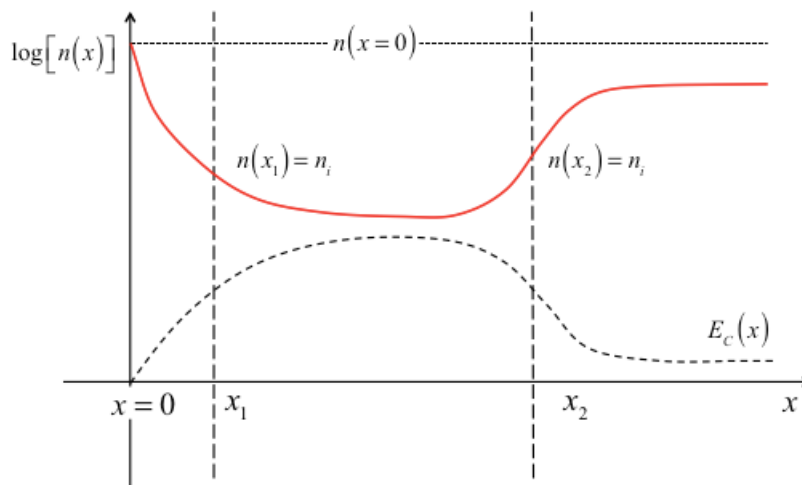The electric field is one over q times the slope of $E_C(x)$ $E_V(x)$, or $E_i(x)$.

2e) Sketch the electron density, $\log\left[n(x)\right]$ vs. position. Make your sketch as accurate as possible without putting numbers on the vertical axis.
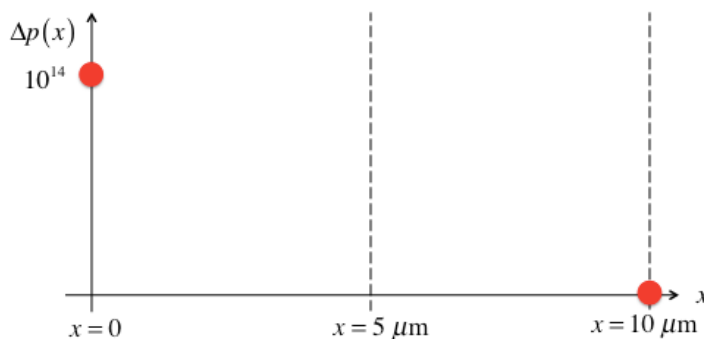
Solution:

Begin with $n_0(x) = n_i e^{(E_F - E_i(x))/k_B T}$

$$\log\left[n_0(x)\right] = \left(\frac{1}{2.3}\right)\ln n_i + \left(\frac{1}{2.3}\right)\frac{\left(E_F - E_i(x)\right)}{k_B T} \quad \text{so}$$

$\log\left[n_0(x)\right] \propto \left(E_F - E_i(x)\right)$ and we can just read it off the energy band diagram.

3)   This problem concerns an n-type semiconductor at room temperature, in steady-state and low level injection with no optical generation. The hole mobility is $\mu_p = 500 \text{ cm}^2/\text{V-s}$ for $0 \le x \le 5 \ \mu\text{m}$ and $\mu_p = 50 \text{ cm}^2/\text{V-s}$ for $5 \le x \le 10 \ \mu\text{m}$. The minority hole lifetime is $\tau_p = 10^{-4} \text{ s}$. The excess hole densities at left and right ends in the figure below are $\Delta p(x=0) = 10^{14} \text{ cm}^{-3}$ and $\Delta p(x=10 \ \mu\text{m}) = 0$. Answer the following questions.

3a)   Simplify the general minority carrier diffusion equation for $0 \le x \le 5 \ \mu\text{m}$. You must show your work and explain how you get your answer. Draw a box around your answer. **You do not need to solve the equation.**

Solution:

Begin with: $\dfrac{\partial \Delta p}{\partial t} = D_p \dfrac{\partial^2 \Delta p}{dx^2} - \dfrac{\Delta p}{\tau_p} + G_L$

Assuming **steady-state** and **no generation**, this simplifies to: $0 = D_p \dfrac{d^2 \Delta p}{dx^2} - \dfrac{\Delta p}{\tau_p} + 0$

Check the diffusion length:

$D_p = \dfrac{k_B T}{q} \mu_p = 0.026 \times 500 = 13 \text{ cm}^2/\text{s}$

$L_p = \sqrt{D_p \tau_p} = \sqrt{13 \times 10^{-4}} = 3.6 \times 10^{-2} \text{ cm} = 360 \ \mu\text{m}$

The region of interest is 5 micrometers long, and the diffusion length is 360 micrometers. We can **ignore recombination** because the diffusion length is very long.

