NAME: Solution PUID:

ECE 305 – Fall 2015

Exam 3 – Monday, November 2, 2015

This is a closed book exam. You may use a calculator and the formula sheet at the end of this exam. Following the ECE policy, the calculator **must** be a Texas Instruments TI---30X IIS scientific calculator.

To receive full credit, you must **show your work** (scratch paper is attached). The exam is designed to be taken in 50 minutes (or less). Be sure to fill in your name and Purdue student ID at the top of the page. DO NOT open the exam until told to do so, and stop working immediately when time is called. The last 2 pages are equation sheets, which you may remove, if you want.

100 points possible,

I) 40 points (8 points per question)
II) 30 points
III) 30 points

Course policy

If I am caught cheating, I will earn an F in the course & be reported to the Dean of Students.

I repeat: ____

____

Signature: ____

Part I: Answer the 5 multiple choice questions below by entering them on your IDP-15 Scantron.

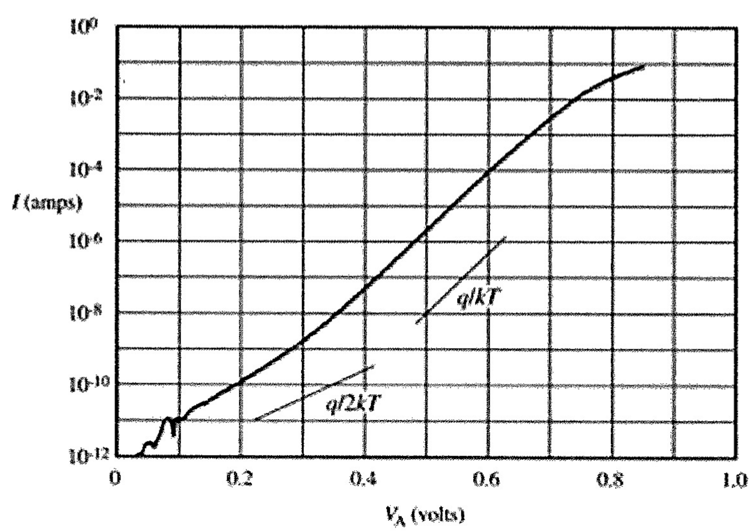
1 (8 points). Which of the following is the Schottky barrier height?

a. The difference between the conduction band at the MS junction of an n-type semiconductor and the Fermi level in the metal
b. The difference between the valence band at the MS junction of an n-type semiconductor and the Fermi level in the metal
c. The difference between the conduction band at the MS junction of a p-type semiconductor and the Fermi level in the metal
d. The difference between the valence band at the MS junction of an p-type semiconductor and the Fermi level in the metal
e. a and d

2 (8 points). The depletion and potential drop in the metal side of the Schottky diode is?

a. Very small, negligible
b. Small, but not negligible
c. Large
d. Must know doping of metal
e. Under steady-state conditions

3 (8 points). In the following diagram, what non-ideality accounts for the current roll-over at large forward biases ($V > 0.7$ V)?



a. Recombination in the space-charge region
b. Avalanche diode
c. Quantum-mechanical tunneling
d. Recombination in the quasi-neutral region
e. Series resistance

4 (8 points). For a metal-semiconductor diode, which of the following is true?

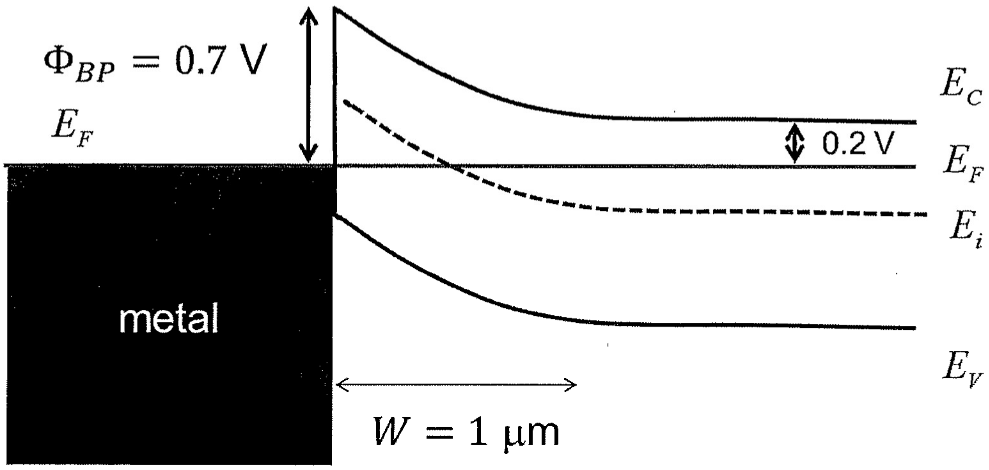
a. The saturation current density is much larger than a PN junction with the same bandgap semiconductor
b. The n=2 current is absent
c. The diode turn-on voltage is reduced compared to a PN junction made from the same semiconductor
d. All of the above
e. None of the above

