

NAME: FULL SOLUTION

PUID: _____

ECE 305 – Fall 2017

Exam 3 – Thursday, October 26, 2017

This is a closed book exam. You may use a calculator and the formula sheet at the end of this exam. Following the ECE policy, the calculator **must** be a Texas Instruments TI-30X IIS scientific calculator.

To receive full credit, you must **show your work** (scratch paper is attached).

The exam is designed to be taken in 60 minutes (or less). Be sure to fill in your name and Purdue student ID at the top of the page. DO NOT open the exam until told to do so, and stop working immediately when time is called. The last 2 pages are equation sheets, which you may remove, if you want.

100 points possible,

I) 40 points (8 points per question)

II) 30 points

III) 30 points

Course policy

If I am caught cheating, I will earn an F in the course & be reported to the Dean of Students.

I repeat: _____

Signature: _____

Part I: Answer the 5 multiple choice questions below by entering them on your IDP-15 Scantron.

1 (8 points). Which recombination mechanism in diode produces current with an ideality factor $n = 2$?

- a. Recombination in the quasi neutral region at forward bias
- b. Recombination in the space charge region at reverse bias
- c. Recombination in the quasi neutral region at reverse bias
- d. Recombination in the space charge region at forward bias**
- e. Recombination at the surface at reverse bias

2 (8 points). For a solar cell obeying superposition, why does the total current decrease with applied voltage?

- a. To maintain conservation of power with time
- b. Photogeneration is reduced at higher applied voltage
- c. The electric field at the junction is reduced at higher applied voltage**
- d. Diffusion at the junction is increased at higher applied voltage
- e. All of the above

3 (8 points). What dominates the current beyond the reverse bias breakdown voltage?

- a. Series resistance
- b. Recombination in the space-charge region
- c. Quantum-mechanical tunneling
- d. Carrier avalanche**
- e. Diffusion current

4 (8 points). Which of the following statements is true?

- a. The carrier depletion on the metal side of a Schottky diode is negligible.
- b. At same forward bias, a Schottky diode has higher current than a PN junction made of same bandgap semiconductor
- c. Quantum mechanical tunneling can affect Schottky diode performance
- d. All of the above**
- e. None of the above

5 (8 points). Which is true of the capacitance of Schottky diodes?

- a. It is proportional to $1/\sqrt{V_{bi} - V_A}$**
- b. It consists of junction capacitance and diffusion capacitance
- c. It is proportional to $1/(V_{bi} - V_A)$
- d. It is proportional to $V_{bi} - V_A$
- e. It is proportional to $\exp(qV_A/kT)$

Part II (Free Response, 30 points)

Consider a crystalline silicon n-p step homojunction ideal diode at room temperature ($T=300$ K). The n-type region has doping $N_D=10^{17}$ /cm³, width $W_N = 10$ μm, minority carrier $\mu_p=450$ cm²/V·s and minority carrier $\tau_p=1$ μs. The p-type region has doping $N_A=3 \cdot 10^{18}$ /cm³, width $W_P = 15$ μm, minority carrier $\mu_n=1400$ cm²/V·s, and minority carrier $\tau_n=3$ μs at $T=300$ K. Assume the dopants are fully ionized.

- a. Calculate whether the diode is long or short, and the dark current density J_0 of the diode.

$$D_n=1400 \cdot 0.0259=36 \text{ cm}^2/\text{s}; L_n=\sqrt{D_n \cdot \tau_n}=104 \text{ } \mu\text{m}$$

$$D_p=450 \cdot 0.0259=11.655 \text{ cm}^2/\text{s}; L_p=\sqrt{D_p \cdot \tau_p}=34 \text{ } \mu\text{m}; \text{ therefore it is short on both sides.}$$

$$J_0=q \cdot n_i^2 \cdot \left[\frac{D_n}{(W_p \cdot N_A)} + \frac{D_p}{(W_n \cdot N_D)} \right] \quad (\text{short})$$

$$=1.6 \cdot 10^{-19} \cdot 1 \cdot 10^{20} \left[\frac{36 \text{ cm}^2/\text{s}}{(1.5 \cdot 10^{-3} \text{ cm} \cdot 3 \cdot 10^{18} / \text{cm}^3)} + \frac{11.6 \text{ cm}^2/\text{s}}{(1 \cdot 10^{-3} \text{ cm} \cdot 1 \cdot 10^{17} / \text{cm}^3)} \right]$$

$$=16 \cdot [8 \cdot 10^{-15} + 1.16 \cdot 10^{-13}] = 1.984 \cdot 10^{-12} \text{ A/cm}^2$$

