NAME:_____Answer Key___________________ **PUID:_________________________**

ECE 305 – Fall 2015

Exam 4 – Monday, November 23, 2015

This is a closed book exam. You may use a calculator and the formula sheet at the end of this exam. Following the ECE policy, the calculator **must** be a Texas Instruments TI---30X IIS scientific calculator.

To receive full credit, you must **show your work** (scratch paper is attached).
The exam is designed to be taken in 50 minutes (or less). Be sure to fill in your name and Purdue student ID at the top of the page. DO NOT open the exam until told to do so, and stop working immediately when time is called. The last 2 pages are equation sheets, which you may remove, if you want.

100 points possible,

I) 40 points (8 points per question)
II) 30 points
III) 30 points

Course policy

If I am caught cheating, I will earn an F in the course & be reported to the Dean of Students.

I repeat: ____

____

Signature: ____

Part I: Answer the 5 multiple choice questions below by entering them on your IDP-15 Scantron.

1 (8 points). An MOS capacitor can be modeled as follows:

a. Two bias-dependent capacitors in series
b. One constant and one bias-dependent capacitor in parallel
c. One constant and one bias-dependent capacitor in series
d. Two constant capacitors in series
e. One constant and two bias-dependent capacitors in parallel

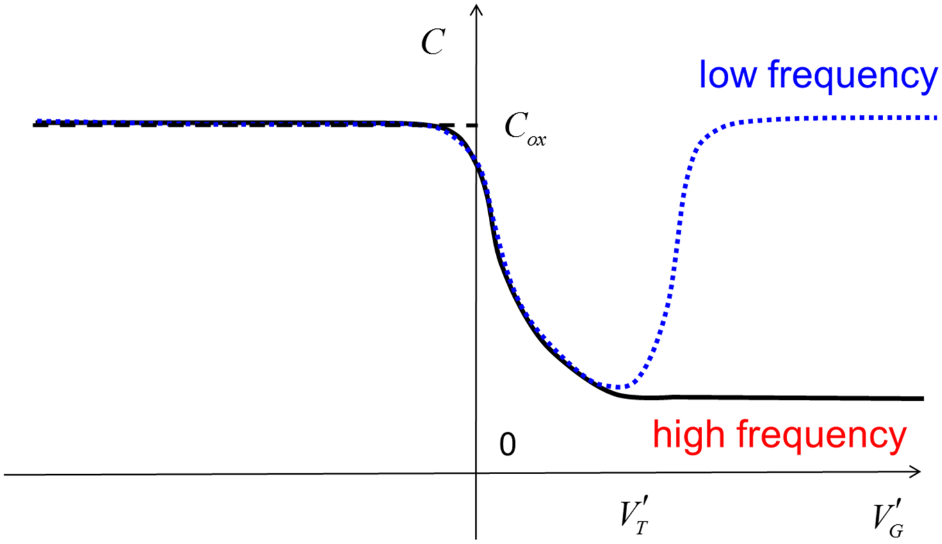
2 (8 points). For a long-channel MOSFET biased above the turn-on voltage V_T, how does the saturated drain current I_D scale with the gate voltage V_{GS}?

a. $1/(V_{GS} - V_T)^2$
b. $\sqrt{V_{GS} - V_T}$
c. $V_{GS} - V_T$
d. $(V_{GS} - V_T)^2$
e. $(V_{GS} - V_T)^3$

3 (8 points). What is the effect of sodium ion transport into a gate oxide on a MOS capacitor?

a. Etch the oxide away
b. Shift the threshold voltage in a time-varying fashion
c. Convert the oxide into a good conductor
d. Change the semiconductor doping from p to n-type
e. Reduce the series resistance to zero

4 (8 points). Consider the current-voltage relationship depicted in the following diagram:



If $K_{ox} = 4$, and $K_{Si} = 12$, and $t_{ox} = 6$ nm, what is a reasonable estimate of W in the high-frequency regime for $V_G' \gg V_T'$?