The final result is:

$$\boxed{\dfrac{d^2 \Delta p}{dx^2} = 0}$$

3b) Simplify the general minority carrier diffusion equation for $5 \le x \le 10 \ \mu m$. You must show your work and explain how you get your answer. Draw a box around your answer. **You do not need to solve the equation.**

Solution

Steady-state and no recombination, so just as in 3a), the MCDE simplifies to:

$$0 = D_p \frac{d^2 \Delta p}{dx^2} - \frac{\Delta p}{\tau_p} + 0$$

Need to check the diffusion length in this region.

$$D_p = \frac{k_B T}{q} \mu_p = 0.026 \times 50 = 1.3 \ \text{cm}^2/\text{s}$$

$$L_p = \sqrt{D_p \tau_p} = \sqrt{1.3 \times 10^{-4}} = 1.14 \times 10^{-2} \ \text{cm} = 114 \ \mu m$$

The region of interest is 5 micrometers long, the diffusion length is 114 micrometers. We can **ignore recombination** in this region too because the diffusion length is very long.

The final result is:
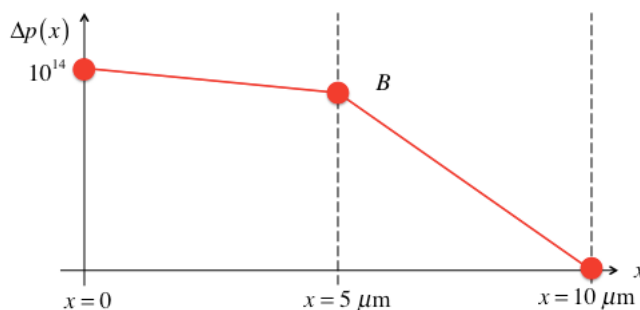
$$\boxed{\frac{d^2 \Delta p}{dx^2} = 0}$$

3c) Sketch and **explain** the solution for $0 \le x \le 10 \ \mu m$.

Solution:

We recognize that the solutions to the simplified MCDE are straight lines. Boundary conditions at the two ends are given, and the solutions must match at $x = 5 \ \mu m$, so we say $\Delta p(x = 5 \ \mu m) = B$, where B will be determined later.

The solution is sketched below.

Note: The precise value of B will be determined later, but the two solutions must match, so that we don't get an infinite gradient (and therefore infinite current) there.

3d) Sketch and **explain** the diffusion current, $J_p = -qD_p \, dp/dx$ vs. position.

Solution:
The diffusion current is $J_p = -qD_p \, dp/dx$, so it is constant for $0 \le x \le 5 \, \mu m$ and for $5 \le x \le 10 \, \mu m$. The current must be continuous at $x = 5 \, \mu m$. If it were not, then there would be more in-flow than out-flow of holes at that point (or more out-flow than in-flow), and the hole density would increase (or decrease) with time and we would not have a steady-state solution. We conclude that $J_p = -qD_p \, dp/dx = \text{constant} > 0$.
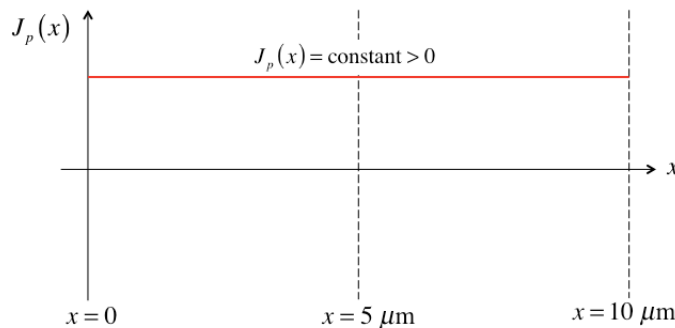
3e)   Solve for $\Delta p(x = 5 \, \mu m)$.

Solution:
Let $A = \Delta p(x = 0) = 10^{14}$.

Current for $0 \le x \le 5 \, \mu m$: $J_{p1} = -qD_{p1} \, dp/dx = -qD_{p1} \left(\dfrac{B - A}{5 \times 10^{-4}} \right)$

Current for $5 \le x \le 10 \, \mu m$: $J_{p2} = -qD_{p2} \, dp/dx = -qD_{p2} \left(\dfrac{0 - B}{5 \times 10^{-4}} \right)$

Current is continuous at $x = 5 \, \mu m$:

$J_{p1} = -qD_{p1} \left(\dfrac{B - A}{5 \times 10^{-4}} \right) = J_{p2} = -qD_{p2} \left(\dfrac{0 - B}{5 \times 10^{-4}} \right)$ $\qquad\qquad D_{p1}(A - B) = D_{p2}(B)$

$B = \left(1 - \dfrac{D_{p2}}{D_{p1}} \right) \dfrac{A}{\left(1 + \dfrac{D_{p2}}{D_{p1}} \right)} = \dfrac{A}{(1 + 0.1)} A = 0.91A$

$\boxed{B = \Delta p(x = 5 \, \mu m) = 0.91A = 0.91 \times 10^{14} \; cm^{-3}}$

Discussion: In the figure of 3a), we see that the slope will be very small for $0 \leq x \leq 5\,\mu$m and much steeper for $5 \leq x \leq 10\,\mu$m. To keep the current constant, we need a steep slope because the diffusion coefficient in the region, $5 \leq x \leq 10\,\mu$m, is 10 times smaller than in the region $0 \leq x \leq 5\,\mu$m.