$\qquad$

## ECE 305 - Fall 2016

Exam 3 - Wednesday, October 26, 2016
This is a closed book exam. You may use a calculator and the formula sheet at the end of this exam. Following the ECE policy, the calculator must be a Texas Instruments TI-30X IIS scientific calculator.

To receive full credit, you must show your work (scratch paper is attached).
The exam is designed to be taken in 50 minutes (or less). Be sure to fill in your name and Purdue student ID at the top of the page. DO NOT open the exam until told to do so, and stop working immediately when time is called. The last 2 pages are equation sheets, which you may remove, if you want.

100 points possible,
I) 40 points ( 8 points per question)
II) 30 points
III) 30 points

## Course policy

If I am caught cheating, I will earn an F in the course \& be reported to the Dean of Students.
I repeat: $\qquad$
$\qquad$
Signature: $\qquad$

Part I: Answer the 5 multiple choice questions below by entering them on your IDP-15 Scantron.
1 (8 points). Which of the following is/are basic requirement(s) for a laser to operate?
a. Gain medium
b. Resonant cavity
c. Low-bandgap semiconductor
d. $a$ and $b$
e. All of the above

2 (8 points). The depletion and potential drop in the metal side of the Schottky diode is?
a. Very small, negligible
b. Small, but not negligible
c. Large
d. Must know doping of metal
e. Under steady-state conditions

Note: also counted A as correct for Question 2, because of possible confusion from review session.
3 (8 points). In the following diagram, what non-ideality accounts for the current roll-over at large reverse biases ( $V_{A}<-38 \mathrm{~V}$ )?

a. Recombination in the space-charge region
b. Avalanche diode effect
c. Generation in the depletion region
d. Diffusion current
e. Series resistance

4 (8 points). For a metal-semiconductor diode, which of the following is true?
a. The saturation current density is much larger than a PN junction with the same bandgap semiconductor
b. The $n=2$ current is absent
c. The diode turn-on voltage is reduced, compared to a PN junction made from the same semiconductor
d. All of the above
e. None of the above

5 (8 points). A small incremental (positive) voltage is applied to a diode. What is the source of the charge on the plates of the depletion capacitor?
a. Majority carriers at the edges of the depletion region
b. Minority carriers at the edges of the depletion region
c. Majority carriers at the metallurgical junction
d. Minority carriers at the metallurgical junction
e. Ionized dopants at the contacts

## Part II (Free Response, 30 points)

Consider the energy band diagrams of a metal and a semiconductor shown below. The semiconductor has a relative dielectric constant of 12 , an electron affinity $\chi=3.25 \mathrm{eV}, \Phi_{s}=3.5 \mathrm{eV}$, and a bandgap $E_{g}=1.0 \mathrm{eV}$. The metal has a workfunction $\Phi_{M}=4.0 \mathrm{eV}$. Answer the following questions.

a. (10 pts) Draw the equilibrium band diagram for an ideal metal-semiconductor (MS) structure formed from the pictured components with the indicated parameter values.


Continued on next page....
b. (4 pts) Is this ideal MS structure rectifying or Ohmic? Justify your answer.

Rectifying, because the metal workfunction exceeds the vacuum electron affinity of the semiconductor.
c. (8 pts) What is the MS barrier height $\Phi_{B}$ ? Label the diagram accordingly.

$$
\Phi_{B}=\Phi_{M}-\chi=4.0 \mathrm{eV}-3.25 \mathrm{eV}=0.75 \mathrm{eV}
$$

d. (8 pts) What is the built-in voltage $V_{b i}$ of this MS structure?

$$
V_{b i}=\Phi_{M}-\Phi_{S}=4 \mathrm{eV}-3.5 \mathrm{eV}=0.5 \mathrm{eV}
$$

## Part III (Free Response, 30 points)

Consider the $\mathrm{p}+/ \mathrm{n} / \mathrm{n}+$ diode illustrated below. Assume that the n region doping $N_{D}=3 \cdot 10^{15} \mathrm{~cm}^{-3}$, while doping values in the $\mathrm{p}+$ and $\mathrm{n}+$ regions are so large that their depletion widths are negligible.


Also assume that $V_{b i}=1.1 \mathrm{~V}, K_{s}=12, J_{o}=0.14 \mathrm{nA} / \mathrm{cm}^{2}, \tau=1 \mu \mathrm{~s}$, and that the capacitance as a function of voltage is measured as below:

a. (10 pts) Calculate the minimum capacitance per unit area.

$$
\frac{C_{J}}{A}=\frac{\epsilon}{d}=\frac{12 \cdot\left(8.854 \cdot 10^{-14} \mathrm{~F} / \mathrm{cm}\right)}{2 \cdot 10^{-4} \mathrm{~cm}}=5.3 \frac{\mathrm{nF}}{\mathrm{~cm}^{2}}
$$

b. (10 pts) What is the diode complex admittance per unit area as a function of frequency $\omega$ [in MHz] when $V_{A}=0$ ? Justify your answer.