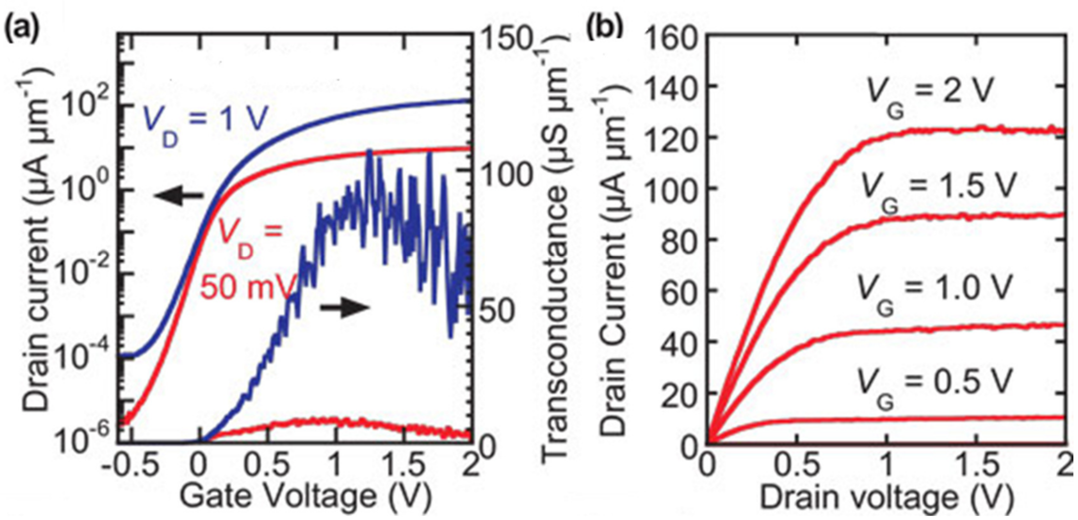
- a. 1 nm
- b. 6 nm
- c. 18 nm
- d. 54 nm
- e. 288 nm

5 (8 points). When minority carriers pile up at the oxide-Si interface of a MOS device, what is the bias condition?

- a. Inversion
- b. Flatband
- c. Depletion
- d. Deep depletion
- e. Accumulation

Part II (Free Response, 30 points)

Consider the transfer and output characteristics observed for an InGaAs MOSFET below. Assume the power supply voltage V_{DD}=1 V and C_{ox}=200 nF/cm^2. Please answer the following questions.



a. What is the on current for this MOSFET? Explain how you find it.

I_{on}=42 μA / μm. Read from the I_d-V_d curves, I_{on}=I_{ds} when V_g=V_g=V_{dd}=1V.

b. What is the off current for this MOSFET? Explain how you find it.

I_{off} = 80 nA / μm. Read from the semilog I_d-V_g curves, I_{off}=I_{ds} when V_g=0 V, V_d=V_{dd}=1V

c. What is the average subthreshold swing at V_{ds}=50 mV? Explain how you find it.

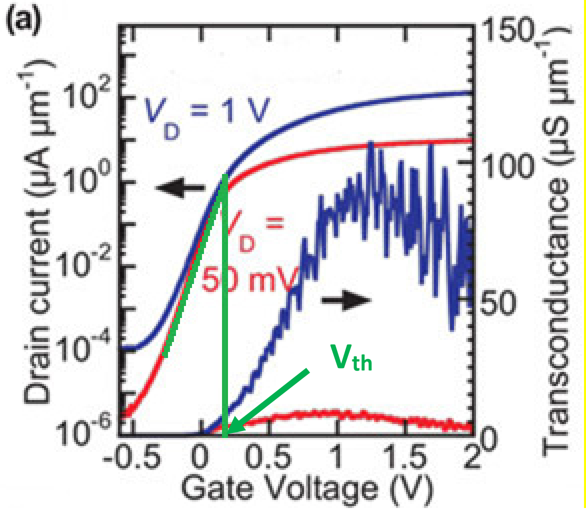
About 110 mV / dec. When I_{ds} changes by 3 orders of magnitude, V_g changes by about 0.33V. So $SS = \frac{\Delta Vg}{3\text{ decades}} = \frac{330\text{ mV}}{3\text{ dec}} = 110\text{ mV/dec}$

d. What is the threshold voltage? Explain how you find it. Please note that the transfer characteristics are plotted on a log scale.