- b. For crystalline silicon, the critical breakdown field is $\mathcal{E}_{cr} = 3 \cdot 10^5$ V/cm, and the built-in voltage is 0.923 V. Use this to calculate the reverse breakdown voltage V_{BR} of the diode.

$$\mathcal{E}_{cr}^2 = 2q(V_{bi} + V_{br}) \cdot N_D / (K_s \cdot \epsilon_0)$$

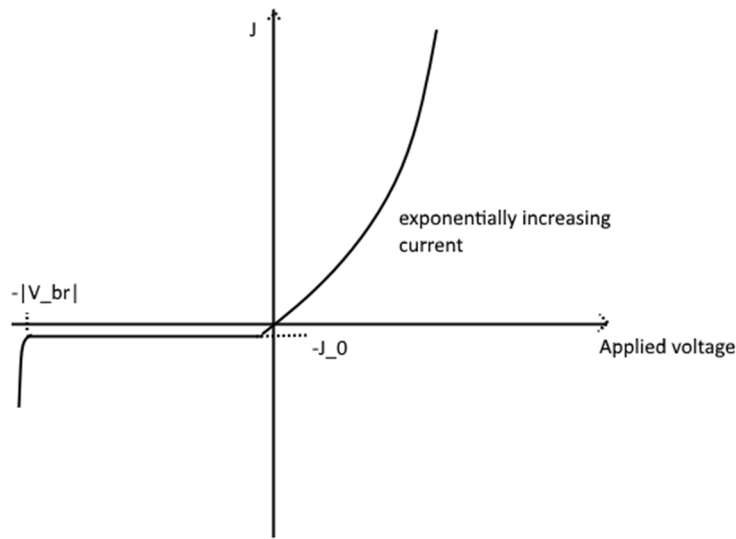
$$V_{bi} + V_{br} = \mathcal{E}_{cr}^2 \cdot K_s \cdot \epsilon_0 / (2q \cdot N_D)$$

$$V_{br} = (3 \cdot 10^5 \text{ V/cm})^2 \cdot (11.8 \cdot 8.854 \cdot 10^{-14} \text{ F/cm}) / (2 \cdot 1.6 \cdot 10^{-19} \text{ C} \cdot 1 \cdot 10^{17} / \text{cm}^3) - 0.923$$

$$V_{br} = 2.938 - 0.923 = 2.015 \text{ V}$$

$$\text{If } \mathcal{E}_{cr}^2 = 2q(V_{bi} + V_{br}) \cdot N_A \cdot N_D / (N_A + N_D) \cdot (K_s \cdot \epsilon_0) \text{ used, } V_{br} = 2.11 \text{ V};$$

- c. Assuming there are no other non-idealities, sketch the current density J of the diode for the range of applied voltage from $-(|V_{BR}| + 0.1)$ to $|V_{BR}| + 0.1$.



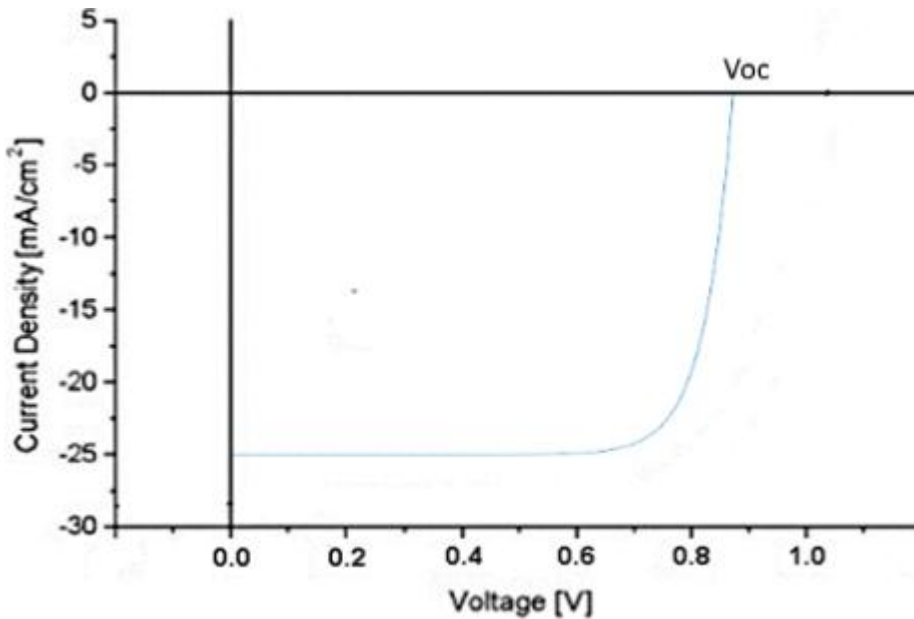
Part III (Free Response, 30 points)