5 (8 points). Which is true of the junction capacitance in both PN and MS diodes?

a. It is proportional to $1/\sqrt{V_{bi} - V_A}$
b. It is proportional to $\sqrt{V_{bi} - V_A}$
c. It is proportional to $1/(V_{bi} - V_A)$
d. It is proportional to $V_{bi} - V_A$
e. It is proportional to $\exp(qV_A/kT)$

Part II (Free Response, 30 points)

Consider an ideal metal-semiconductor with the band diagram shown below. The semiconductor has a relative dielectric constant of 12 and an electron affinity of 4.0 eV. Answer the following questions.



a. What is the built-in voltage of this junction?

$$qV_{bi} = \Phi_{BP} - (E_C - E_F)_{QN}$$
$$= 0.7 - 0.2 \text{ eV}$$
$$\boxed{V_{bi} = 0.5 \text{ V}}$$

b. What is the numerical value of the maximum electric field $\mathcal{E}(0)$?

Since $\frac{1}{2}\mathcal{E}(0)W = V_{bi}$,

$$\mathcal{E}(0) = \frac{2V_{bi}}{W} = \frac{2 \cdot 0.5 \text{ V}}{10^{-4} \text{ cm}}$$
$$\boxed{\mathcal{E}(0) = 10^4 \text{ V/cm}}$$

c. What is the doping density of the semiconductor?

From Poisson's equation, $\frac{d\mathcal{E}}{dx} = \frac{\overset{0}{\mathcal{E}(W)} - \mathcal{E}(0)}{W} = -\frac{\rho}{\epsilon}$

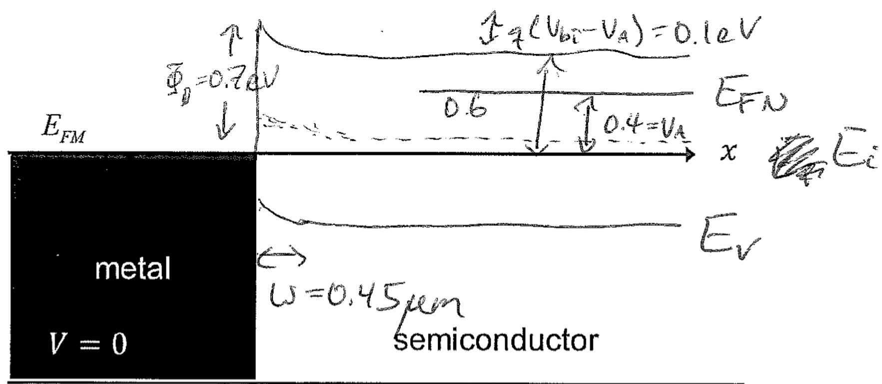
$$-\frac{\mathcal{E}(0)}{W} = -\frac{qN_D}{K_s\epsilon_0} \Rightarrow N_D = \frac{K_s\epsilon_0\mathcal{E}(0)}{qW} = \frac{12\cdot(8.854\cdot10^{-14}\frac{F}{cm})(10^4\frac{V}{cm})}{(1.6\cdot10^{-19}C)(10^{-4}cm)}$$
$$= 6.6\times10^{1-14+4+19+4}/cm^3$$
$$\boxed{N_D = 6.6\times10^{14}/cm^3}$$

d. What is the numerical value of the small signal junction capacitance ($V_A = 0$)? Assume the cross-sectional area $A = 0.1\ \text{cm}^2$.

$$C_j = A\sqrt{\frac{qK_s\epsilon_0 N_D}{2V_{bi}}} = (0.1\,cm^2)\sqrt{\frac{(1.6\cdot10^{-19}C)(12)(8.854\cdot10^{-14}\frac{F}{cm})(6.6\cdot10^{14}/cm^3)}{2\cdot0.5\,V}}$$

$$= 0.1\,cm^2\sqrt{1.122\cdot10^{-19-14+14+3}\,\frac{F^2}{cm^4}} = 1.06\cdot10^{-9}F = \boxed{1.06\,nF}$$

e. Assume the metal is grounded and a voltage of 0.4 V is applied to the right-hand side of the junction. Draw the resulting energy band diagram, and label any changed energy levels with numerical values.



Part III (Free Response, 30 points)

Assume that we have a solar cell with dark current per unit area $J_o = 2$ nA/cm², short circuit current per unit area $J_{SC} = 30$ mA/cm², and ideality factor $n = 2$.