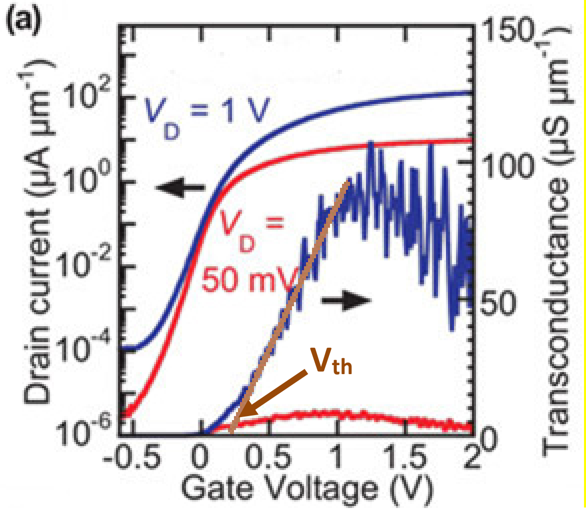
The threshold voltage V_{th} is about 0.2V - 0.25 V.

There are two ways to find the threshold voltage without having the linear I_d-V_g curves:

The first way is to use the semi-log I_d-V_g plot. The sub-threshold current depends linearly on gate voltage in such a semi-log plot. The gate voltage at which the plot departs from linearity is taken as the threshold voltage. Usually, the V_{th} determined using this method is somewhat lower than that by the linear extrapolation method. Here, it around 0.2 V, as shown below:



The second way is called a transconductance derivative, which is less commonly used. The derivative of the transconductance with gate voltage is determined at low drain voltage and plotted versus gate voltage. The second derivative d^2I_d/dV_{gs}^2 will tend to infinity at $V_{gs}=V_{th}$, around 0.25 V in this case, as shown in the following figure:



e. What is the inversion layer charge density in the on state (in nC/cm^2)?

$|Q| = C_{ox} (V_g - V_{th}) = 150$ nC/cm^2

Part III (Free Response, 30 points)

1. Assume a p$^+$ polysilicon gate with an n-type silicon substrate, with K$_s$=11.8 and $N_D = 10^{14}$ /cm^3. The oxide thickness can be taken as 100 nm and dielectric constant K$_{ox}$=4.

   a. What is ϕ_F here?

   $$\phi_F = -\frac{kT}{q}\ln\left(\frac{N_D}{n_i}\right) = -0.239\ V\ \text{(assuming } T = 300\text{ K).}$$

   b. What is the "metal"-semiconductor workfunction difference?

   $$\Delta W = \frac{1}{2}E_g + (-q)\phi_F = 0.56\text{ eV} + 0.2398\text{ eV} = 0.799\text{ eV}$$

   c. What is W, when $\phi_S = \phi_F$?

   $$W = \sqrt{\frac{2K_s\epsilon_o(-\phi_S)}{qN_D}} = \sqrt{\frac{2 * 11.8 * 8.854\cdot 10^{-14} * 0.239}{1.6\cdot 10^{-19} * 10^{14}}} = 1.77 * 10^{-4}\text{ cm}$$

   d. What is the electric field in the oxide, when $\phi_S = \phi_F$?

   $$|\mathcal{E}(s)| = \sqrt{\frac{2qN_D(-\phi_S)}{K_s\epsilon_o}} = 2.7\cdot 10^3\frac{\text{V}}{\text{cm}} = 2.7\frac{\text{kV}}{\text{cm}}$$

   $$|\mathcal{E}_{ox}| = \frac{K_S}{K_{ox}}|\mathcal{E}(s)| = 8.26\cdot 10^3\frac{\text{V}}{\text{cm}} = 8.26\frac{\text{kV}}{\text{cm}}$$

   e. What is the potential drop across the oxide?

   $$\Delta V = t_{ox} * \mathcal{E}_{ox} = 100\text{ nm} * \left(-8.26\frac{\text{kV}}{\text{cm}}\right) = -0.0826\ V$$

ECE 305 Exam 4 Formula Sheet (Fall 2015)

You may remove these pages from the exam packet, and take them with you.