Assume that we have a solar cell with dark current per unit area $J_o = 0.5 \text{ nA/cm}^2$, short circuit current per unit area $J_{sc} = 25 \text{ mA/cm}^2$, ideality factor $n = 1.8$, and a cross sectional area $A=10^{-4} \text{ cm}^2$.

- a. Calculate the open-circuit voltage (V_{oc}) and fill factor of this cell. Hint: use $FF = \frac{z_{oc} - \ln(z_{oc} + 0.72)}{z_{oc} + 1}$, where $z_{oc} = qV_{oc}/nk_B T$ is the reduced open-circuit voltage.

$V_{oc}=0.826$; $z_{oc}=17.7$ $FF=0.791$

- b. Draw the current density-voltage relation for this cell, for voltages between 0 and V_{oc} . Label the x- and y-intercepts.



- c. Assuming that the cell is illuminated by the AM1.5 solar spectrum ($P_{in} = 100 \text{ mW/cm}^2$), calculate the maximum power conversion efficiency. Also, calculate the efficiency if the doping of both sides of the solar cell is increased by a factor of ten, while the short circuit current remains the same, assuming the ideal diode dark current holds, and the mobility is unchanged.

$\eta_1 = J_{sc} \cdot V_{oc} \cdot FF / P_{in} = 16.34\%$

$J_{0_new} = 10 \cdot J_{0_old}$

$\eta_2 = J_{sc} \cdot V_{oc}' \cdot FF' / P_{in} = 25 \cdot 0.934 \cdot 0.808 / 100 = 18.86\%$

ECE 305 Exam 3 Formula Sheet (Fall 2017)

You may remove these pages from the exam packet, and take them with you.

Physical Constants	Silicon parameters ($T = 300$ K)
$h/2\pi = \hbar = 1.055 \times 10^{-34}$ J·s	$N_C = 3.23 \times 10^{19}$ cm ⁻³
$m_0 = 9.109 \times 10^{-31}$ kg	$N_V = 1.83 \times 10^{19}$ cm ⁻³
$k_B = 1.38 \times 10^{-23}$ J/K	$n_i = 1.1 \times 10^{10}$ cm ⁻³
$q = 1.602 \times 10^{-19}$ C	$K_S = 11.8$
$\epsilon_0 = 8.854 \times 10^{-12}$ F/m	$E_g = 1.12$ eV; $\chi = 4.03$ eV

Miller Indices: (hkl) {hkl} [hkl] <hkl>

Density of states $g_C(E) = \frac{(m_n^*)^{3/2} \sqrt{2(E-E_C)}}{\pi^2 \hbar^3}$

Fermi function $f(E) = \frac{1}{1+e^{(E-E_F)/kT}}$

Intrinsic carrier concentration $n_i = \sqrt{N_C N_V} e^{-E_g/2kT}$

Equilibrium carrier densities: $N_C = \frac{1}{4} \left(\frac{2m_n^* kT}{\pi \hbar^2} \right)^{3/2}$ $N_V = \frac{1}{4} \left(\frac{2m_p^* kT}{\pi \hbar^2} \right)^{3/2}$

$n_0 = N_C e^{(E_F-E_C)/kT} = n_i e^{(E_F-E_i)/kT}$ $p_0 = N_V e^{(E_V-E_F)/kT} = n_i e^{(E_i-E_F)/kT}$

Space charge neutrality: $p - n + N_D^+ - N_A^- = 0$ **Law of Mass Action:** $n_0 p_0 = n_i^2$