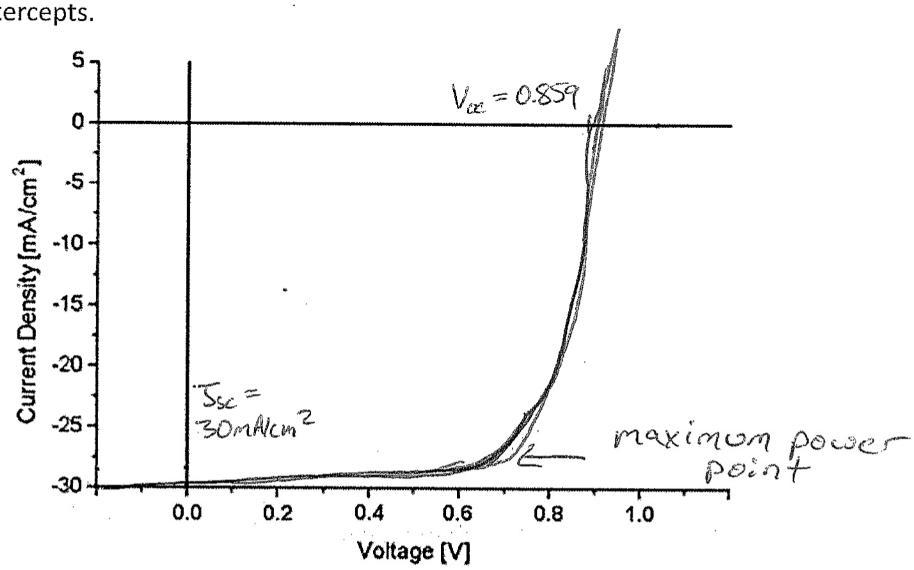
a. Calculate the open-circuit voltage and fill factor of this cell. Hint: use $\mathrm{FF} = \frac{z_{oc} - \ln(z_{oc}+0.72)}{z_{oc}+1}$, where $z_{oc} = qV_{oc}/nk_BT$ is the reduced open-circuit voltage.

$$V_{oc} = \frac{nkT}{q} \ln\left(\frac{J_{sc}}{J_o}\right) = 2 \cdot (0.026V) \ln\left(\frac{3 \cdot 10^{-2} A/cm^2}{2 \cdot 10^{-9} A/cm^2}\right)$$

$V_{oc} = 0.859\,V$ $\quad z_{oc} = 16.52$

$$FF = \frac{16.52 - \ln(16.52 + 0.72)}{16.52 + 1} = 0.78$$

b. Draw the current-voltage relation for this cell, for voltages between 0 and 1 V. Label the x- and y-intercepts.



c. Assuming that the cell is illuminated by the AM1.5 solar spectrum ($P_{in} = 100$ mW/cm²), calculate the maximum power conversion efficiency.

$$P_{max} = J_{sc} \cdot V_{oc} \cdot FF = \left(30 \frac{mA}{cm^2}\right)(0.859V)(0.78) = 20.1 \frac{mW}{cm^2}$$

$$\eta = \frac{P_{max}}{P_{in}} = \frac{20.1\, mW/cm^2}{100\, mW/cm^2} = 20.1\%$$

ECE 305 Exam 3 Formula Sheet (Fall 2015)

You may remove these pages from the exam packet, and take them with you.

Physical Constants	Silicon parameters ($T = 300$ K)
$h/2\pi = \hbar = 1.055 \times 10^{-34}$ J·s	$N_C = 3.23 \times 10^{19}$ cm^{-3}
$m_0 = 9.109 \times 10^{-31}$ kg	$N_V = 1.83 \times 10^{19}$ cm^{-3}
$k_B = 1.38 \times 10^{-23}$ J/K	$n_i = 1.1 \times 10^{10}$ cm^{-3}
$q = 1.602 \times 10^{-19}$ C	$K_s = 11.8$
$\epsilon_0 = 8.854 \times 10^{-12}$ F/m	$E_g = 1.12$ eV; $\chi = 4.03$ eV

Miller Indices: (hkl) {hkl} [hkl] <hkl>

Density of states $g_C(E) = \frac{(m_n^*)^{3/2}\sqrt{2(E-E_C)}}{\pi^2\hbar^3}$

Fermi function $f(E) = \frac{1}{1+e^{(E-E_F)/kT}}$

Intrinsic carrier concentration $n_i = \sqrt{N_C N_V} e^{-E_g/2kT}$

Equilibrium carrier densities: $N_C = \frac{1}{4}\left(\frac{2m_n^* kT}{\pi\hbar^2}\right)^{3/2}$ $\qquad N_V = \frac{1}{4}\left(\frac{2m_p^* kT}{\pi\hbar^2}\right)^{3/2}$