Physical Constants	**Silicon parameters ($T = 300$ K)**
$h/2\pi = \hbar = 1.055 \times 10^{-34}$ J·s	$N_C = 3.23 \times 10^{19}$ cm^{-3}
$m_0 = 9.109 \times 10^{-31}$ kg	$N_V = 1.83 \times 10^{19}$ cm^{-3}
$k_B = 1.38 \times 10^{-23}$ J/K	$n_i = 1.1 \times 10^{10}$ cm^{-3}
$q = 1.602 \times 10^{-19}$ C	$K_s = 11.8$
$\epsilon_0 = 8.854 \times 10^{-12}$ F/m	$E_g = 1.12$ eV; $\chi = 4.03$ eV

Miller Indices: (hkl) {hkl} [hkl] <hkl>

Density of states $g_C(E) = \frac{(m_n^*)^{3/2}\sqrt{2(E-E_C)}}{\pi^2\hbar^3}$

Fermi function $f(E) = \frac{1}{1+e^{(E-E_F)/kT}}$

Intrinsic carrier concentration $n_i = \sqrt{N_C N_V} e^{-E_g/2kT}$

Equilibrium carrier densities: $N_C = \frac{1}{4}\left(\frac{2m_n^* kT}{\pi\hbar^2}\right)^{3/2}$ $\quad N_V = \frac{1}{4}\left(\frac{2m_p^* kT}{\pi\hbar^2}\right)^{3/2}$

$n_0 = N_C e^{(E_F-E_C)/kT} = n_i e^{(E_F-E_i)/kT}$ $\quad p_0 = N_V e^{(E_V-E_F)/kT} = n_i e^{(E_F-E_i)/kT}$

Space charge neutrality: $p - n + N_D^+ - N_A^- = 0$

Law of Mass Action: $n_0 p_0 = n_i^2$

Non-equilibrium carriers: $n = N_C e^{(F_N-E_C)/kT}$ $\quad p = N_V e^{(E_V-F_P)/kT}$ $\quad np = n_i^2 e^{(F_N-F_P)/kT}$

Conductivity/resistivity: $\sigma = \sigma_n + \sigma_n = q(n\mu_n + p\mu_p) = 1/\rho$

Drift-diffusion current equations:

$$J_n = nq\mu_n \mathcal{E}_x + qD_n\frac{dn}{dx} = n\mu_n\frac{dF_n}{dx} \qquad \frac{D_n}{\mu_n} = \frac{kT}{q}$$

$$J_p = pq\mu_p \mathcal{E}_x - qD_p\frac{dp}{dx} = p\mu_p\frac{dF_p}{dx} \qquad \frac{D_p}{\mu_p} = \frac{kT}{q}$$

Carrier conservation equations:

$$\frac{\partial n}{\partial t} = +\nabla\cdot\left(\frac{J_n}{q}\right) + G_n - R_n$$

$$\frac{\partial p}{\partial t} = -\nabla\cdot\left(\frac{J_p}{q}\right) + G_p - R_p$$

Poisson's equation: $\nabla\cdot(\epsilon\mathcal{E}) = \rho$

SRH carrier recombination: $R = \Delta n/\tau_n$ or $R = \Delta p/\tau_p$

Minority carrier diffusion equation: $\frac{\partial\Delta n}{\partial t} = D_n\frac{\partial^2\Delta n}{\partial x^2} - \frac{\Delta n}{\tau_n} + G_L$ $\quad L_{D,n} = \sqrt{D_n\tau_n}$

PN homojunction electrostatics: $V_{bi} = \frac{kT}{q}\ln\left(\frac{N_D N_A}{n_i^2}\right)$ $\quad \frac{d\mathcal{E}}{dx} = \frac{\rho(x)}{K_s\epsilon_o}$

$$W = \sqrt{\frac{2K_s\epsilon_o V_{bi}}{q}\left(\frac{N_A+N_D}{N_A N_D}\right)} \qquad x_n = \left(\frac{N_A}{N_A+N_D}\right)W \qquad x_p = \left(\frac{N_D}{N_A+N_D}\right)W \qquad \mathcal{E}(0) = \sqrt{\frac{2qV_{bi}}{K_s\epsilon_o}\left(\frac{N_A N_D}{N_A+N_D}\right)}$$