Non-equilibrium carriers: $n = N_C e^{(F_N-E_C)/kT}$ $p = N_V e^{(E_V-F_P)/kT}$ $np = n_i^2 e^{(F_N-F_P)/kT}$

Conductivity/resistivity: $\sigma = \sigma_n + \sigma_p = q(n\mu_n + p\mu_p) = 1/\rho$

Drift-diffusion current equations: $J_n = nq\mu_n \mathcal{E}_x + qD_n \frac{dn}{dx} = n\mu_n \frac{dF_n}{dx}$ $\frac{D_n}{\mu_n} = \frac{kT}{q}$

$J_p = pq\mu_p \mathcal{E}_x - qD_p \frac{dp}{dx} = p\mu_p \frac{dF_p}{dx}$ $\frac{D_p}{\mu_p} = \frac{kT}{q}$

Carrier conservation equations: $\frac{\partial n}{\partial t} = +\nabla \cdot \left(\frac{J_n}{q} \right) + G_n - R_n$

$\frac{\partial p}{\partial t} = -\nabla \cdot \left(\frac{J_p}{q} \right) + G_p - R_p$

Poisson's equation: $\nabla \cdot (\epsilon \mathcal{E}) = \rho$

SRH carrier recombination: $R = \Delta n / \tau_n$ or $R = \Delta p / \tau_p$

Minority carrier diffusion equation: $\frac{\partial \Delta n}{\partial t} = D_n \frac{\partial^2 \Delta n}{\partial x^2} - \frac{\Delta n}{\tau_n} + G_L$ $L_{D,n} = \sqrt{D_n \tau_n}$

PN homojunction electrostatics: $V_{bi} = \frac{kT}{q} \ln \left(\frac{N_D N_A}{n_i^2} \right)$ $\frac{d\mathcal{E}}{dx} = \frac{\rho(x)}{K_S \epsilon_0}$

$W = \sqrt{\frac{2K_S \epsilon_0 V_{bi}}{q} \left(\frac{N_A + N_D}{N_A N_D} \right)}$ $x_n = \left(\frac{N_A}{N_A + N_D} \right) W$ $x_p = \left(\frac{N_D}{N_A + N_D} \right) W$ $\mathcal{E}(0) = \sqrt{\frac{2qV_{bi}}{K_S \epsilon_0} \left(\frac{N_A N_D}{N_A + N_D} \right)}$

PN diode current: $\Delta n(0) = \frac{n_i^2}{N_A} (e^{qV_A/kT} - 1)$ $\Delta p(0) = \frac{n_i^2}{N_D} (e^{qV_A/kT} - 1)$

$J_D = J_o (e^{qV_A/kT} - 1)$ $J_o = q \left(\frac{D_n n_i^2}{L_n N_A} + \frac{D_p n_i^2}{L_p N_D} \right)$ (long) $J_o = q \left(\frac{D_n n_i^2}{W_p N_A} + \frac{D_p n_i^2}{W_n N_D} \right)$ (short)

Non-ideal diodes: $I = I_o (e^{q(V_A - IR_s)/kT} - 1)$ $J_{gen} = -q \frac{n_i}{2\tau_o} W$

Photovoltaics: $V_{oc} = \frac{nkT}{q} \ln \left(\frac{J_{sc}}{J_o} \right)$ $J_{PV} = J_o (e^{qV_A/kT} - 1) - J_{sc}$

Small signal model: $G_d = \frac{I_D + I_o}{kT/q}$ $C_J(V_R) = \frac{K_S \epsilon_o A}{\sqrt{\frac{2K_S \epsilon_o V_{bi}}{q N_A}}} = A \sqrt{\frac{q K_S \epsilon_o N_A}{2V_{bi}}}$ $C_D = G_d \tau_n$

MS diode properties: $qV_{bi} = |\Phi_M - \Phi_S|$ $\Phi_{BP} = \chi + E_G - \Phi_M$ $\Phi_{BN} = \Phi_M - \chi$

$J_D = J_o (e^{qV_A/kT} - 1)$ $J_o = A^* T^2 e^{-\Phi_B/kT}$ $A^* = \frac{4\pi q m^* k_B^2}{h^3} = 120 \frac{m^*}{m_o} \frac{A}{cm^2 \cdot K^2}$