$n_0 = N_C e^{(E_F-E_C)/kT} = n_i e^{(E_F-E_i)/kT}$

$p_0 = N_V e^{(E_V-E_F)/kT} = n_i e^{(E_F-E_i)/kT}$

Space charge neutrality: $p - n + N_D^+ - N_A^- = 0$

Law of Mass Action: $n_0 p_0 = n_i^2$

Non-equilibrium carriers: $n = N_C e^{(F_N-E_C)/kT}$ $\quad p = N_V e^{(E_V-F_P)/kT}$ $\quad np = n_i^2 e^{(F_N-F_P)/kT}$

Conductivity/resistivity: $\sigma = \sigma_n + \sigma_n = q(n\mu_n + p\mu_p) = 1/\rho$

Drift-diffusion current equations:

$$J_n = nq\mu_n \mathcal{E}_x + qD_n \frac{dn}{dx} = n\mu_n \frac{dF_n}{dx} \qquad \frac{D_n}{\mu_n} = \frac{kT}{q}$$

$$J_p = pq\mu_p \mathcal{E}_x - qD_p \frac{dp}{dx} = p\mu_p \frac{dF_p}{dx} \qquad \frac{D_p}{\mu_p} = \frac{kT}{q}$$

Carrier conservation equations:

$$\frac{\partial n}{\partial t} = +\nabla \cdot \left(\frac{J_n}{q}\right) + G_n - R_n$$

$$\frac{\partial p}{\partial t} = -\nabla \cdot \left(\frac{J_p}{q}\right) + G_p - R_p$$

Poisson's equation: $\nabla \cdot (\epsilon\mathcal{E}) = \rho$

SRH carrier recombination: $R = \Delta n/\tau_n$ or $R = \Delta p/\tau_p$

Minority carrier diffusion equation: $\frac{\partial \Delta n}{\partial t} = D_n \frac{\partial^2 \Delta n}{\partial x^2} - \frac{\Delta n}{\tau_n} + G_L$ $\qquad L_{D,n} = \sqrt{D_n \tau_n}$

PN homojunction electrostatics: $V_{bi} = \frac{kT}{q}\ln\left(\frac{N_D N_A}{n_i^2}\right)$ $\qquad \frac{d\mathcal{E}}{dx} = \frac{\rho(x)}{K_s\epsilon_o}$

$$W = \sqrt{\frac{2K_s\epsilon_o V_{bi}}{q}\left(\frac{N_A+N_D}{N_A N_D}\right)} \qquad x_n = \left(\frac{N_A}{N_A+N_D}\right)W \qquad x_p = \left(\frac{N_D}{N_A+N_D}\right)W \qquad \mathcal{E}(0) = \sqrt{\frac{2qV_{bi}}{K_s\epsilon_o}\left(\frac{N_A N_D}{N_A+N_D}\right)}$$

PN diode current: $\Delta n(0) = \frac{n_i^2}{N_A}\left(e^{qV_A/kT} - 1\right)$ $\Delta p(0) = \frac{n_i^2}{N_D}\left(e^{qV_A/kT} - 1\right)$

$J_D = J_o\left(e^{qV_A/kT} - 1\right)$ $J_o = q\left(\frac{D_n}{L_n}\frac{n_i^2}{N_A} + \frac{D_p}{L_p}\frac{n_i^2}{N_D}\right)$ *(long)* $J_o = q\left(\frac{D_n}{W_p}\frac{n_i^2}{N_A} + \frac{D_p}{W_n}\frac{n_i^2}{N_D}\right)$ *(short)*

Non-ideal diodes: $I = I_o\left(e^{q(V_A - IR_S)/kT} - 1\right)$ $J_{gen} = -q\frac{n_i}{2\tau_o}W$

Photovoltaics: $V_{oc} = \frac{nkT}{q}\ln\left(\frac{J_{sc}}{J_o}\right)$ $J_{PV} = J_o\left(e^{qV_A/kT} - 1\right) - J_{sc}$

Small signal model: $G_d = \frac{I_D + I_o}{kT/q}$ $C_J(V_R) = \frac{K_s\epsilon_o A}{\sqrt{\frac{2K_s\epsilon_o V_{bi}}{qN_A}}} = A\sqrt{\frac{qK_s\epsilon_o N_A}{2V_{bi}}}$ $C_D = G_d\tau_n$

MS diode properties: $qV_{bi} = |\Phi_M - \Phi_S|$ $\Phi_{BP} = \chi + E_G - \Phi_M$ $\Phi_{BN} = \Phi_M - \chi$

$J_D = J_o\left(e^{qV_A/kT} - 1\right)$ $J_o = A^*T^2e^{-\Phi_B/kT}$ $A^* = \frac{4\pi q m^* k_B^2}{h^3} = 120\frac{m^*}{m_o}\ \frac{\mathrm{A}}{\mathrm{cm}^2\cdot\mathrm{K}^2}$