PN diode current: $\Delta n(0) = \frac{n_i^2}{N_A}\left(e^{qV_A/kT} - 1\right)$ $\Delta p(0) = \frac{n_i^2}{N_D}\left(e^{qV_A/kT} - 1\right)$

$J_D = J_o\left(e^{qV_A/kT} - 1\right)$ $J_o = q\left(\frac{D_n}{L_n}\frac{n_i^2}{N_A} + \frac{D_p}{L_p}\frac{n_i^2}{N_D}\right)$ *(long)* $J_o = q\left(\frac{D_n}{W_p}\frac{n_i^2}{N_A} + \frac{D_p}{W_n}\frac{n_i^2}{N_D}\right)$ (*short*)

Non-ideal diodes: $I = I_o\left(e^{q(V_A - IR_S)/kT} - 1\right)$ $J_{gen} = -q\frac{n_i}{2\tau_o}W$

Photovoltaics: $V_{oc} = \frac{nkT}{q}\ln\left(\frac{J_{sc}}{J_o}\right)$ $J_{PV} = J_o\left(e^{qV_A/kT} - 1\right) - J_{sc}$

Small signal model: $G_d = \frac{I_D + I_o}{kT/q}$ $C_J(V_R) = \frac{K_s\epsilon_o A}{\sqrt{\frac{2K_s\epsilon_o V_{bi}}{qN_A}}} = A\sqrt{\frac{qK_s\epsilon_o N_A}{2V_{bi}}}$ $C_D = G_d\tau_n$

MS diode properties: $qV_{bi} = |\Phi_M - \Phi_S|$ $\Phi_{BP} = \chi + E_G - \Phi_M$ $\Phi_{BN} = \Phi_M - \chi$

$J_D = J_o\left(e^{qV_A/kT} - 1\right)$ $J_o = A^*T^2e^{-\Phi_B/kT}$ $A^* = \frac{4\pi qm^*k_B^2}{h^3} = 120\frac{m^*}{m_o}\ \frac{\mathrm{A}}{\mathrm{cm^2\cdot K^2}}$

MOS capacitors: $W = \sqrt{\frac{2K_s\epsilon_o\phi_s}{qN_A}}\ \mathrm{cm}$ $\mathcal{E}_s = \sqrt{\frac{2qN_A\phi_s}{K_s\epsilon_o}}\ \frac{\mathrm{V}}{\mathrm{cm}}$

$$Q_B = -qN_AW(\phi_s) = -\sqrt{2qK_s\epsilon_oN_A\phi_s}\ \frac{\mathrm{C}}{\mathrm{cm^2}}$$

$$V_G = V_{FB} + \phi_s + \Delta\phi_{ox} = V_{FB} + \phi_s - \frac{Q_s(\phi_s)}{C_{ox}}$$

$$C_{ox} = K_o\epsilon_o/x_o \qquad V_{FB} = \Phi_{ms}/q - Q_F/C_{ox}$$

$C = C_{ox}/\left[1 + \frac{K_oW(\phi_s)}{K_sx_o}\right]$ $V_T = -Q_B(2\phi_F)/C_{ox} + 2\phi_F$ $Q_n = -C_{ox}(V_G - V_T)$

MOSFETs: $I_D = -WQ_n(y=0)\langle v_y(y=0)\rangle$

$$I_D = \frac{W}{L}\mu_nC_{ox}(V_{GS} - V_T)V_{DS} \qquad I_D = WC_{ox}v_{sat}(V_{GS} - V_T)$$

Square Law (for $V_{GS} \geq V_T$): $I_D = \begin{cases}\frac{W}{L}\mu_nC_{ox}[(V_{GS} - V_T)V_{DS} - V_{DS}^2/2], & 0 \leq V_{DS} \leq V_{GS} - V_T \\ \frac{W}{2L}\mu_nC_{ox}(V_{GS} - V_T)^2, & V_{DS} \geq V_{GS} - V_T\end{cases}$