$$
\begin{aligned}
Y / A=j \omega\left(\frac{C_{J}}{A}+\frac{C_{D}}{A}\right)+\frac{G_{S}}{A} & =j \omega\left(5.3 \frac{\mathrm{nF}}{\mathrm{~cm}^{2}}+\frac{0.14 \mathrm{nA} / \mathrm{cm}^{2}}{0.0259 \mathrm{~V}} \cdot(1 \mu \mathrm{~s})\right)+\frac{0.14 \mathrm{nA} / \mathrm{cm}^{2}}{0.0259 \mathrm{~V}} \\
\frac{Y}{A} & =\left(j \omega \cdot 5.3+5.4 \cdot 10^{-6}\right) \frac{\mathrm{mS}}{\mathrm{~cm}^{2}}
\end{aligned}
$$

c. (10 pts) At what value of applied voltage $V_{A}$ does the capacitance flatten out (i.e., at which voltage $V_{1}$ does the capacitance drop to 1.01 times its minimum value, as you go from right to left on the voltage axis)?

For diffusion capacitance to be non-negligible, we must have $C_{D}\left(V_{A}\right) \geq C_{D}(0)$, which implies $I_{D}+I_{o} \geq 0.01 I_{o}$, so that $I_{o} e^{q V_{1} / k T}=0.01 I_{o}$, or $q V_{1} / k T=\ln (0.01)$, which finally results in $V_{1}=(0.0259 \mathrm{~V}) \cdot(-4.605)=-0.12 \mathrm{~V}$.

## ECE 305 Exam 3 Formula Sheet (Fall 2016)

You may remove these pages from the exam packet, and take them with you.

| Physical Constants | Silicon parameters $(\boldsymbol{T}=\mathbf{3 0 0} \mathbf{K})$ |
| :---: | :---: |
| $h / 2 \pi=\hbar=1.055 \times 10^{-34} \mathrm{~J} \cdot \mathrm{~s}$ | $N_{C}=3.23 \times 10^{19} \mathrm{~cm}^{-3}$ |
| $m_{0}=9.109 \times 10^{-31} \mathrm{~kg}$ | $N_{V}=1.83 \times 10^{19} \mathrm{~cm}^{-3}$ |
| $k_{B}=1.38 \times 10^{-23} \mathrm{~J} / \mathrm{K}$ | $n_{i}=1.1 \times 10^{10} \mathrm{~cm}^{-3}$ |
| $q=1.602 \times 10^{-19} \mathrm{C}$ | $K_{S}=11.8$ |
| $\epsilon_{0}=8.854 \times 10^{-12} \mathrm{~F} / \mathrm{m}$ | $E_{g}=1.12 \mathrm{eV} ; \chi=4.03 \mathrm{eV}$ |

Miller Indices: (hkl) $\{\mathrm{hkl}\}[\mathrm{hkl}]<\mathrm{hkl}>\quad$ Density of states $g_{C}(E)=\frac{\left(m_{n}^{*}\right)^{3 / 2} \sqrt{2\left(E-E_{C}\right)}}{\pi^{2} \hbar^{3}}$
Fermi function $f(E)=\frac{1}{1+e^{\left(E-E_{F}\right) / k T}} \quad$ Intrinsic carrier concentration $n_{i}=\sqrt{N_{C} N_{V}} e^{-E_{g} / 2 k T}$
Equilibrium carrier densities: $N_{C}=\frac{1}{4}\left(\frac{2 m_{n}^{*} k T}{\pi \hbar^{2}}\right)^{3 / 2} \quad N_{V}=\frac{1}{4}\left(\frac{2 m_{p}^{*} k T}{\pi \hbar^{2}}\right)^{3 / 2}$
$n_{0}=N_{C} e^{\left(E_{F}-E_{C}\right) / k T}=n_{i} e^{\left(E_{F}-E_{i}\right) / k T}$
$p_{0}=N_{V} e^{\left(E_{V}-E_{F}\right) / k T}=n_{i} e^{\left(E_{F}-E_{i}\right) / k T}$
Space charge neutrality: $p-n+N_{D}^{+}-N_{A}^{-}=0 \quad$ Law of Mass Action: $n_{0} p_{0}=n_{i}^{2}$
Non-equilibrium carriers: $\quad n=N_{C} e^{\left(F_{N}-E_{C}\right) / k T} \quad p=N_{V} e^{\left(E_{V}-F_{P}\right) / k T} \quad n p=n_{i}^{2} e^{\left(F_{N}-F_{P}\right) / k T}$
Conductivity/resistivity: $\sigma=\sigma_{n}+\sigma_{n}=q\left(n \mu_{n}+p \mu_{p}\right)=1 / \rho$
Drift-diffusion current equations: $\quad J_{n}=n q \mu_{n} \varepsilon_{x}+q D_{n} \frac{d n}{d x}=n \mu_{n} \frac{d F_{n}}{d x} \quad \frac{D_{n}}{\mu_{n}}=\frac{k T}{q}$

$$
J_{p}=p q \mu_{p} \varepsilon_{x}-q D_{p} \frac{d p}{d x}=p \mu_{p} \frac{d F_{p}}{d x} \quad \frac{D_{p}}{\mu_{p}}=\frac{k T}{q}
$$

Carrier conservation equations: $\quad \frac{\partial n}{\partial t}=+\nabla \cdot\left(\frac{J_{n}}{q}\right)+G_{n}-R_{n}$
$\frac{\partial p}{\partial t}=-\nabla \cdot\left(\frac{J_{p}}{q}\right)+G_{p}-R_{p}$
Poisson's equation:
$\nabla \cdot(\epsilon \mathcal{E})=\rho$
SRH carrier recombination: $\quad R=\Delta n / \tau_{n} \quad$ or $\quad R=\Delta p / \tau_{p}$
Minority carrier diffusion equation: $\frac{\partial \Delta n}{\partial t}=D_{n} \frac{\partial^{2} \Delta n}{\partial x^{2}}-\frac{\Delta n}{\tau_{n}}+G_{L} \quad L_{D, n}=\sqrt{D_{n} \tau_{n}}$
PN homojunction electrostatics: $\quad V_{b i}=\frac{k T}{q} \ln \left(\frac{N_{D} N_{A}}{n_{i}^{2}}\right) \quad \frac{d \varepsilon}{d x}=\frac{\rho(x)}{K_{s} \epsilon_{o}}$
$W=\sqrt{\frac{2 K_{S} \epsilon_{0} V_{b i}}{q}\left(\frac{N_{A}+N_{D}}{N_{A} N_{D}}\right)} \quad x_{n}=\left(\frac{N_{A}}{N_{A}+N_{D}}\right) W \quad x_{p}=\left(\frac{N_{D}}{N_{A}+N_{D}}\right) W \quad \varepsilon(0)=\sqrt{\frac{2 q V_{b i}}{K_{s} \epsilon_{o}}\left(\frac{N_{A} N_{D}}{N_{A}+N_{D}}\right)}$

PN diode current: $\quad \Delta n(0)=\frac{n_{i}^{2}}{N_{A}}\left(e^{q V_{A} / k T}-1\right) \quad \Delta p(0)=\frac{n_{i}^{2}}{N_{D}}\left(e^{q V_{A} / k T}-1\right)$
$J_{D}=J_{o}\left(e^{q V_{A} / k T}-1\right) \quad J_{o}=q\left(\frac{D_{n}}{L_{n}} \frac{n_{i}^{2}}{N_{A}}+\frac{D_{p}}{L_{p}} \frac{n_{i}^{2}}{N_{D}}\right)$ (long) $\quad J_{o}=q\left(\frac{D_{n}}{W_{p}} \frac{n_{i}^{2}}{N_{A}}+\frac{D_{p}}{W_{n}} \frac{n_{i}^{2}}{N_{D}}\right)$ (short)
Non-ideal diodes: $\quad I=I_{o}\left(e^{q\left(V_{A}-I R_{S}\right) / k T}-1\right) \quad J_{g e n}=-q \frac{n_{i}}{2 \tau_{o}} W$
Photovoltaics: $\quad V_{o c}=\frac{n k T}{q} \ln \left(\frac{J_{s c}}{J_{o}}\right) \quad J_{P V}=J_{o}\left(e^{q V_{A} / k T}-1\right)-J_{s c}$
Small signal model: $\quad G_{d}=\frac{I_{D}+I_{o}}{k T / q} \quad C_{J}\left(V_{A}\right)=\frac{K_{s} \epsilon_{o} A}{\sqrt{\frac{2 K_{s} \epsilon_{o}\left(V_{b i}-V_{A}\right)}{q N_{A}}}}=A \sqrt{\frac{q K_{S} \epsilon_{o} N_{A}}{2\left(V_{b i}-V_{A}\right)}} \quad C_{D}=G_{d} \tau_{n}$
MS diode properties: $q V_{b i}=\left|\Phi_{M}-\Phi_{S}\right| \quad \Phi_{B P}=\chi+E_{G}-\Phi_{M} \quad \Phi_{B N}=\Phi_{M}-\chi$
$J_{D}=J_{o}\left(e^{q V_{A} / k T}-1\right) \quad J_{o}=A^{*} T^{2} e^{-\Phi_{B} / k T} \quad A^{*}=\frac{4 \pi q m^{*} k_{B}^{2}}{h^{3}}=120 \frac{m^{*}}{m_{o}} \frac{\mathrm{~A}}{\mathrm{~cm}^{2} \cdot \mathrm{~K}^{2}}$

